Composition of functions

Inverse functions

Today's objectives

- Define composition of functions
- Give examples of composing functions algebraically and by graphing
- Define inverse function
- Practice finding inverse function algebraically and by graphing

Beads and necklaces

- Few years ago I took up beading for fun. I would buy a bag of varied beads and found that I can make 14 necklaces from it.

- As my beading skills got better, I found that people liked my designs and are willing to pay for my necklaces. I started selling them at a local farmers' market for $9.50.

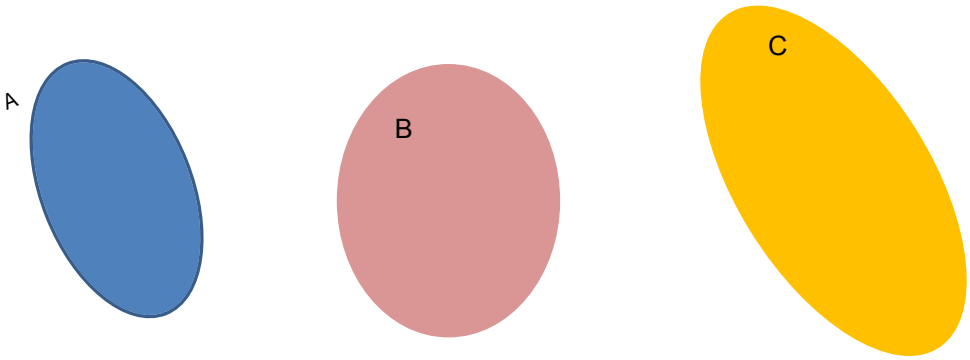
- I would like to know how much money I will make based on the number of bags of beads I buy.

Definition

- Let $f: A \rightarrow B$, $g: B \rightarrow C$ be two functions.

Composition of f and g is a function, denoted by $g \circ f$, defined by:

$$g \circ f(x) = g(f(x))$$



Find $g \circ f$ if

$f(x) = 7x - 2$

$g(x) = x^2 - 2x$

$f(x) = x^2 - 2x$

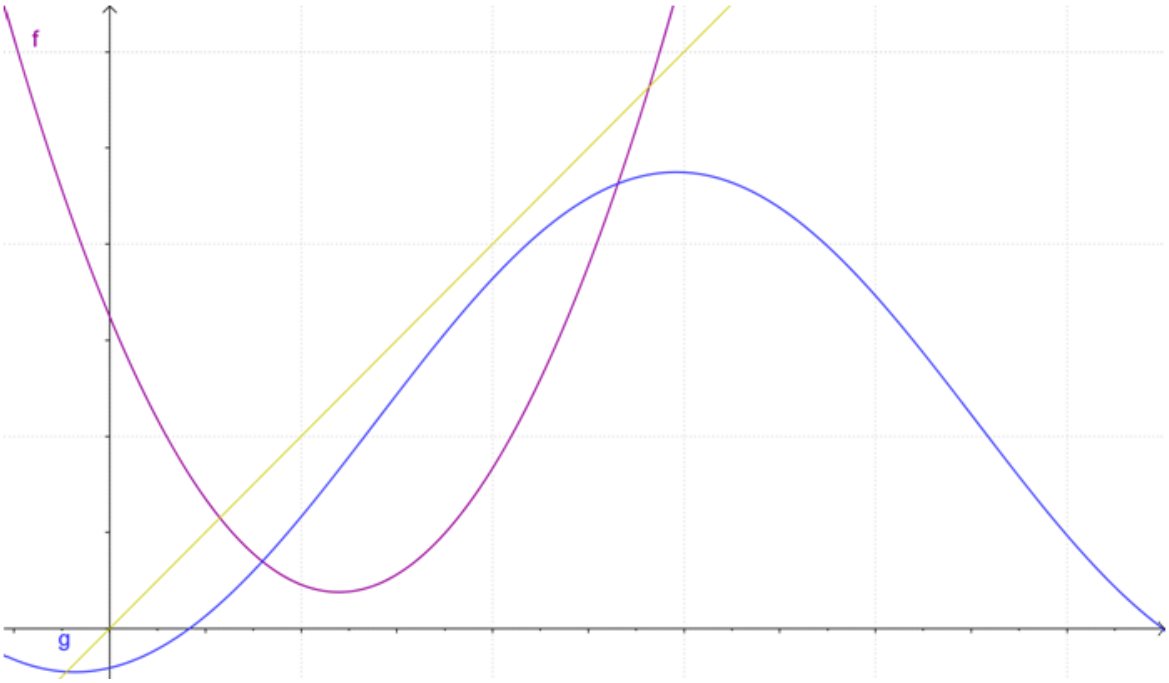
$g(x) = 7x - 2$

The following functions can be written as $g \circ f$. What are f and g?

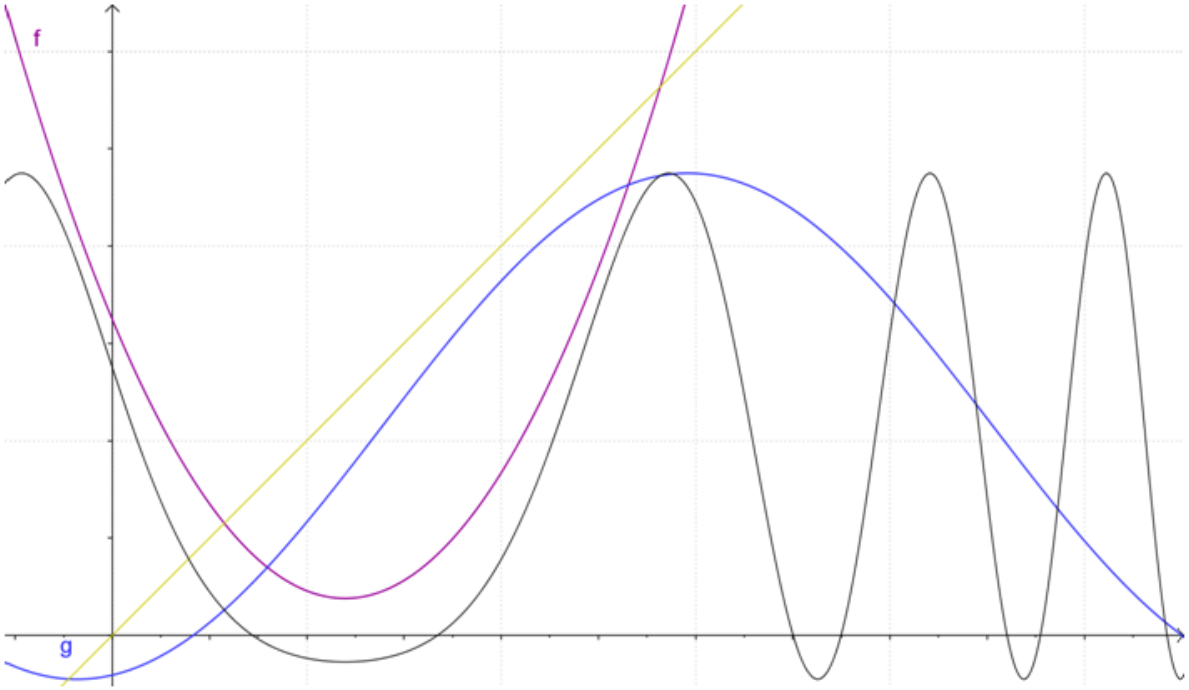
$$F(x) = \sqrt{x^2 - 2x + 1}$$

$$F(x) = \frac{x+2}{x+7}$$

Graphing composition of functions



If we did a whole bunch of points



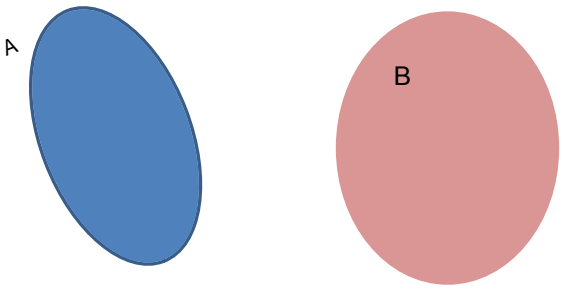
Remember my beading problem?

- As my beading skills got better, I found that people liked my designs and are willing to pay for my necklaces. I started selling them at a local farmers' market for $9.50.

- I would like to know how many necklaces I need to make in order to earn $779.

Interesting question

- If I have a function *f* can I find function *g* so that $g \circ f(x) = x$?



Inverse function

- If a function $f: A \rightarrow B$ has the property that each element of B is the image of exactly one element of A (we say f is *injective*), then f has an *inverse function,* f^{-1}

$$f \circ f^{-1}(x) = x$$
$$f^{-1} \circ f(x) = x$$

- *Horizontal line test:* Function f has an inverse if each horizontal line intersects the graph of f in **exactly** one point.

Finding the inverse function

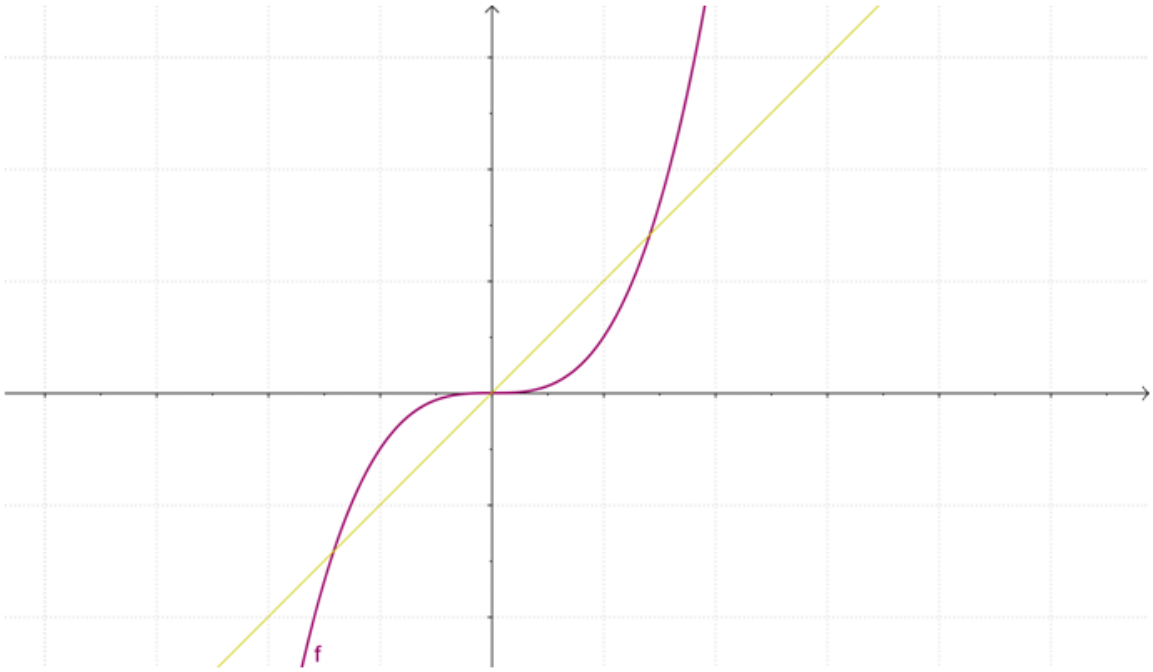
Finding expression for inverse

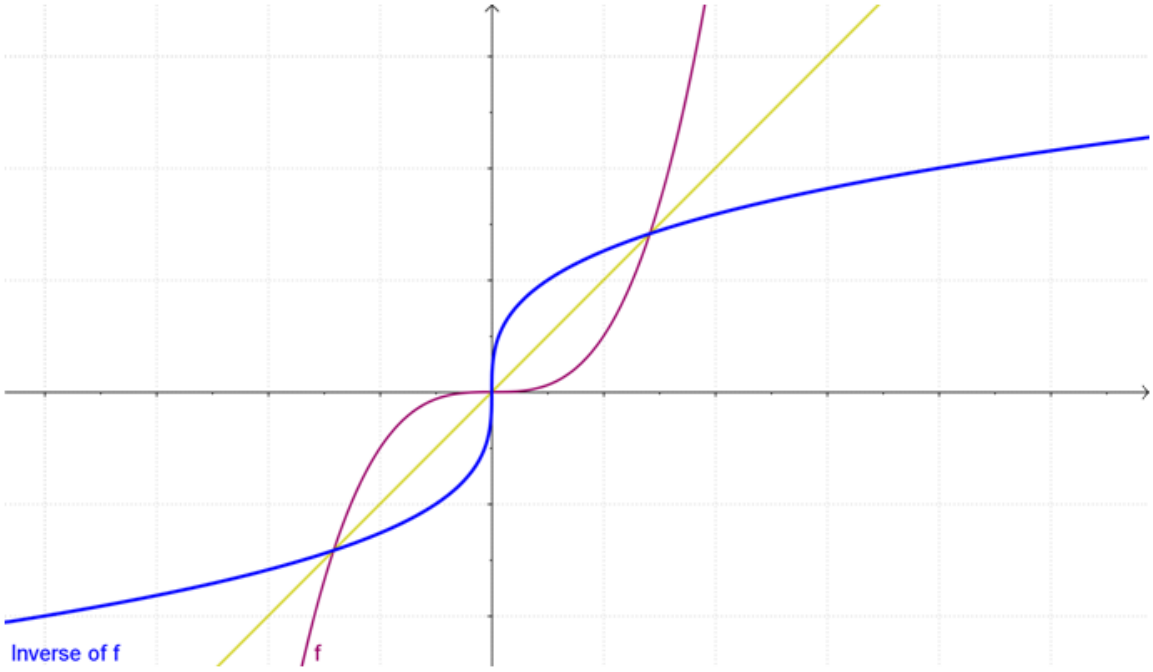
$f(x) = 2x + 1$

$g(x) = \dfrac{x - 3}{2x + 1}$

$h(x) = 2x^2 + 1$

Finding the graph of inverse function




Inverse of f
f