

MATH 1010 ~ Intermediate Algebra

Chapter 6: RATIONAL EXPRESSIONS,  
EQUATIONS AND FUNCTIONS

## Section 6.2: Multiplying and Dividing Rational Expressions

Objectives:

- ☆ Multiply rational expressions and simplify.
- ☆ Divide rational expressions and simplify.

$$\frac{2}{3x} * \frac{15}{7x}$$

$$\frac{3x}{24} \div \frac{15x}{8}$$

Multiply these. Simplify the answer.

$$\begin{aligned}
 \text{a) } \frac{\cancel{2}x^4y^2}{\cancel{3}xy^3} \cdot \frac{\cancel{-6}xy^2}{\cancel{14}x^3} &= \frac{\cancel{-2}\overset{x}{\cancel{x}}\overset{y}{\cancel{y}}}{\cancel{7}\cancel{x^3}\cancel{y^3}} \\
 &= \frac{-2xy}{7}, \quad x \neq 0, y \neq 0
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } \frac{(5x^2 - 5x)(x^2 + 8x + 12)}{x^2 + 5x - 6} \cdot \frac{10x}{10x} \\
 = \frac{\cancel{5x}(x-1)\cancel{(x+6)}(x+2)}{\cancel{(x+6)}\cancel{(x-1)}\cancel{10x}} \leftarrow \\
 = \boxed{\frac{x+2}{2}, \quad x \neq -6, 1, 0}
 \end{aligned}$$

Divide these. Simplify the answer.

$$\begin{aligned}
 \text{a) } \frac{x^2 - 4}{3x^3} \div \frac{2x + 2}{9x^4} &= \frac{x^2 - 4}{3x^3} \cdot \frac{9x^4}{2x + 2} \\
 &= \frac{(x-2)(x+2) \cancel{9} \cancel{x^4}^3 x}{\cancel{3} \cancel{x^3} (2)(x+1)} \\
 &= \frac{3x(x-2)(x+2)}{2(x+1)}, x \neq 0
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } \frac{x^2 y^3}{3x^3} \div \frac{x^3 y}{2x + x^2} &= \frac{\cancel{x^2} \cancel{y^3}^2}{3 \cancel{x^3}^2 x} \cdot \frac{2x + x^2}{x^3 \cancel{y}} \\
 &= \frac{y^2 (\cancel{x}) (2+x)}{3 \cancel{x} (x^3)} = \frac{y^2 (2+x)}{3x^3}, y \neq 0, x \neq -2
 \end{aligned}$$

$x \neq 0$   
 $y \neq 0$   
 $x \neq -2$

## ① EXAMPLE

Fill in the missing factor.

$$\text{a) } \frac{14x(x-3)^2}{(x-3)(?) } = \frac{2x}{x-3}$$

$$\frac{\cancel{14}x^2 \cancel{(x-3)} \cancel{(x-3)}}{\cancel{7}(x-3) \cancel{(x-3)} \cancel{(x-3)}} = \frac{2x}{x-3}$$

$$? = 7(x-3)^2$$

$$\text{b) } \frac{(3x+5)(?)}{5x^2(3x-5)} = \frac{3x+5}{x}$$

$$\frac{5x(3x+5) \cancel{(3x-5)}}{\cancel{5}x^2 \cancel{(3x-5)}} = \frac{3x+5}{x}$$

$$? = 5x(3x-5)$$

## ② EXAMPLE

Simplify the answer to these.

$$a) \frac{x^2 - 3x + 2}{x + 2} \cdot \frac{3x}{x - 2} \cdot \frac{2x + 4}{x^2 - 5x}$$

$$= \frac{(x-2)(x-1)(\cancel{3x})2(\cancel{x+2})}{(\cancel{x+2})(\cancel{x-2})(\cancel{x})(x-5)}$$

$$= \frac{6(x-1)}{(x-5)}, \quad x \neq 0, 2, -2$$

$$b) \frac{7x}{4x-16} \div \frac{14x^2 + 21x}{2x^2 - 7x - 4}$$

$$= \frac{7x}{4(x-4)} \div \frac{7x(2x+3)}{(2x+1)(x-4)}$$

$$= \frac{\cancel{7x}}{4(\cancel{x-4})} \cdot \frac{(2x+1)(\cancel{x-4})}{\cancel{7x}(2x+3)}$$

$$= \frac{2x+1}{4(2x+3)}, \quad x \neq 0, 4, -\frac{1}{2}$$

c) 
$$\frac{x+3}{x^2+7x+10} \div \frac{x^2+6x+9}{x^2+5x+6}$$

$$= \frac{(x+3)}{(x+2)(x+5)} \div \frac{(x+3)(x+3)}{(x+2)(x+3)}$$

$$= \frac{1(\cancel{x+3})}{(\cancel{x+2})(x+5)} \cdot \frac{(\cancel{x+2})(\cancel{x+3})}{(\cancel{x+3})(\cancel{x+3})}$$

$$= \frac{1}{x+5}, x \neq -2, -3$$

domain:  
 $x \neq -2, -5, -3$

d) 
$$\frac{y^2-100}{4y^2} \cdot \frac{y^3-5y^2-50y}{y^4+10y^3} \div \frac{(y-10)^2}{5y}$$

$$= \frac{(y-10)(y+10)}{4y^2} \cdot \frac{y(y^2-5y-50)}{y^3(y+10)} \div \frac{(y-10)(y-10)}{5y}$$

$$= \frac{(\cancel{y-10})(\cancel{y+10})y(\cancel{y-10})(y+5)5\cancel{y}}{4\cancel{y}^2(\cancel{y}^3)(\cancel{y+10})(\cancel{y-10})(\cancel{y-10})}$$

$$= \frac{5(y+5)}{4y^3}, y \neq 10, 10$$

domain:  
 $y \neq 0, -10, 10$

$$\begin{aligned}
 \text{e) } \frac{\frac{5x}{x+7}}{\frac{10}{x^2+8x+7}} &= \frac{5x}{x+7} \div \frac{10}{x^2+8x+7} \\
 &= \frac{5x}{x+7} \div \frac{10}{(x+7)(x+1)} \quad x \neq -1, -7 \\
 &= \frac{\cancel{5x}}{\cancel{(x+7)}} \cdot \frac{\cancel{(x+7)}(x+1)}{\cancel{10}} \\
 &= \frac{x(x+1)}{2}, \quad x \neq -1, -7
 \end{aligned}$$