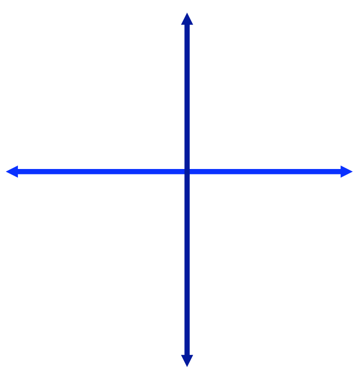
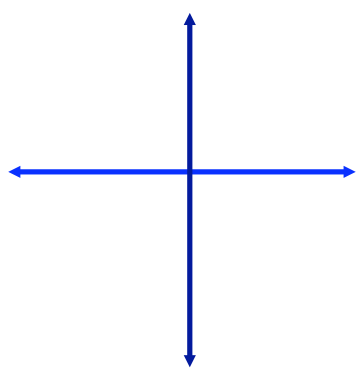
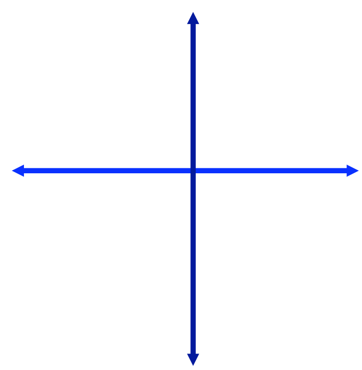
Linear and non-linear systems of equations

A system of equations is simply a set of two or more equations in two or more variables that we solve simultaneously.

A system of linear equations in two variables has three possible outcomes:

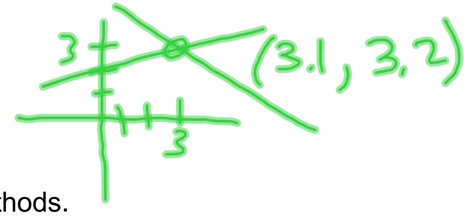






You already know two strategies to solve two equations in two unknowns.

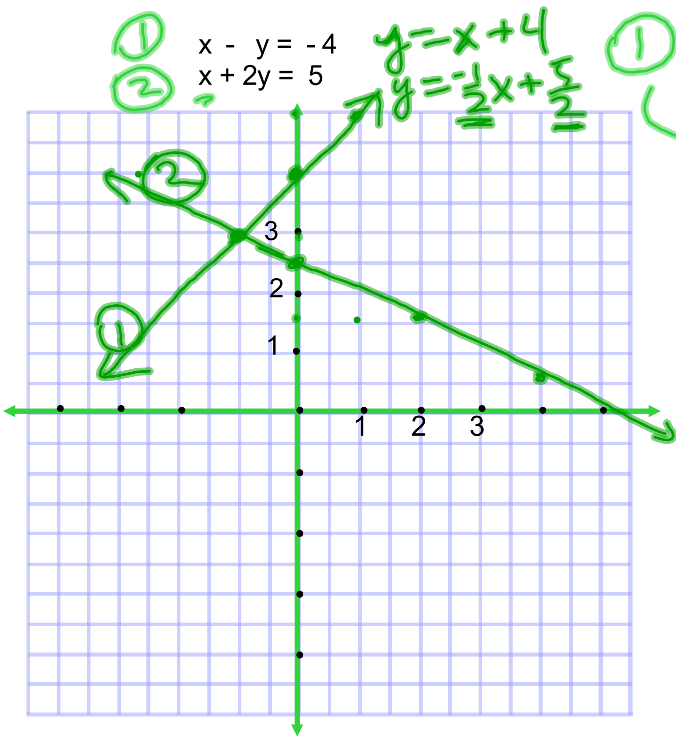
1. Graphically - Not reliable, but useful.
2. Substitution - A method that will always work.



1) Solve this set of linear equations using both of the methods.

① $x - y = -4$ $\quad y = x + 4$

② $x + 2y = 5$ $\quad y = -\frac{1}{2}x + \frac{5}{2}$



① $x = y - 4$

② $y - 4 + 2y = 5$

$3y - 4 = 5$

$3y = 9$

$y = 3$

① $x = 3 - 4 = -1$

$(-1, 3)$

2) Solve using substitution.

linear ① $3x + y = 2$

nonlinear ② $x^3-2+y=0$

① $y = -3x+2$

② $x^3 - 2 + -3x + 2 = 0$

$x^3 - 3x = 0$

$x(x^2-3) = 0$

$x = 0$ $\quad$ $x^2 - 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

① $x=0,\ y = -3(0)+2$
$= 2$

$(0, 2)$

$x = \sqrt{3},\ y = -3\sqrt{3}+2$

$(\sqrt{3}, -3\sqrt{3}+2)$

$x = -\sqrt{3},\ y = -3(-\sqrt{3})+2 = 3\sqrt{3}+2$

$(-\sqrt{3}, 3\sqrt{3}+2)$

3) Solve by graphing.

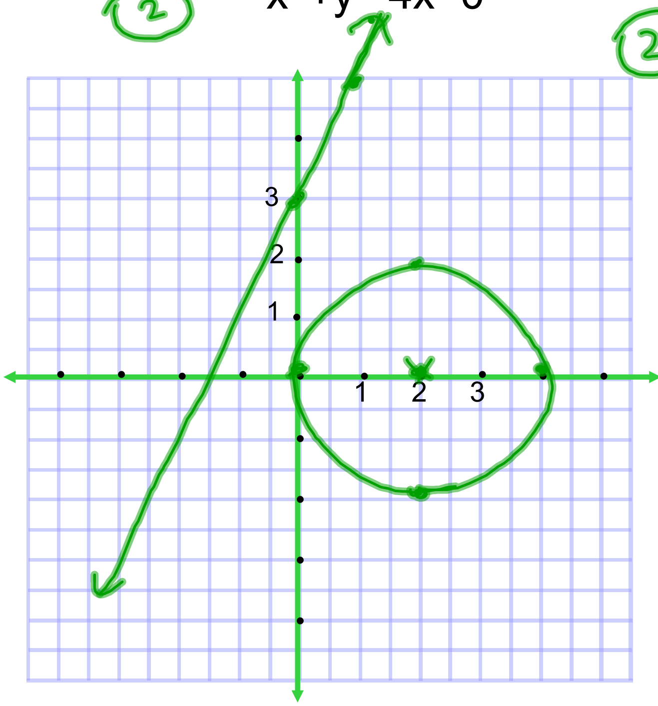
circle $\underline{(x-h)^2+(y-k)^2=r^2}$

(h,k) ctr

r = radius

① $2x-y+3=0$ $\quad y=2x+3$

② $x^2+y^2-4x=0$



② $x^2+y^2-4x=0$

$(x^2-4x)+y^2=0$

$(x^2-\underline{4}x+4)-4+y^2=0 \quad \left(\frac{-4}{2}\right)^2=4$

$(x-\underline{2})^2+y^2=4$

circle ctr $(2,0)$

rad = 2

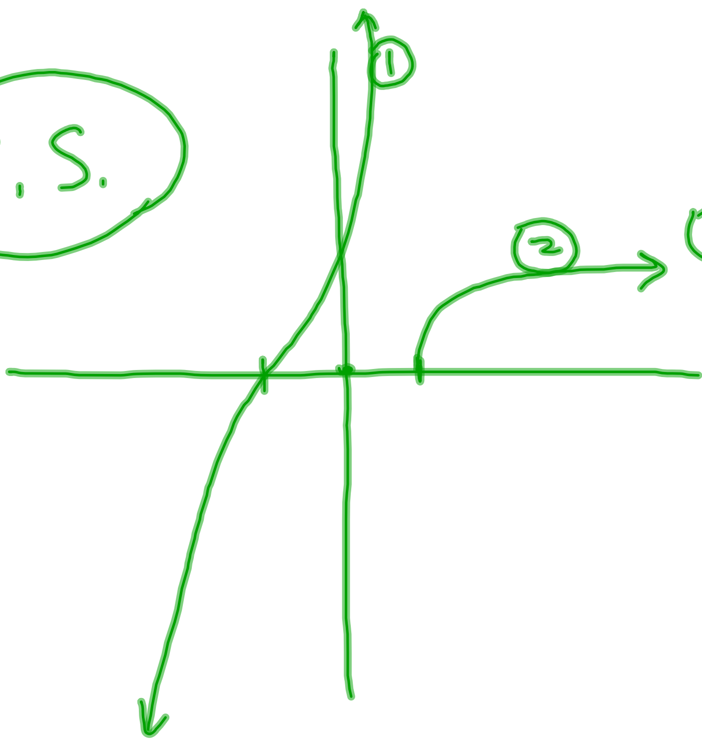
$\Rightarrow$ N.S.

4) Solve

1. $y = (x+1)^3$
2. $y = \sqrt{x-1}$

① base graph $y = x^3$
shift graph
left 1

N.S.



② base graph
$y = \sqrt{x}$

shift graph
1 unit to
right

5) Solve

① $y = x^3 - 2x^2 + x - 1$

② $y = -x^2 + 3x - 1$

$-x^2 + 3x - 1 = x^3 - 2x^2 + x - 1$

$0 = x^3 - x^2 - 2x$

$0 = x(x^2 - x - 2)$

$0 = x(x+1)(x-2)$

$x - 2x = -x$

$x = 0 \quad x + 1 = 0 \quad x - 2 = 0$

$x = -1 \quad x = 2$

$x = 0$

$y = -0 + 0 - 1 = -1$

$(0, -1)$

$x = -1$

$y = -(-1)^2 + 3(-1) - 1$

$= -1 - 3 - 1 = -5$

$(-1, -5)$

$x = 2$

$y = -2^2 + 3(2) - 1$

$= -4 + 6 - 1 = 1$

$(2, 1)$