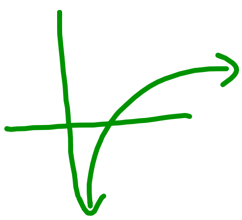
Logarithmic Functions and Their Graphs

In section 3.2 you will learn to:

- Recognize, evaluate and graph logarithmic functions with whole number bases.
- Recognize, evaluate and graph natural logarithmic functions.
- Evaluate logarithms without using a calculator.
- Use logarithmic functions to model and solve real-life problems.

$\log_8 x = y$



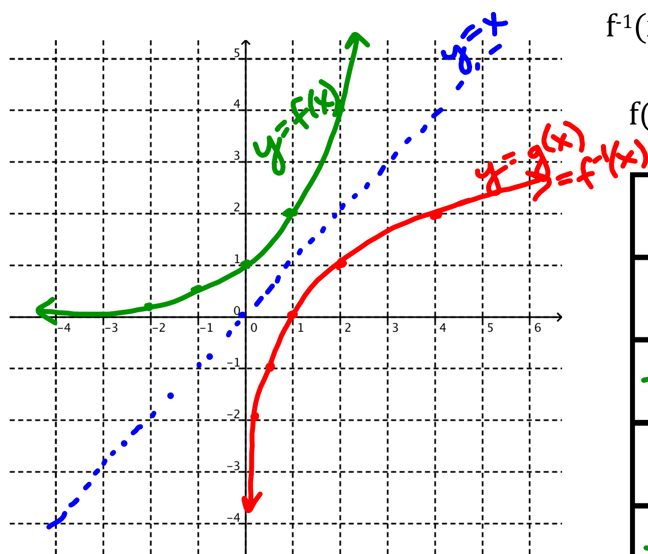
Review: Exponential Functions

$f(x) = 2^x$

What is the inverse of this function?

$f^{-1}(x) = \log_2 x$

$f^{-1}(x) = ?$



$f^{-1}(x) =$

$f(x) = 2^x$

0	1
1	2
-1	$\frac{1}{2} = 2^{-1}$
2	4
-2	$\frac{1}{4}$

$g(x) =$

1	0
2	1
$\frac{1}{2}$	-1
4	2
$\frac{1}{4}$	-2

$y = f(x) = 2^x$

HA: $y = 0$

no VA

domain: $x \in \mathbb{R}$

$(-\infty, \infty)$

range: $y > 0$

$(0, \infty)$

$y = f^{-1}(x) = g(x)$

Asymptotes: VA: $x = 0$

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Evaluating logarithmic functions:

<u>Exponential form</u> is equivalent to <u>logarithmic form</u>

$\boxed{b^x = y \quad \Longleftrightarrow \quad \log_b y = x}$

prompt: $\log_b y = x$ "b to what power gives y?"

ex $8^{-1} = 1/8 \quad \Longleftrightarrow \quad \log_8(1/8) = -1$

$b = 8,\ x = -1,\ y = 1/8$

$\Longleftrightarrow$ analogy

ex $15 \div 3 = ?$

$3 \cdot ? = 15$

Example 1: Notice that a logarithm is always equal to an exponent.

Determine the answer and write each one in the other form.

$10^4 = 10{,}000 \quad \Longleftrightarrow \quad \log_{10} 10000 = 4$

$\log_3(1/27) = ? \quad \Longleftrightarrow \quad 3^{?} = \frac{1}{27} \qquad ? = -3$

$= -3$

$(9/100)^{-1/2} = \left(\frac{9}{100}\right)^{-1/2} = \left(\frac{100}{9}\right)^{1/2} = \sqrt{\frac{100}{9}} = \frac{10}{3}$

$\Longleftrightarrow \quad \log_{9/100}\left(\frac{10}{3}\right) = -\frac{1}{2}$

$\log_2(2\sqrt{2}\ ) =$

$\log_2(2 \cdot 2^{1/2})$

$= \log_2(2^{3/2}) = \frac{3}{2} \quad \Longleftrightarrow \quad 2^{3/2} = 2^{3/2}$

$\log_5(1) =$

$? = 0$

$\Longleftrightarrow \quad 5^{?} = 1 \qquad ? = 0 \qquad 5^0 = 1$

Example 2: Find the value of $F(x)$ in each of the following if $F(x) = \log_8 x$

a) $F(8) = \log_8 8 = 1$

b) $F(64) = \log_8(64) = 2$ $\quad (8^2 = 64)$

c) $F(8^{10}) = \log_8(8^{10}) = 10$

d) $F(2) = \log_8(2) = \log_8(\sqrt[3]{8}) = \log_8(8^{1/3}) = \frac{1}{3}$

e) $F(0) = \log_8(0)$ $\quad \star\ 8^{?} = 0$

undefined $\quad$ no answer

Evaluating logarithmic expressions on a calculator:

Base 10 logarithms are called common logarithms. They are written (without base) as $\log x = \log_{10} x$.

$\log 1000 =$ $\log_{10} 10^3 = 3$

$\log .001 =$ $\log \frac{1}{1000} = \log 10^{-3} = -3$

$\log (1) =$ 0

$\log 15$ is asking the question, 10 to what power will yield 15?
Your calculator will tell you this is about 1.176.

Base *e* logarithms are called natural logarithms. They are written as $\log_e x = \ln x$ (the natural log of x.) You may want to write them as an exponential expression to evaluate these.

$\ln (e^3) =$ 3

$\ln (1/e) =$ $\ln (e^{-1}) = -1$

$\ln (e^0) =$ $\ln (1) = 0$

$\ln(100)$ is asking what power of e will yield 100.
Your calculator will tell you this is about 4.605

Example 3:
Use a calculator to evaluate these logs to four significant digits:

$\log 72 \approx 1.857$

$\log_{10} 0.000387 \approx -3.412$

$\ln 218 \approx 5.384$

$\log_e 10 \approx 2.303$

Do these without a calculator, then check with a calculator.

$\log 100 = 2$

$\ln e^5 = 5$

$\log 0 =$? $\Leftrightarrow 10^{?} = 0$

undefined

$\ln 1 = 0$

Four initial properties of logarithms:

1. $\log_a 1 = 0$ (because $a^0 = 1$, $a \neq 0$)

2. $\log_a a = 1$ (because $a^1 = a$)

3. $\log_a a^x = x$ Inverse property (because $a^x = a^x$)

(log base a "undoes" exponential with base a)

4. If $\log_a x = \log_a y$, then $x = y$ One-to-one property

Example 4: Evaluate these:

$\log_5 1 =$ 0

$\log_6 6 =$ 1

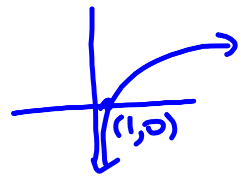
$\log_2 2^{1.7} =$ 1.7

$\ln e^{12} =$ 12

Finally, suppose $\log_3 x = \log_3 100$. What can you conclude?

$x = 100$

general shape $y = \log_a x$



Graphs of logarithmic functions:

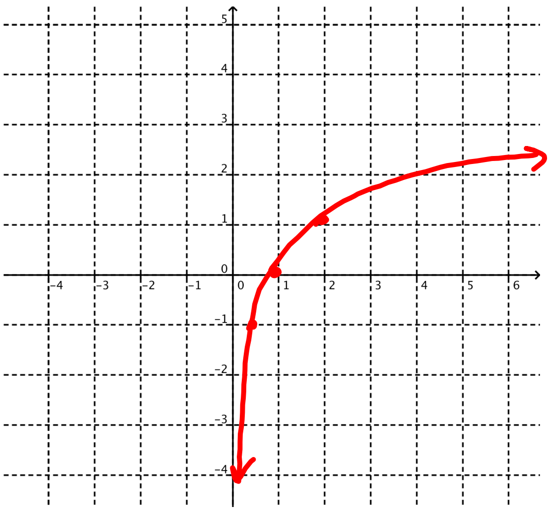
$f(x) = \log_2 x$

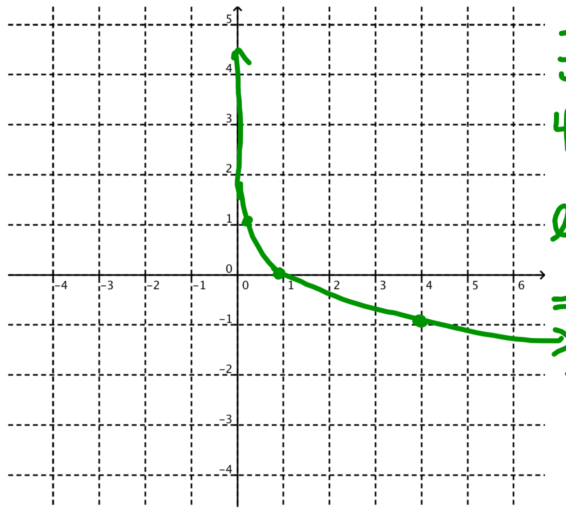
x	y
2	1
$\frac{1}{2}$	-1

$g(x) = \log_{(1/4)} x$

x	y
$\frac{1}{4}$	1
4	-1

$\log_{\frac{1}{4}} 4 = \log_{\frac{1}{4}} \left(\frac{1}{4}\right)^{-1} = -1$





Asymptotes: VA: $x = 0$

Domain: $x > 0$ (or $(0, \infty)$)

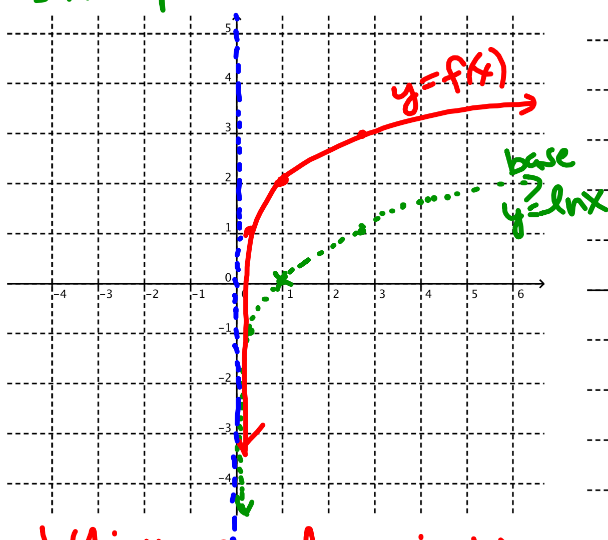
Range: $y \in \mathbb{R}$ (or $(-\infty, \infty)$)

Transformations of logarithmic functions:

$f(x) = 2 + \ln x$

shift up 2

base fn: $y = \ln x$

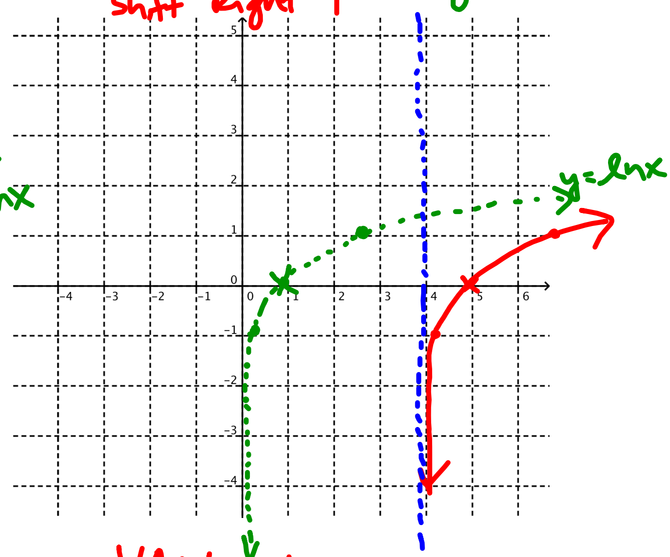


VA: $x = 0$ domain: $x > 0$

$g(t) = \ln(t-4)$

shift right 4

base fn: $y = \ln t$



VA: $t = 4$

domain: $t > 4$

$g(t) = \ln(t-4)$

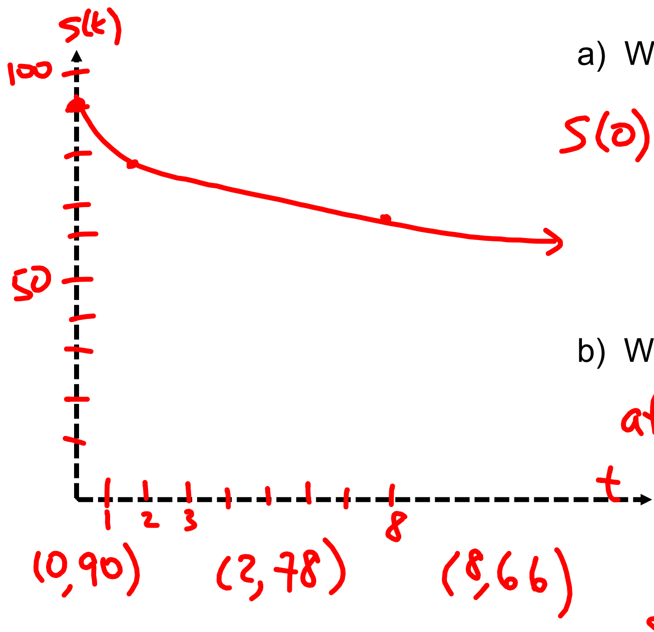
$t - 4 > 0$

$t > 4$

Example 5: An application of logarithms:

Scientific studies show that in many cases, human memory of certain information seems to deteriorate over time and can be modeled by decreasing logarithmic functions. For example, suppose a student learns to speak French so well that on an initial exam she scores 90. Over time and without practice her score on comparable exams decreases.

$s(t) = 90 - 12\log_3(t+1)$



a) What is her score on the initial exam?

$s(0) = 90 - 12\log_3(0+1)$

$= 90 - 12(0) = 90$

b) What is her score after 2 days? After 8 days?

after 2 days, $t=2$

$s(2) = 90 - 12\log_3(3)$

$= 90 - 12(1) = 78$

$s(8) = 90 - 12\log_3(9)$

$= 90 - 12(2) = 66$