

Two variable linear systems

To solve linear equations, we can use a third method, Elimination.

Example:

$$\begin{array}{r}
 \bullet 2(3x - 2y = 7) \Rightarrow 6x - 4y = 14 \\
 \bullet 8x + 4y = 0 \\
 \hline
 \rightarrow 14x = 14 \\
 x = 1
 \end{array}$$

$$\begin{array}{l}
 \Rightarrow 3(1) - 2y = 7 \\
 3 - 2y = 7 \\
 -2y = 4 \\
 y = -2
 \end{array}
 \left. \vphantom{\begin{array}{l} 3(1) - 2y = 7 \\ 3 - 2y = 7 \\ -2y = 4 \\ y = -2 \end{array}} \right\} (1, -2)$$

algebraically
 ① elimination
 ② substitution

Elimination

- Multiply the equations by numbers to make the coefficients of one of the variables add to 0.
- Add the two equations together.
- Solve for x or y.
- Back substitute to get the value of the other variable.

Example 2:

$$3y = 4x - 5 \quad \textcircled{1} \quad -2(-4x + 3y = -5)$$

$$-8x + 6y = 1 \quad \textcircled{2} \quad -8x + 6y = 1$$

$$\begin{array}{r} 8x - 6y = 10 \\ + -8x + 6y = 1 \\ \hline 0 = 11 \quad \text{false} \end{array}$$

\Rightarrow N.S.

Example 3:

$$2x - y = 9 \quad \textcircled{1} \quad \text{eliminate } y$$

$$-10x + 5y = -45 \quad \textcircled{2}$$

$$\begin{array}{r} 5(2x - y = 9) \Leftrightarrow 10x - 5y = 45 \\ + -10x + 5y = -45 \\ \hline 0 = 0 \end{array}$$

true

\Rightarrow same line \leftrightarrow
infinitely many solution

Example 4:

Two planes start from LAX and fly in opposite directions. The second plane starts 1/2 hour after the first plane, but its speed is 80 km/h faster. Find the airspeed of each plane if 2 hours after the first plane departs the planes are 3200 km apart.

$(v+80) \frac{\text{km}}{\text{hr}}$
 $v \frac{\text{km}}{\text{hr}}$
 $t \text{ hrs}$
 $(t - \frac{1}{2}) \text{ hrs}$
 $t = 2 \text{ hrs}$
 3200 km
 want to know $v = ?$
 $v + 80 = ?$

$d = rt$
 dist. rate time
 speed velocity

① $d_1 = v(2)$
 ② $d_2 = (v+80)(\frac{3}{2})$

$V = 880 \frac{\text{km}}{\text{hr}}$
 ①
 $v + 80 = 880$
 $+ 80$
 ② $= 960 \frac{\text{km}}{\text{hr}}$

$2v + (v+80)\frac{3}{2} = 3200$
 $2v + \frac{3}{2}v + 120 = 3200$
 $\frac{2}{7} \cdot \frac{7}{2}v = 3080 \cdot \frac{2}{7} \Rightarrow v = 880 \frac{\text{km}}{\text{hr}}$

Example 5

A total of \$32,000 is invested in two municipal bonds that pay 5.75% and 6.25% simple interest. The investor wants an annual interest income of \$1900 from the investments. What amount should be invested in the 5.75% bond? (assume 1 year)

$x = \text{amt } \$ \text{ in bond 1}$

$y = \text{amt } \$ \text{ in bond 2}$

	rate	amt \$	interest
B1	5.75%	x	$0.0575x$
B2	6.25%	y	$0.0625y$
total		32000	1900
		(1)	(2)

① $x + y = 32000$

② $0.0575x + 0.0625y = 1900$

① $y = 32000 - x$

② $0.0575x + 0.0625(32000 - x) = 1900$

