MATH 1010 ~ Intermediate Algebra

Chapter 7: RADICALS AND COMPLEX NUMBERS

Section 7.2: Simplifying Radical Expressions

Objectives:

- Use the Product and Quotient Rules for Radicals to simplify radical expressions.
- Use rationalization techniques to simplify radical expressions.
- Use the Pythagorean Theorem in application problems.

$\sqrt{64x^3}$

$\sqrt{648}$

$\sqrt[3]{(-64)x^2y^5}$

$\sqrt[3]{24x^3y^5}$

① EXAMPLE

Simplify these rational expressions.

$a)\ \sqrt{75} = \sqrt{25\cdot 3} = \sqrt{25}\,\sqrt{3}$

$= \boxed{5\sqrt{3}}$

$\sqrt{25\cdot 3}$
$= (25\cdot 3)^{1/2}$
$= 25^{1/2}\,3^{1/2}$

$b)\ \sqrt{162} = \sqrt{81\cdot 2} = \sqrt{81}\,\sqrt{2}$

$= \boxed{9\sqrt{2}}$

assume $x \geq 0$, $y \geq 0$

$c)\ \sqrt{72x^3y^2} = \sqrt{36\cdot 2x^2\, x\, y^2} = \sqrt{36}\,\sqrt{2}\,\sqrt{x^2}\,\sqrt{x}\,\sqrt{y^2}$

$= \boxed{6xy\sqrt{2x}}$

$\sqrt{x^3} = x^{3/2}$
$= x^{1\frac{1}{2}} = x^1 x^{1/2}$
$= x^1\sqrt{x}$

$d)\ \sqrt{0.0027}$

$= \sqrt{\dfrac{27}{10000}} = \dfrac{\sqrt{9\cdot 3}}{\sqrt{100^2}} = \dfrac{3\sqrt{3}}{100}$ or $\dfrac{3}{100}(\sqrt{3})$

$= 0.03(\sqrt{3})$

WARNING:

$\sqrt{x^2} = |x|$

$\sqrt[4]{x^4} = |x|$

$\sqrt{x^2} = x$, if $x \geq 0$

ex ① $\sqrt{(-5)^2} = \sqrt{25} = 5$

$\neq -5$

② $\sqrt{5^2} = \sqrt{25} = 5$

② EXAMPLE

Simplify these rational expressions.

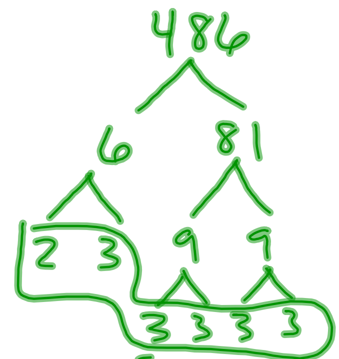
$a)\ \sqrt{18x^4} = \sqrt{9\cdot 2x^4} = 3x^2\sqrt{2}$

$b)\ \sqrt[3]{81} = \sqrt[3]{27\cdot 3} = \sqrt[3]{27}\,\sqrt[3]{3} = 3\sqrt[3]{3}$

$c)\ \sqrt[5]{486x^7} = \sqrt[5]{3^5\cdot 2x^5x^2}$

$= \sqrt[5]{3^5}\,\sqrt[5]{2}\,\sqrt[5]{x^5}\,\sqrt[5]{x^2}$

$= \boxed{3x\sqrt[5]{2x^2}}$



$486 = 3^5\cdot 2$

$d)\ \sqrt{\dfrac{18x^2}{w^6}} = \dfrac{\sqrt{9}\,\sqrt{2}\,\sqrt{x^2}}{\sqrt{w^6}} = \dfrac{3|x|\sqrt{2}}{|w^3|}$

or if $x \geq 0, w > 0$

$\dfrac{3x\sqrt{2}}{w^3}$

$e)\ \sqrt[5]{128u^4v^7}$

$= \sqrt[5]{64\cdot 2u^4v^7} = \sqrt[5]{2^7u^4v^7}$

$= \sqrt[5]{2^5}\,\sqrt[5]{2^2}\,\sqrt[5]{u^4}\,\sqrt[5]{v^5}\,\sqrt[5]{v^2}$

$= \boxed{2v\sqrt[5]{4u^4v^2}}$

③ EXAMPLE

Rationalize the denominator.

Make the denominator have no radical sign.

TRICK: multiply by one

a) $\sqrt{\frac{1}{3}} = \frac{\sqrt{1}}{\sqrt{3}} = \frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$

$= \frac{\sqrt{3}}{\sqrt{3^2}} = \boxed{\frac{\sqrt{3}}{3}}$

b) $\sqrt{\frac{4}{x^3}} = \frac{\sqrt{4}}{\sqrt{x^3}} = \frac{2}{\sqrt{x^2}\sqrt{x}} = \frac{2}{|x|\sqrt{x}}\left(\frac{\sqrt{x}}{\sqrt{x}}\right)$

$= \frac{2\sqrt{x}}{|x|\,|x|} = \boxed{\frac{2\sqrt{x}}{x^2}}$

c) $\frac{10}{\sqrt[5]{6}} = \frac{10}{\sqrt[5]{6}}\left(\frac{\sqrt[5]{6^4}}{\sqrt[5]{6^4}}\right) = \frac{10\sqrt[5]{6^4}}{\sqrt[5]{6^5}} = \frac{\overset{5}{\cancel{10}}\sqrt[5]{6^4}}{\underset{3}{\cancel{6}}} = \boxed{\frac{5\sqrt[5]{6^4}}{3}}$

d) $\sqrt[3]{\frac{9}{25}} = \frac{\sqrt[3]{9}}{\sqrt[3]{5^2}}\left(\frac{\sqrt[3]{5}}{\sqrt[3]{5}}\right) = \frac{\sqrt[3]{45}}{\sqrt[3]{5^3}} = \boxed{\frac{\sqrt[3]{45}}{5}}$

e) $\sqrt[3]{\dfrac{20x^2}{9y^4}} = \dfrac{\sqrt[3]{20x^2}}{\sqrt[3]{9y^4}} = \dfrac{\sqrt[3]{20x^2}}{y\sqrt[3]{3^2 y}}\left(\dfrac{\sqrt[3]{3y^2}}{\sqrt[3]{3y^2}}\right)$

$\sqrt[3]{y^4} = \sqrt[3]{y^3}\,\sqrt[3]{y}$

$= y\sqrt[3]{y}$

$= \dfrac{\sqrt[3]{60x^2y^2}}{y\left(\sqrt[3]{3^3y^3}\right)}$

$= \dfrac{\sqrt[3]{60x^2y^2}}{y(3y)} = \dfrac{\sqrt[3]{60x^2y^2}}{3y^2}$

f) $\dfrac{5}{\sqrt{8x^5}}$

$\sqrt{8}$

$= \sqrt{4}\sqrt{2}$

$= 2\sqrt{2}$

$\sqrt{x^5}$

$= \sqrt{x^4}\sqrt{x}$

$= x^2\sqrt{x}$

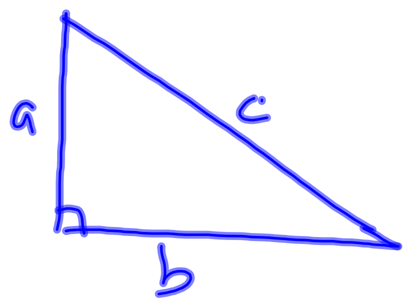
$= \dfrac{5}{2x^2\sqrt{2x}}\left(\dfrac{\sqrt{2x}}{\sqrt{2x}}\right)$

$= \dfrac{5\sqrt{2x}}{2x^2(2x)}$

$= \boxed{\dfrac{5\sqrt{2x}}{4x^3}}$

Pythagorean Thm

(only applies to right triangles)



$$a^2+b^2=c^2$$

ex $a=4$, $c=9$, $b=?$

$$4^2+b^2=9^2$$

$$16+b^2=81$$

$$b^2=65$$

$$\boxed{b=\sqrt{65}}$$