

2.5 Finding the zeros of polynomial functions

We will learn how to:

- Determine the number of zeros of polynomial functions
- Find rational zeros of polynomial functions
- Find conjugate pairs of complex zeros
- Find zeros of polynomials by factoring
- Write a polynomial function given the roots.

$$P(x) = a_1x^n + a_2x^{n-1} + \dots + a_{n-1}x + a_n$$

$$\text{Factored form } a(x - r_1)(x - r_2) \dots (x - r_n)$$

The roots are r_1, r_2, \dots, r_n

Rational Root Theorem:

$$P(x) = (2x-5)(3x+2)(x-1)$$

Has roots:

$$P(x) = 6x^3 - 17x^2 + x + 10$$

set of p_s :

set of q_s :

$P(x)$ must have integer coefficients.

If $P(x)$ has any rational roots, they will be of the form: p/q where p and q have no common factors other than 1 and where p is a factor of the constant term and q is a factor of the leading coefficient.

This allows us to attempt to break higher degree polynomials down into their factored form and determine the roots of a polynomial.

Example 1: Factor completely and determine the roots of this polynomial.

$$P(x) = x^3 + 3x^2 + x - 2$$

- 1) set of p_s
- 2) set of q_s
- 3) possible roots of $P(x)$
- 4) Test each possible root using synthetic division:

Example 2: Find the roots and write in factored form:

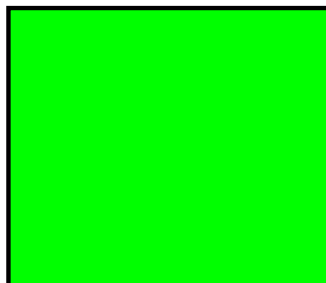
$$y = 9x^4 - 3x^3 + x^2 - 8x + 4$$



Example 3:

Determine the roots and write in factored form:

$$y = x^3 - 7x - 6$$

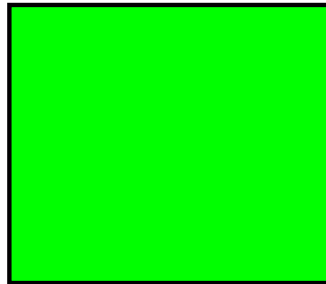


Notice: as soon as you can get the factored form down to a quadratic, use the quadratic formula to find the other two roots. They may be complex.

Complex roots will come in conjugate pairs. If $a + bi$ is a root, then $a - bi$ will be a root if the polynomial has integer coefficients.

Example 4: Factor and determine the roots:

$$y = x^3 + 4x^2 + 14x + 20$$



Example 5:

Write a polynomial function with real coefficients of degree 4 which has these roots:

$$2i, -3, 1$$

