Logarithmic Functions and Their Graphs

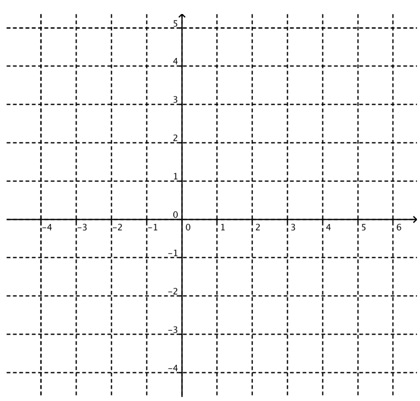
In section 3.2 you will learn to:

- Recognize, evaluate and graph logarithmic functions with whole number bases.
- Recognize, evaluate and graph natural logarithmic functions.
- Evaluate logarithms without using a calculator.
- Use logarithmic functions to model and solve real-life problems.

Review: Exponential Functions

$f(x) = 2^x$

What is the inverse of this function?



$f^{-1}(x)=?$

$f(x) = 2^x$

$g(x)=$

Asymptotes:
Domain:
Range:

Evaluating logarithmic functions:

<u>Exponential form</u> is equivalent to <u>logarithmic form</u>

$b^x=y$ $\log_b y=x$

$8^{-1}=1/8$ $\log_8(1/8)=-1$

Example 1: Notice that a logarithm is always equal to an exponent.

Determine the answer and write each one in the other form.

$10^4 =$

$\log_3(1/27) =$

$(9/100)^{-1/2} =$

$\log_2(2\sqrt{2}) =$

$\log_5(1)=$

Example 2: Find the value of $F(x)$ in each of the following if $F(x) = \log_8 x$

a) $F(8) =$

b) $F(64) =$

c) $F(8^{10}) =$

d) $F(2) =$

e) $F(0)=$

Evaluating logarithmic expressions on a calculator:

Base 10 logarithms are called common logarithms. They are written (without base) as $\log x = \log_{10} x$.

$\log 1000 =$

$\log .001 =$

$\log (1) =$

$\log 15$ is asking the question, 10 to what power will yield 15? Your calculator will tell you this is about 1.176.

Base *e* logarithms are called natural logarithms. They are written as $\log_e x = \ln x$ (the natural log of x.) You may want to write them as an exponential expression to evaluate these.

$\ln (e^3) =$

$\ln (1/e) =$

$\ln (e^0) =$

$\ln(100)$ is asking what power of e will yield 100. Your calculator will tell you this is about 4.605

Example 3:
Use a calculator to evaluate these logs to four significant digits:

$\log 72 =$

$\log_{10} 0.000387 =$

$\ln 218 =$

$\log_e 10 =$

Do these without a calculator, then check with a calculator.

$\log 100 =$

$\ln e^5 =$

$\log 0 =$

$\ln 1 =$

Four initial properties of logarithms:

1. $\log_a 1 = 0$

2. $\log_a a = 1$

3. $\log_a a^x = x$ Inverse property

4. If $\log_a x = \log_a y$, then $x = y$ One-to-one property

Example 4: Evaluate these:

$\log_5 1 =$

$\log_6 6 =$

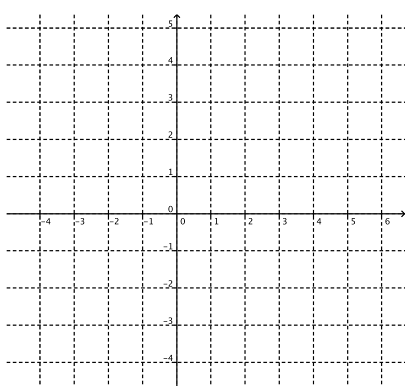
$\log_2 2^{1.7} =$

$\ln e^{12} =$

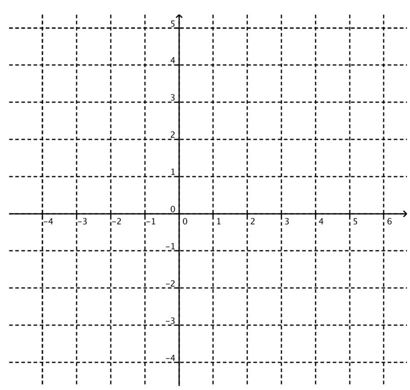
Finally, suppose $\log_3 x = \log_3 100$. What can you conclude?

Graphs of logarithmic functions:

$f(x) = \log_2 x$



$g(x) = \log_{(1/4)} x$

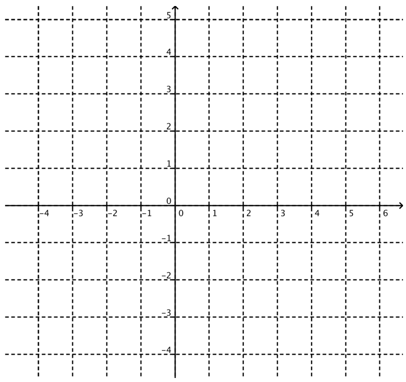


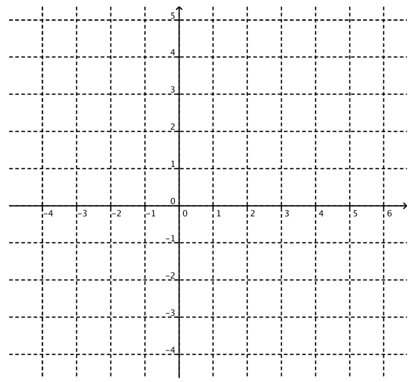
Asymptotes:
Domain:
Range:

Transformations of logarithmic functions:

$f(x) = 2 + \ln x$

$g(t) = \ln(t-4)$

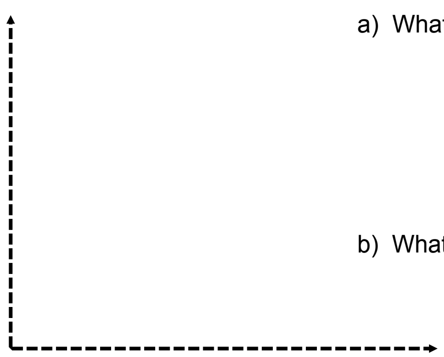




Example 5: An application of logarithms:

Scientific studies show that in many cases, human memory of certain information seems to deteriorate over time and can be modeled by decreasing logarithmic functions. For example, suppose a student learns to speak French so well that on an initial exam she scores 90. Over time and without practice her score on comparable exams decreases.

$s(t) = 90 - 12 \log_3 (t+1)$



a) What is her score on the initial exam?

b) What is her score after 2 days? After 8 days?