In section 7.3 you will learn to:

- Use back substitution to solve linear systems in row-echelon form.
- Use Gaussian elimination to solve systems of linear equations,
- Solve non-square systems of linear equations.
- Model and solve real-life problems by setting up systems of linear equations in three or more variables.

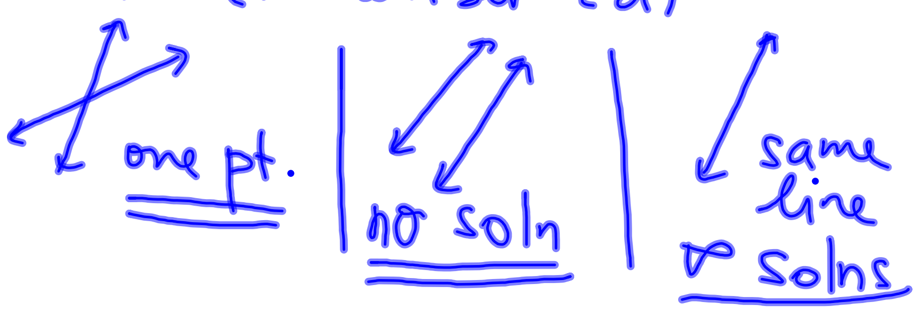
Multivariable linear systems

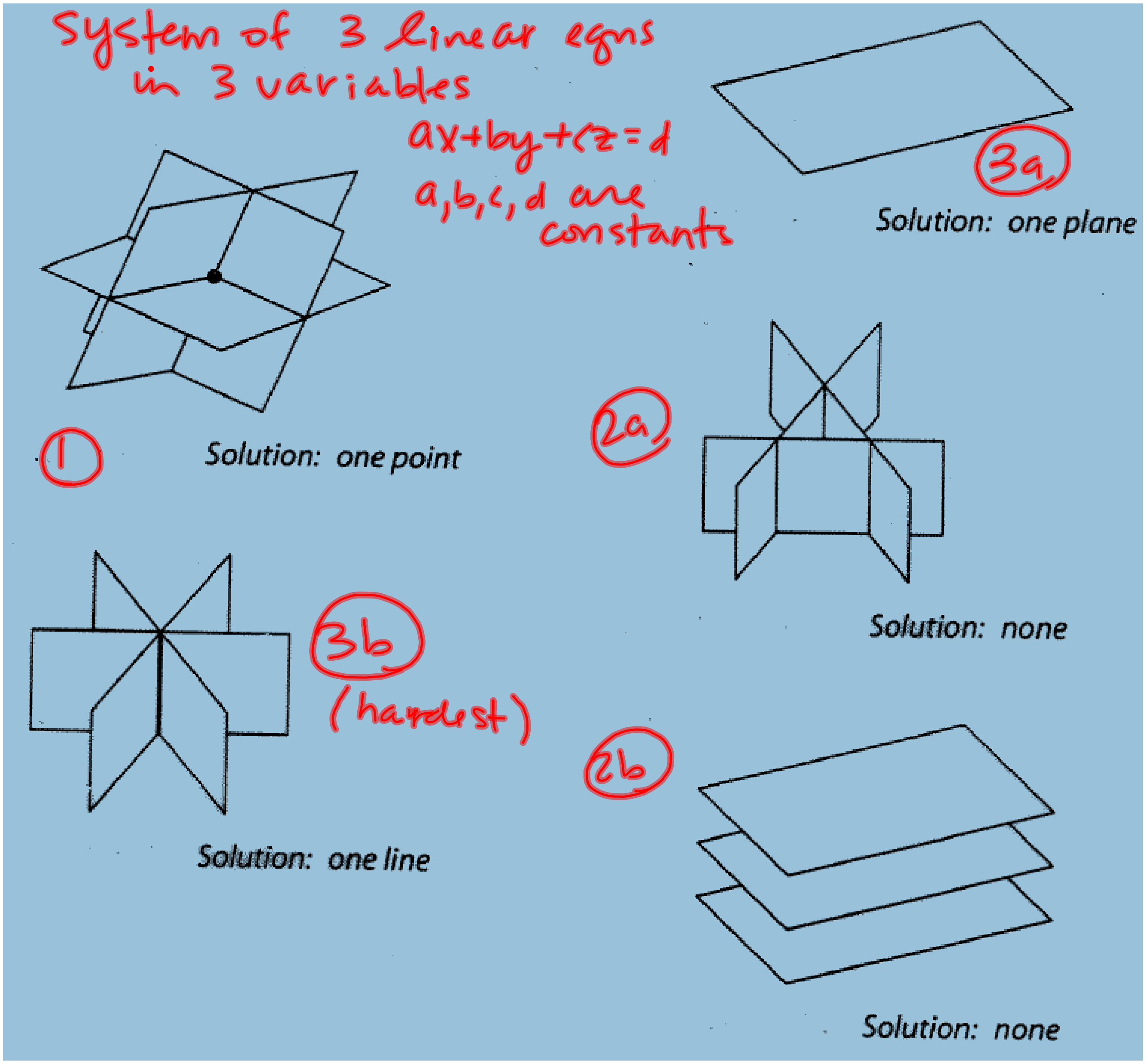
An equation with three variables represents what? a shape in 3-d

(a linear eqn in 3d)

What are the possibilities for solutions to three equations in three unknowns?

Note: (Remember 2-d)




System of 3 linear eqns
in 3 variables
$ax+by+cz=d$
a,b,c,d are
constants
3a
Solution: one plane
1
Solution: one point
2a
Solution: none
3b
(hardest)
Solution: one line
2b
Solution: none

<u>Method of Gaussian Elimination</u> -- You may: (solving a system of <u>linear</u> eqns)

1. Exchange two rows.
2. Multiply a row by a nonzero constant.
3. Temporarily multiply a row by a nonzero constant, add it to another row and replace one of the rows with the result. (elimination step)

(Note: row = an entire eqn)

Example 1

$-x + y - z = -4$

(1) $x - y + z = 4$ (-1)

(2) $x + 3y - 2z = -3$

(3) $3x + 2y + 2z = 6$

You may:
- Exchange rows
- Multiply row
- Multiply row, add (elimination)

$-3x + 3y - 3z = -12$

(-3) $x - y + z = 4$

$4y - 3z = -7$

$3x + 2y + 2z = 6$

$x - y + z = 4$

$4y - 3z = -7$

(-3) $5y - z = -6$

$-15y + 3z = 18$

(1) $x - y + z = 4$

(2) $-11y = 11 \longrightarrow y = -1$

(3) $5y - z = -6$

(3) $5(-1) - z = -6$

$+5 \qquad +5$

$-z = -1$

$z = 1$

(1) $x - (-1) + 1 = 4$

$x + 2 = 4 \Rightarrow x = 2$

pt of intersectn
$(2, -1, 1)$

Example 2 Solve

$$-3x+6y-3z=-12$$

(-3)

$$\begin{aligned} x - 2y + z &= 4 \\ 3x - 6y + 3z &= 7 \\ 2x + y + 4z &= 2 \end{aligned}$$

$x-2y+z=4$

$\boxed{0=-5}$

$2x+y+4z=2$

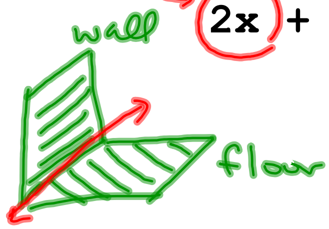
$0 \neq -5$

$\Rightarrow$ no solution

$-2x + 4y + 2z = 10$

$(-2)\quad x - 2y - z = -5$

$2x + y + z = 5$



(Note: this is underdetermined system) (in 3d)

(fewer eqns than variables)

① $x - 2y - z = -5$

② $5y + 3z = 15$

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② $3z = -5y + 15$

$$\boxed{z = \frac{-5}{3}y + 5}$$

① $x - 2y - \left(\frac{-5}{3}y + 5\right) = -5$

$x - 2y + \frac{5}{3}y - 5 = -5$

$\qquad\qquad\qquad +5 \quad +5$

$x - \frac{1}{3}y = 0$

$$\boxed{x = \frac{1}{3}y}$$

line of intersection

A

$x = \frac{1}{3}y$

$z = \frac{-5}{3}y + 5$

$y = y$

pts on line

x	y	z
0	0	5
$\frac{1}{3}$	1	10/3
2/3	2	5/3

re-parameterize

B

$y = 3t$

$x = \frac{1}{3}(3t) = t$

$z = \frac{-5}{3}(3t) + 5 = -5t + 5$

Example 4

Find the equation of the parabola $y = ax^2 + bx + c$ that passes through the points

$x_1\ y_1 \quad x_2\ y_2 \quad x_3\ y_3$

(0,3), (1,4) and (2,3).

① $3 = c$

② $4 = a + b + c$

③ $3 = 4a + 2b + c$

so

$$\left.\begin{aligned} 4 &= a + b + 3 \\ 3 &= 4a + 2b + 3 \end{aligned}\right\} \quad \begin{aligned} a + b &= 1 \\ 4a + 2b &= 0 \end{aligned}$$

$$\left.\begin{aligned} -2a - 2b &= -2 \\ 4a + 2b &= 0 \end{aligned}\right\} \quad \begin{aligned} 2a &= -2 \\ a &= -1 \\ b &= 2 \end{aligned}$$

EQ: $y = -x^2 + 2x + 3$