MATH 1010 ~ Intermediate Algebra

Chapter 6: RATIONAL EXPRESSIONS, EQUATIONS AND FUNCTIONS

Section 6.6: Solving Rational Equations

Objectives:

- Solve rational equations containing constant denominators.
- Solve rational equations containing variable denominators.

$$\frac{2x}{x+1} + \frac{3}{x-2} = 2$$

① EXAMPLES

a) $\frac{1}{3}+\frac{x}{10}=-1$

LCD = 30

$30\left(\frac{1}{3}+\frac{x}{10}\right)=-1(30)$

$\frac{30}{3}+\frac{30x}{10}=-30$

$10+3x=-30 \iff \frac{3x}{3}=\frac{-40}{3} \iff x=-40/3$

★ WARNING:
In an equation, we can get rid of fractions by multiplying both sides of eqn by LCD.

NOT applicable in an expression

$x\neq 0$ b) $\frac{3}{2x}+\frac{1}{5x}=6$

LCD = 10x

$\cancel{10x}^{5}\left(\frac{3}{\cancel{2x}}\right)+\cancel{10x}^{2}\left(\frac{1}{\cancel{5x}}\right)=6(10x)$

$15+2=60x$

$\frac{17}{60}=\frac{60x}{60}$

$17/60=x$

② EXAMPLES

a) $\frac{3x}{x-2}+\frac{4}{x^2-4}=-1$

LCD = $(x-2)(x+2)$

$x \neq 2, -2$

$\frac{(x-2)(x+2)3x}{(x-2)}+\frac{4(x-2)(x+2)}{(x-2)(x+2)}=-1(x-2)(x+2)$

$3x^2+6x+4=-(x^2-4)$

$3x^2+6x+4=-x^2+4$

$+x^2 \quad -4 \quad +x^2 \quad -4$

$4x^2+6x=0$

$2x(2x+3)=0$

$2x=0 \quad 2x+3=0$

$x=0 \quad 2x=-3$

$x=-3/2$

b) $\frac{3x}{x+5}=8-\frac{15}{x+5}$

LCD = $\frac{(x+5)}{1}$

$x \neq -5$

$\frac{3x(x+5)}{(x+5)}=8(x+5)-\frac{15(x+5)}{(x+5)}$

$3x=8x+40-15$

$3x=8x+25$

$-8x \quad -8x$

$\frac{-5x}{-5}=\frac{25}{-5}$

$x=-5$

N.S.

③ EXAMPLES

a) $\frac{2}{x^2+2x-8} - \frac{1}{x^2+9x+20} = \frac{4}{x^2+3x-10}$

$LCD = (x+4)(x-2)(x+5)$

$x \neq -4, 2, -5$

$(x+4)(x-2) \quad (x+4)(x+5) \quad (x+5)(x-2)$

$$\frac{2(x+4)(x-2)(x+5)}{(x+4)(x-2)} - \frac{1(x+4)(x-2)(x+5)}{(x+4)(x+5)} = \frac{4(x+4)(x-2)(x+5)}{(x+5)(x-2)}$$

$$2(x+5) - (x-2) = 4(x+4)$$

$$2x+10-x+2 = 4x+16$$

$$x+12 = 4x+16$$

$$-x \quad -16 \quad -x \quad -16$$

$$\frac{-4}{3} = \frac{3x}{3} \iff x = -4/3$$

b) $\frac{12}{x+5} + \frac{5}{x} = \frac{20}{x}$

$LCD = x(x+5)$

$x \neq 0, -5$

$$\frac{12x(x+5)}{(x+5)} + \frac{5x(x+5)}{x} = \frac{20x}{x}(x+5)$$

$$12x + 5(x+5) = 20(x+5)$$

$$12x+5x+25 = 20x+100$$

$$17x+25 = 20x+100$$

$$-20x \quad -25 \quad -20x \quad -25$$

$$-3x = 75$$

$$x = \frac{-75}{3} = -25$$