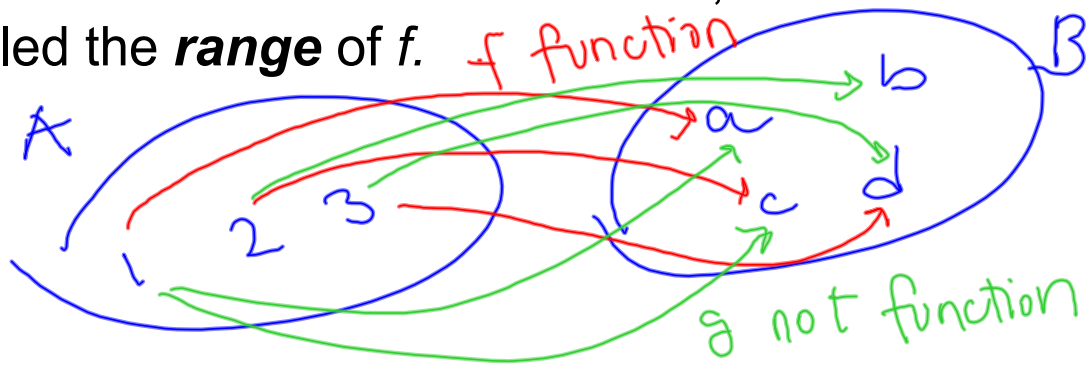
# Functions

And a whole bunch of other goodies

# Functions

- A ***function f*** from set A to set B is a rule that to each element (INDEPENDENT) of the set A assigns EXACTLY one element of the set B (DEPENDENT).

- Set A is called the ***domain*** of *f*, while B is called the ***range*** of *f.*



# Different ways to describe a function

- Verbally – sentence describing how the dependent and independent variable are related

- Numerically – using a table or list of ordered pairs

- Graphically – drawing all the ordered pairs on a coordinate system (the independent variable corresponds to the horizontal axis, and dependent to vertical)

- Algebraically – writing an expression that describes how one variable depends on the other

## Are these functions? Find the domains and ranges.

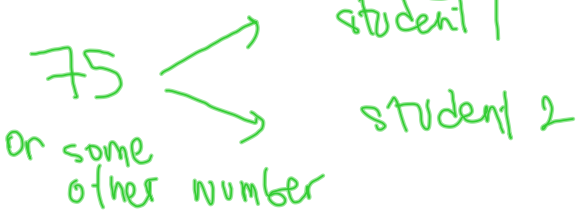
- There are 120 students in the class M1050.

- To each student in the class M1050 we associate their grade on the final exam.

Domain: {students in M1050}
Range: {1, . . . , 100}
Function: yes no

- To each score 1 to 100 we associate a student with that score.

Domain: {1, . . . , 100}
Range: {students in M1050}
Function: yes no



## Are these functions? Find the domains and ranges.

- {(1,2), (1,3),(2,4),(2,5),(3,6),(3,7)

  Domain: {1, 2, 3}
  Range: {2, 3, 4, 5, 6, 7}
  Function: yes **no**

- {(2,8),(3,7), (4,6),(5,7),(6,8)}

  Domain: {2, 3, 4, 5, 6}
  Range: {8, 7, 6}
  Function: **yes** no

# Is this a function? Find the domain and range.

| x | y |
|---|---|
| 1 | 13 |
| 2 | 21 |
| 3 | 17 |
| 3 | 17 |
| 4 | 12 |
| 5 | 15 |

Yes, this is a function

Domain: {1, 2, 3, 4, 5}

Range: {13, 21, 17, 12, 15}

## Is this a function? Find the domain and range.

- Is $y$ a function of $x$ if we have $3x + 5y = 2$

- Question: **"Do we have only one y for each x?**

To find that out we should express y in terms of x, and see if we get a unique (only one) value of y for each individual x:

$3x + 5y = 2 \quad / -3x$

$5y = 2 - 3x \quad / \div 5$

$y = \frac{2 - 3x}{5}$

Yes

Domain: $\mathbb{R}$

Range: $\mathbb{R}$

## Is this a function? Find the domain and range.

- Is $x$ a function of $y$? We have $3x + 5y = 2$
- Question: **"Do we have only one $x$ for each $y$?"**

To find that out we should express x in terms of y, and see if we get a unique (only one) value of x for each individual y:

$3x+5y=2 \;/\; -5y$

$3x=2-5y \;/\div 3$

$x=\frac{2-5y}{3}$

Domain: $\mathbb{R}$

Range: $\mathbb{R}$

Yes.

# Function notation and evaluating functions

$y = g(x) = 2x + 4$

Evaluate function *g* at 2, 4, -3, 1/2

| x | g(x) |
|---|---|
| 2 | $g(2) = 2 \cdot 2 + 4 = 8$ |
| 4 | $g(4) = 2 \cdot 4 + 4 = 8 + 4 = 12$ |
| −3 | $g(-3) = 2 \cdot (-3) + 4 = -6 + 4 = -2$ |
| 1/2 | $g(\frac{1}{2}) = 2 \cdot \frac{1}{2} + 4 = 1 + 4 = 5$ |
| | |
| | |

## Piecewise defined functions

$$f(x)=\begin{cases} x^2-1, & x\le 3 \\ x+3, & x>3 \end{cases}$$

- Evaluate *f* at 6, -12 and 0
- Draw a table of values for $x \in [-1, 5]$

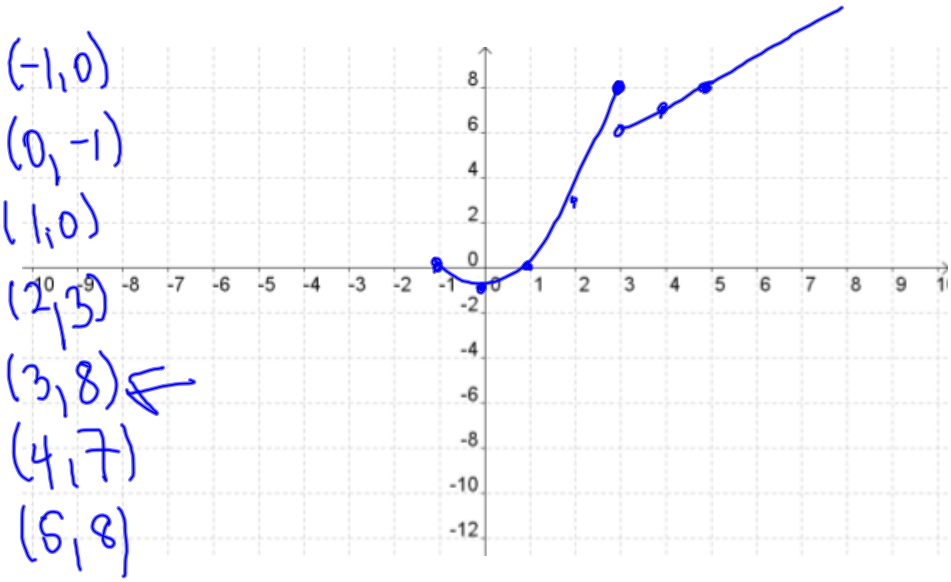
$f(6) = 6+3 = 9$

$f(-12) = (-12)^2 - 1 = 144 - 1 = 143$

$f(0) = 0^2 - 1 = -1$

$$f(x) = \begin{cases} x^2 - 1, & x \leq 3 \end{cases}$$

| x | f(x) |
|---|---|
| -1 | $(-1)^2-1=0$ $(-1,0)$ |
| 0 | $0-1=-1$ $(0,-1)$ |
| 1 | $1^2-1=0$ $(1,0)$ |
| 2 | $4-1=3$ $(2,3)$ |
| 3 | $9-1=8$ $(3,8)$ |
| 4 | $4+3=7$ $(4,7)$ |
| 5 | $5+3=8$ $(5,8)$ |



# Find the domains of the following functions

$$g(x) = \sqrt{1-2x}$$

$1-2x \geq 0 \quad / +2x$

$1 \geq 2x \quad / \div 2$

$\boxed{\frac{1}{2} \geq x}$

All real numbers smaller than or eqval to $\frac{1}{2}$.

$x \in (-\infty, \frac{1}{2}]$

# Find the domains of the following functions

$$h(s) = \frac{s(s+3)}{(s-2)(s+4)}$$

$s-2 \neq 0 \qquad s \neq 2$

$s+4 \neq 0 \qquad s \neq -4$

$D: \mathbb{R} \setminus \{-4, 2\}$

$(-\infty, -4) \cup (-4, 2) \cup (2, \infty)$

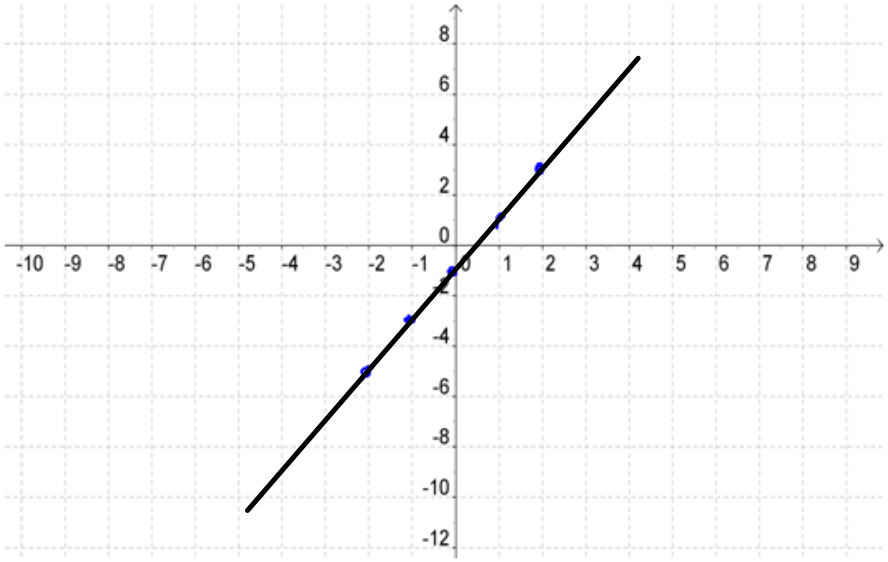
# Find the domains of the following functions

$$h(x) = \sqrt[3]{1-2x}$$

$D = \mathbb{R}$

## Graph of a function f is the set of all points *(x, f(x))* in the coordinate plane.

- Graph $f(x)=2x-1$




| x | f(x) |
|---|---|
| −2 | −5 |
| −1 | −3 |
| 0 | −1 |
| 1 | 1 |
| 2 | 3 |

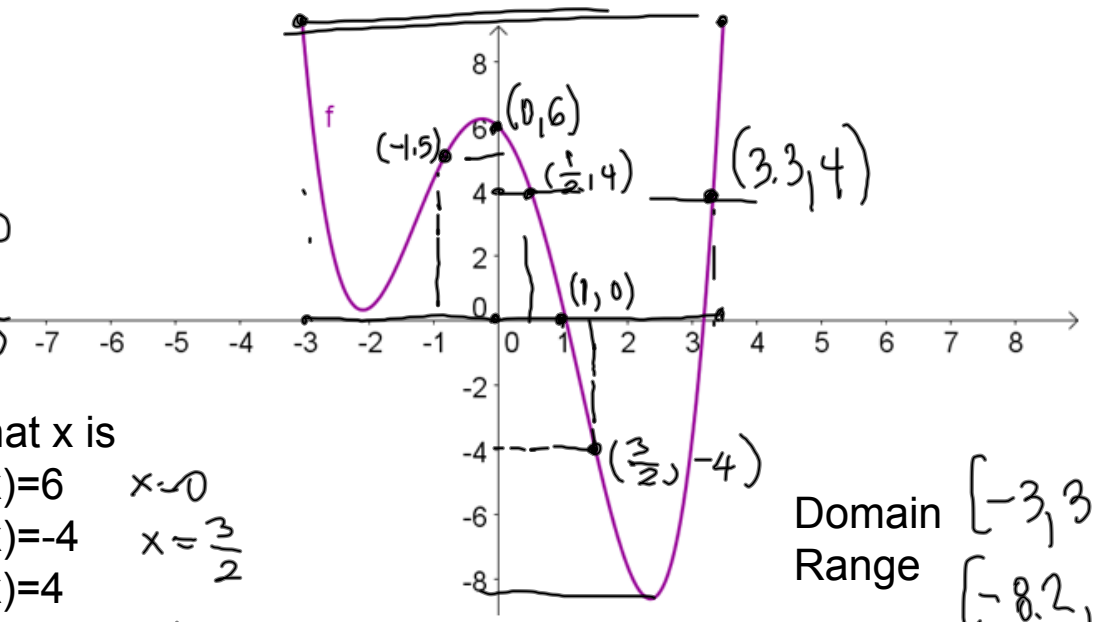
# What can the graph tell us?

- Can I read the value of a function at a given point?

- If I know the value of the function, can I find its origin (the value of independent variable this value corresponds to)?

- Can I read the domain and range?

# What can the graph tell us?

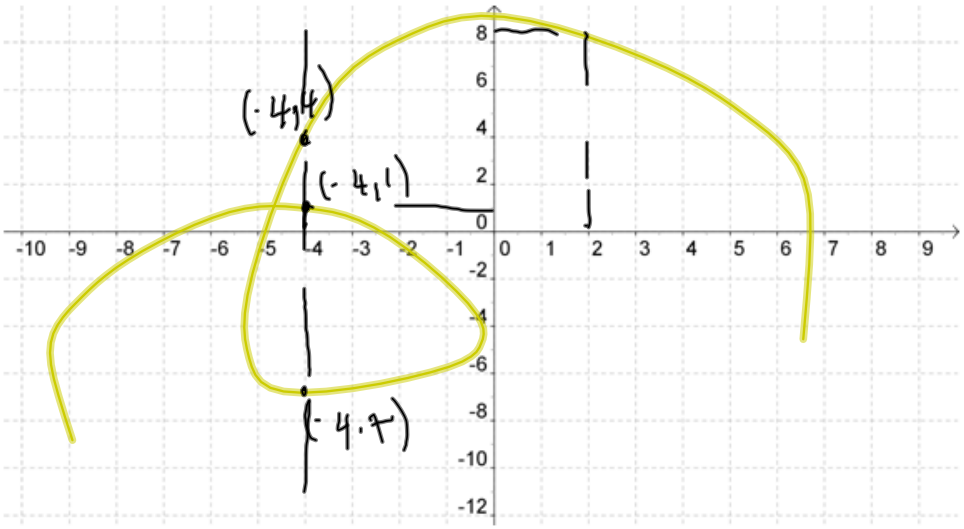



f(1) = 0
f(0)= 6
f(-1)= 5

For what x is
f(x)=6 x=0
f(x)=-4 x=3/2
f(x)=4
x1=-1/2
x2=3.3

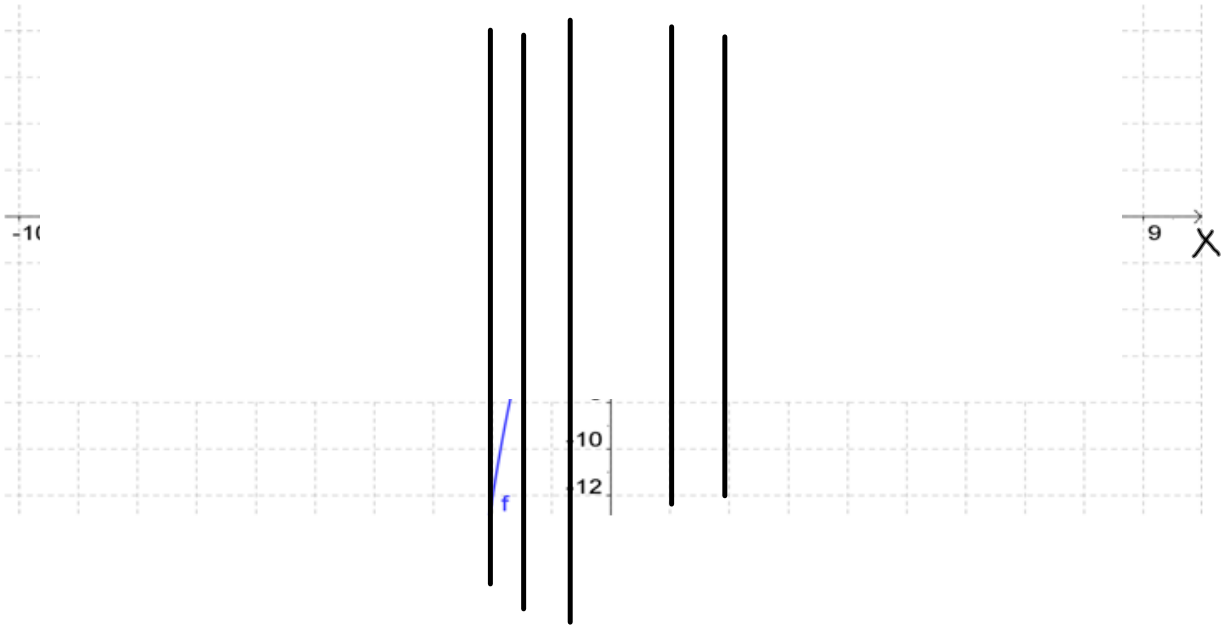
Domain [-3, 3.5]
Range [-8.2, 9]

# Is this a function? Find its domain and range



# Vertical line test

- A curve in the plane is a graph of a function of *x* only if every vertical line intersects that curve in at most one point.



# Review

- Let the function *f* be defined by $f(x) = \frac{1}{\sqrt{1-x^2}}$

- Indicate whether the following statements are true or false:

**1.** *f(x)* is never positive.
**2.** *f(x)* is never zero.
**3.** 0 is in the domain of *f*
**4.** All negative real numbers are in the domain of *f*
**5.** All positive real numbers are in the domain of *f*
**6.** 1 is in the domain of *f*
**7.** *f* is never negative.

**http://matti.usu.edu/grapher/**