2.6 RATIONAL FUNCTIONS

In this section you will learn how to:

- Find the domain of rational functions
- Find horizontal, vertical and slant asymptotes of rational functions
- Analyze and sketch the graph of a rational function
- Use rational functions to model and solve real-life problems

A rational function is $Q(x) = \dfrac{N(x)}{D(x)}$

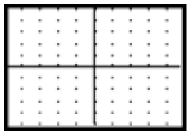
where N(x) is a polynomial function of any degree and D(x) must be a polynomial of degree 1 or greater.

The Numerator determines the roots and the Denominator determines the vertical asymptotes.

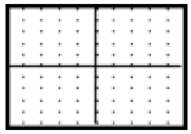
Vertical Asymptotes are caused by zero values in the denominator.

A look at $y = \frac{1}{x}$ and some transformations

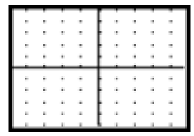
$y = \frac{1}{x}$



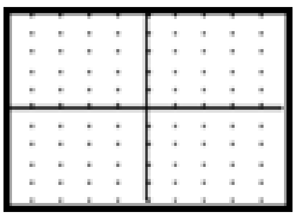
$y = \frac{1}{x+2}$



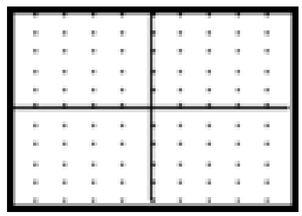
$y = \frac{1}{x-3} + 1$



$y = \frac{2x - 3}{(x-1)(x+2)}$

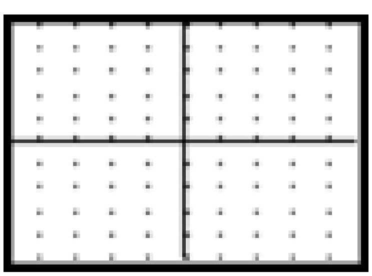


$y = \frac{2x(x+2)}{(x-1)(x+3)}$

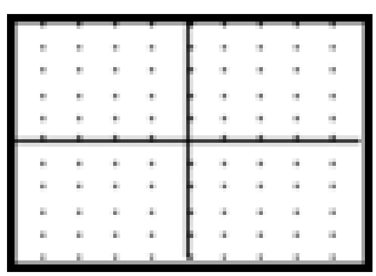


The numerator tell us the roots (x-intercepts) of the function.
To find the y-intercept, let x=0.

$y = \frac{2x - 3}{(x-1)(x+2)}$



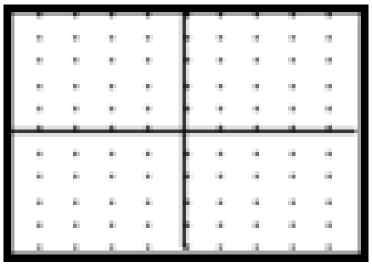
$y = \frac{2x(x+2)}{(x-1)(x+3)}$

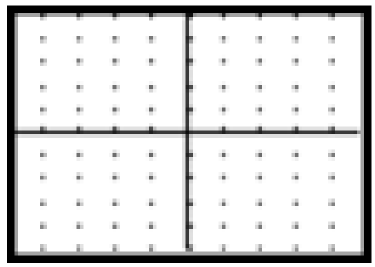


End behavior is determined by the quotient of the leading terms.

$$y = \frac{2x - 3}{(x-1)(x+2)}$$

$$y = \frac{2x(x+2)}{(x-1)(x+3)}$$

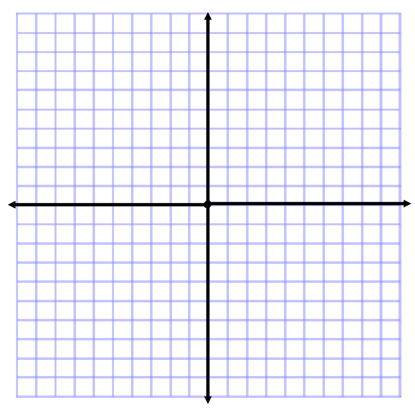




How do we know what the function looks like? We need to make a sign line:

What happens if there is a common factor in the numerator and the denominator?

$$y = \frac{x^3 - 3x^2 + 2x}{x^2 - 1}$$



hole in the function:

Roots:

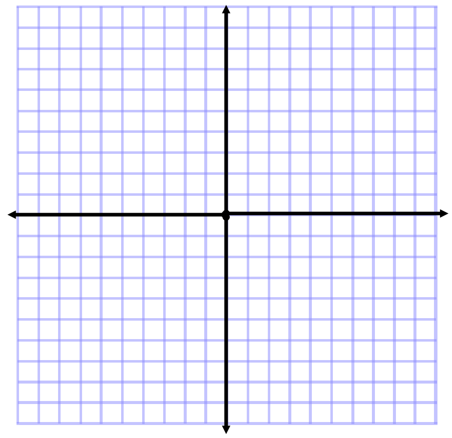
y-int

V. Asymptotes

End Behavior

IN SUMMARY:

Factor numerator and denominator



Reduce any common factors and note the hole(s) in the function.

Determine x and y intercepts.

Determine end behavior.

Determine vertical asymptotes.

Make a sign line: