2.5 Finding the zeros of polynomial functions

We will learn how to:

- Determine the number of zeros of polynomial functions
- Find rational zeros of polynomial functions
- Find conjugate pairs of complex zeros
- Find zeros of polynomials by factoring
- Write a polynomial function given the roots.

$$P(x) = a_1x^n + a_2x^{n-1} + . . . +a_{n-1}x + a_n$$

Factored form $a(x - r_1)(x - r_2) \ldots (x - r_n)$

The roots are $r_1, r_2, ..., r_n$

Rational Root Theorem:

$P(x) = (2x-5)(3x+2)(x-1)$ Has roots:

$P(x) = 6x^3 - 17x^2 + x + 10$

set of p_s :

set of q_s :

P(x) must have integer coefficients.

If P(x) has any rational roots, they will be of the form: p/q where p and q have no common factors other than 1 and where p is a factor of the constant term and q is a factor of the leading coefficient.

This allows us to attempt to break higher degree polynomials down into their factored form and determine the roots of a polynomial.

Example 1: Factor completely and determine the roots of this polynomial.

$P(x) = x^3 + 3x^2 + x - 2$

1) set of p_s

2) set of q_s

3) possible roots of P(x)

4) Test each possible root using synthetic division:

Example 2: Find the roots and write in factored form:

$y = 9x^4 - 3x^3 + x^2 - 8x + 4$

Example 3:

Determine the roots and write in factored form:

$y = x^3 - 7x - 6$

Notice: as soon as you can get the factored form down to a quadratic, use the quadratic formula to find the other two roots. They may be complex.
Complex roots will come in conjugate pairs. If a + bi is a root, then a - bi will be a root if the polynomial has integer coefficients.

Example 4: Factor and determine the roots:

$y = x^3 + 4x^2 + 14x + 20$

Example 5:

Write a polynomial function with real coefficients of degree 4 which has these roots:

2i, -3, 1