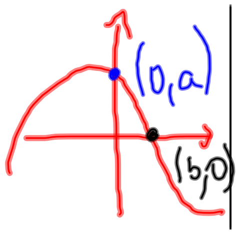
Graphing and analyzing functions

Intercepts, zeros



- *(0,a)* is called y-intercept of *f* if *f(0)=a.* $f(0)=a$

  Find y-intercept of $\mathbf{g(x)}=\sqrt{\mathbf{4-x}}$

  $$g(0)=\sqrt{4-0}=\sqrt{4}=2$$

  $$(0,2)$$

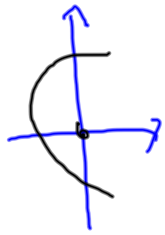
- *(a,0)* is called x-intercept of *f* if *f(a)=0.* In this case, *a* is also called a *zero* of the function *f.*

  Find x-intercept of $\mathbf{g(x)}=\sqrt{\mathbf{4-x}}$

  $$\sqrt{4-x}=0$$
  $$4-x=0$$
  $$4=x$$

  $$(4,0)$$

Find x and y intercepts



$$h(x) = |x - 2| - 2$$

x-intercept : x st $h(x) = 0$

$$|x-2| - 2 = 0$$

$$|x-2| = 2$$

1) $x - 2 = 2 \quad / +2$

$x = 4$

$(4, 0)$

2) $x - 2 = -2 \quad / +2$

$x = 0$

$(0, 0)$. y. int.

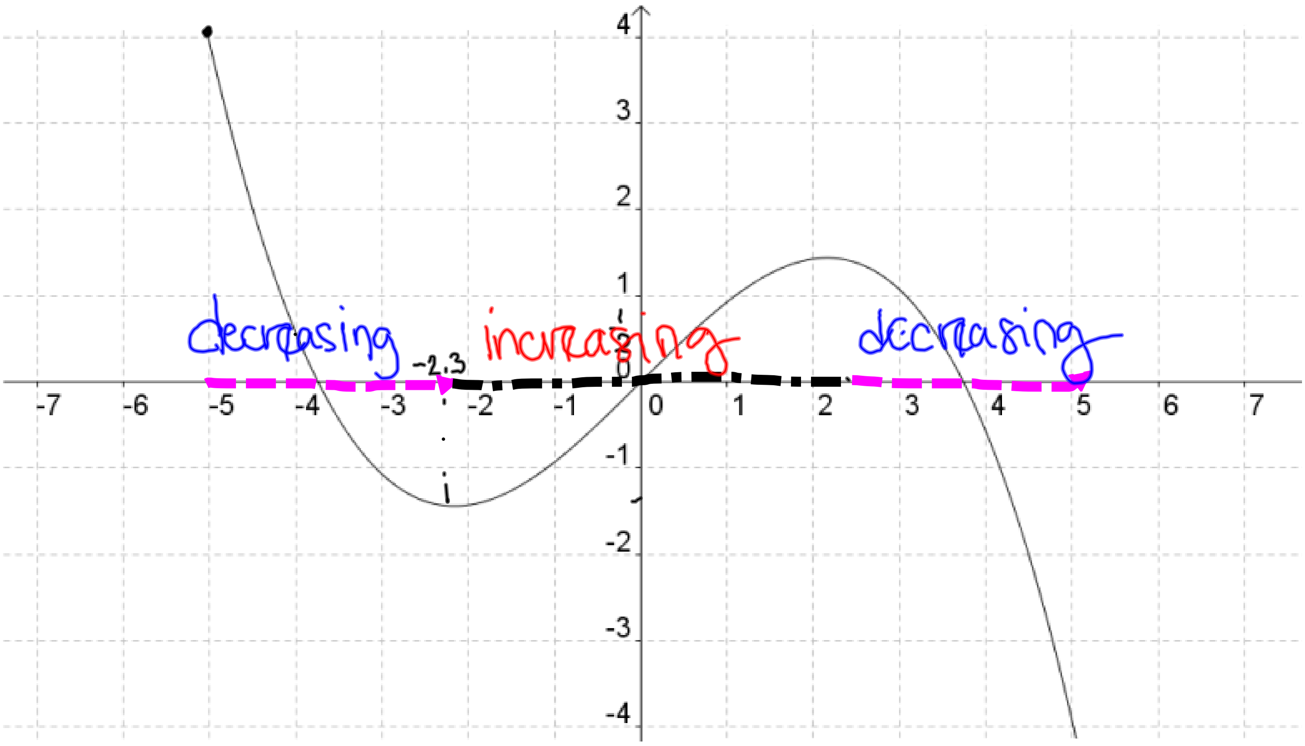
y. intercept : $h(0) = |0-2| - 2$

$$= |-2| - 2 = 2 - 2 = 0$$

Increasing and decreasing functions

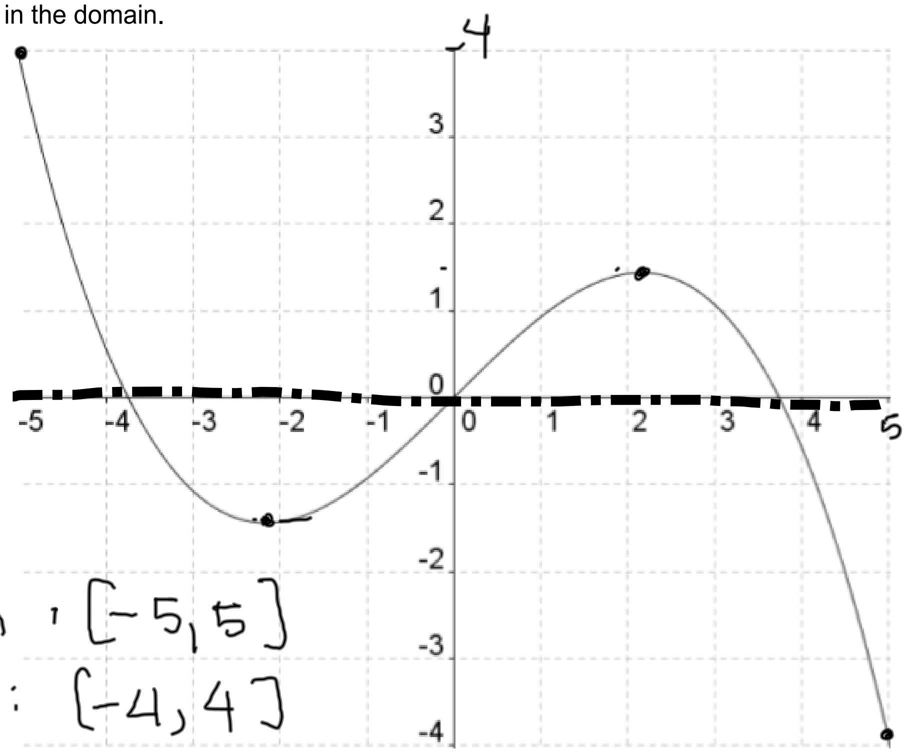
- A function f is **increasing** on an interval if for any two points a and b in the interval for which $a<b$ we have that $f(a)<f(b)$.

- A function f is **decreasing** on an interval if for any two points a and b in the interval for which $a<b$ we have that $f(a)<f(b)$.



Minimum and maximum

- We say that the function f has a **relative (local) minimum** at a point a if $f(a) \leq f(x)$ for all x in some open interval around a.
- We say that the function f has an **absolute (global) minimum** at a point a if $f(a) \leq f(x)$ for all x in the domain.
- We say that the function f has a **relative (local) maximum** at a point a if $f(x) \leq f(a)$ for all x in some open interval around a.
- We say that the function f has an **absolute (global) maximum** at a point a if $f(x) \leq f(a)$ for all x in the domain.



Domain : [-5, 5]

Range: [-4, 4]

How are these different?

$f(x)=x^2$
$h(x)=x^2-3$
$g(x)=x^2+3$
$k(x)=(x-3)^2$
$l(x)=(x+3)^2$
$m(x)=-(x^2)$

x	$f(x)$	$h(x)$	$g(x)$	$k(x)$	$l(x)$	$m(x)$
-5	25	22	28	64	4	-25
-4	16	13	19	49	1	-16
-3	9	6	12	36	0	-9
-2	4	1	7	25	1	-4
-1	1	-2	4	16	4	-1
0	0	-3	3	9	9	0
1	1	-2	4	4	16	-1
2	4	1	7	1	25	-4
3	9	6	12	0	36	-9
4	16	13	19	1	49	-16

5 4

http://www.coolmath.com/graphit

http://www.geogebra.org

$f(x)=x^2$

$h(x)=x^2-3 = f(x)-3$

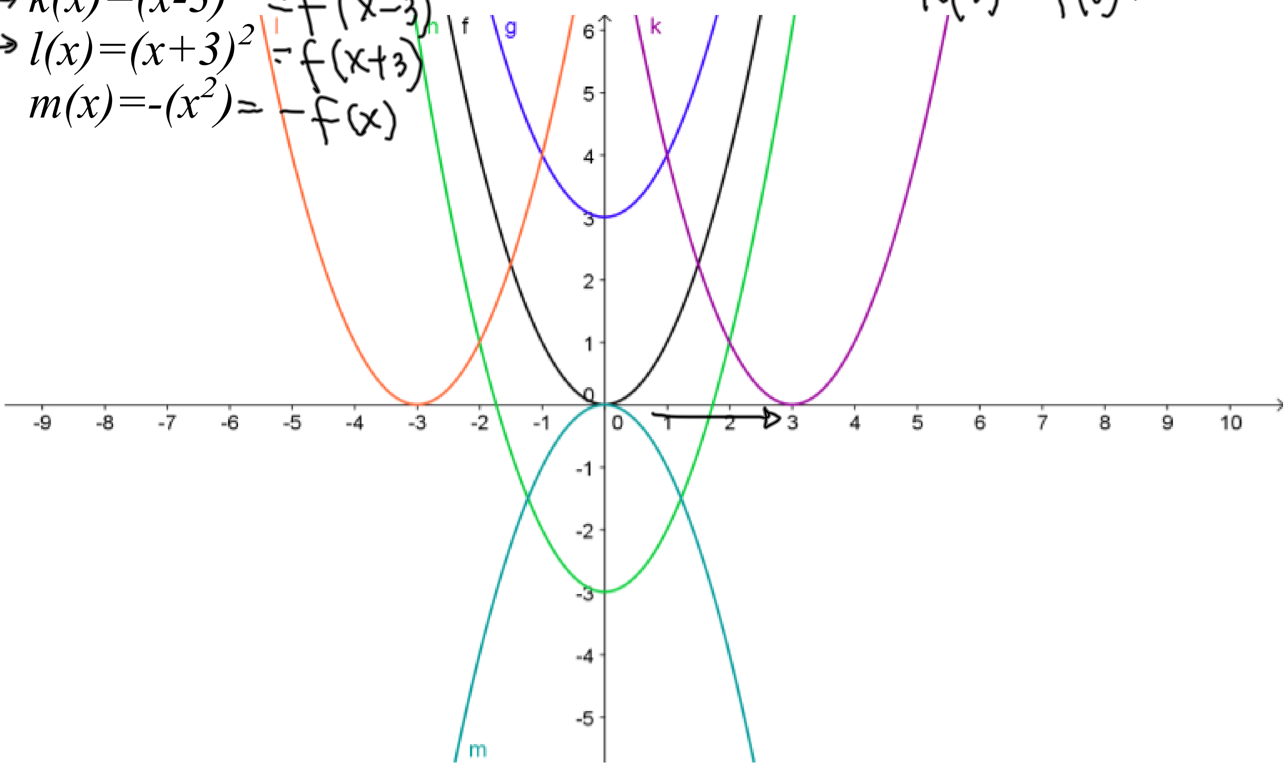
$g(x)=x^2+3 = f(x)+3$

→ $k(x)=(x-3)^2 = f(x-3)$

→ $l(x)=(x+3)^2 = f(x+3)$

$m(x)=-(x^2) = -f(x)$

$K(3) = f(0)$



Graph transformations

- If c is any positive number and $f(x)$ any function then:

- The graph of $h(x)=f(x)+c$ is that of f shifted c units upward
- The graph of $g(x)=f(x)-c$ is that of f shifted c units downward
- The graph of $k(x)=f(x-c)$ is that of f shifted c units to the right
- The graph of $l(x)=f(x+c)$ is that of f shifted c units to the left
- The graph of $m(x)=-f(x)$ is that of f reflected along x-axis.

Non-rigid transformations

$f(x)=x^2$
$h(x)=3x^2-2$
$g(x)=3(x^2-2)$
$k(x)=3(x-2)^2$

x	$f(x)$	$h(x)$	$g(x)$	$k(x)$
-5	25	73	69	
-4	16	46	42	
-3	9	25	21	75
-2	4	10	6	48
-1	1	1	-3	27
0	0	-2	-6	12
1	1	1	-3	3
2	4	10	6	0
3	9	25	21	3
4	16	46	42	12

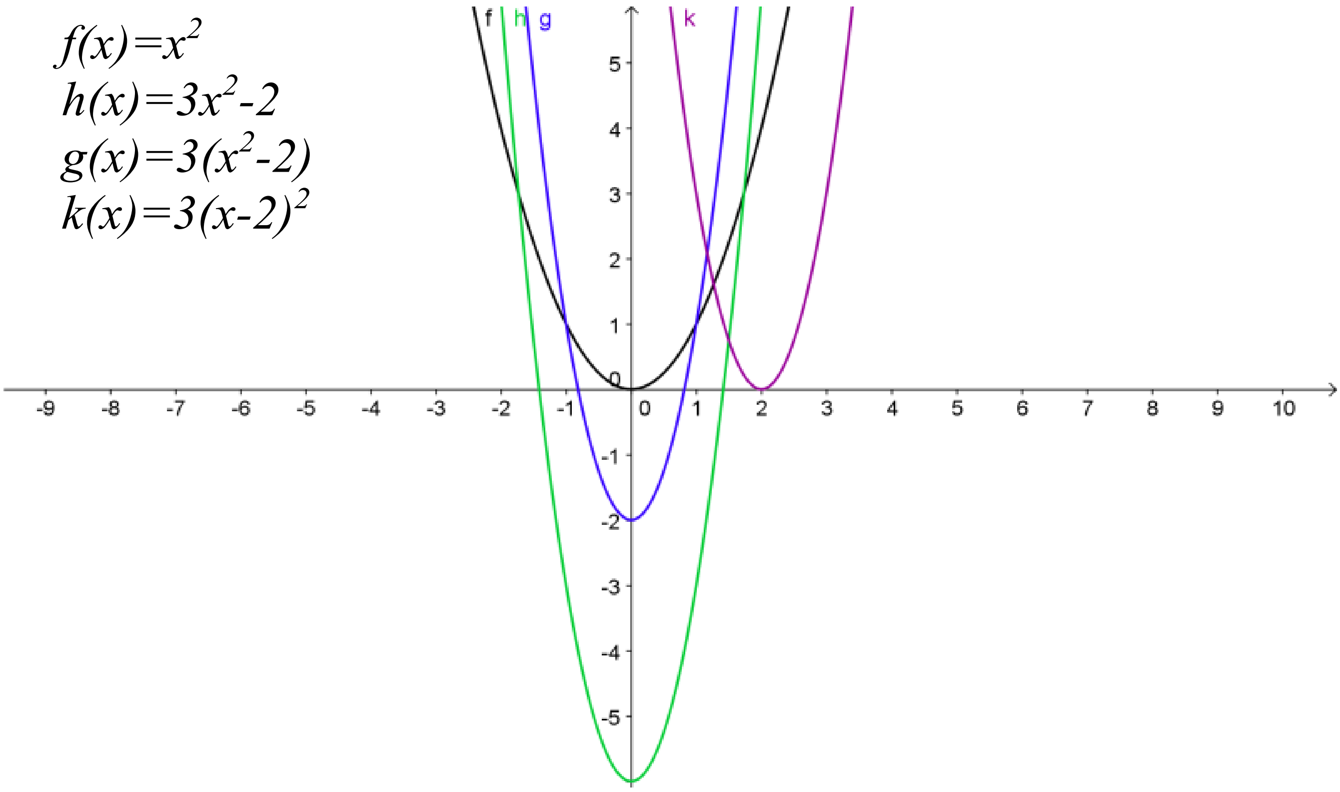
$h(x)=3f(x)-2$
$g(x)=3(f(x)-2)=$
$=3f(x)-6$
$k(x)=3f(x-2)$

$f(x)=x^2$

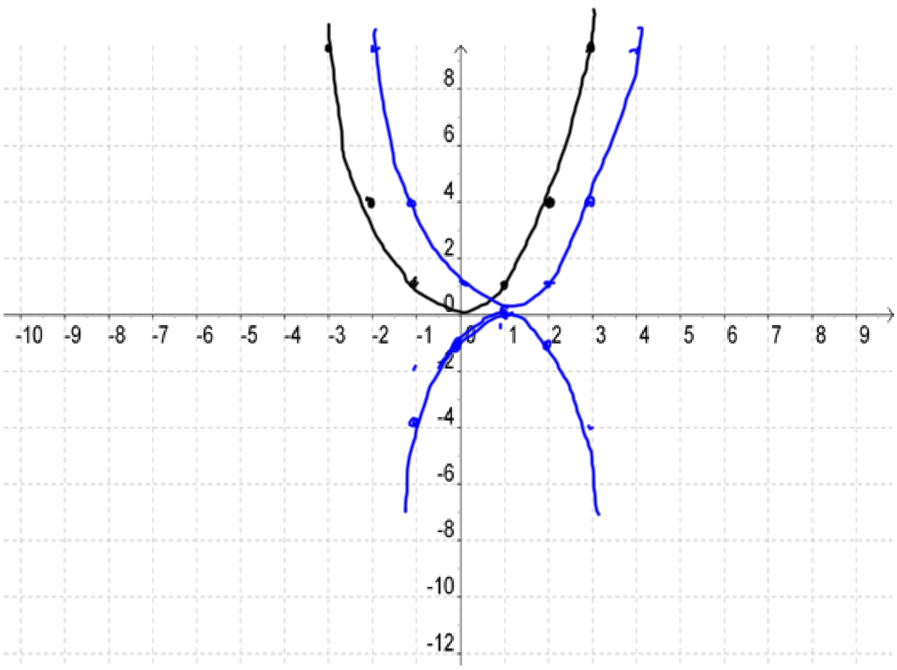
$h(x)=3x^2-2$

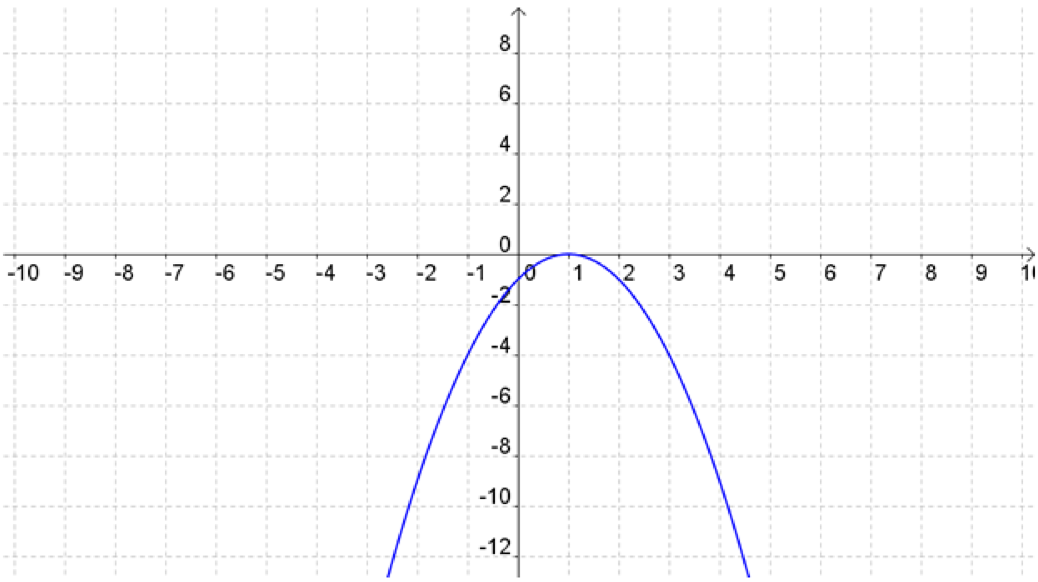
$g(x)=3(x^2-2)$

$k(x)=3(x-2)^2$



Graph $f(x) = -(x-1)^2$





Library of parent functions

- Linear $f(x)=ax+b$
- Constant $f(x)=c$
- Identity $f(x)=x$
- Quadratic $f(x)=x^2$
- Square root $f(x)=\sqrt{x}$
- Cubic $f(x)=x^3$
- Absolute value $f(x)=|x|$
- Reciprocal $f(x)=1/x$

$h(x)=\sqrt{x-4}$

Draw graphs of each of these functions using symmetries, intercepts, and table of values you learned. Then check your solutions using one of the graphing tools.