

MATH 1010 ~ Intermediate Algebra

Chapter 1 Fundamentals of Algebra

## Section 1.3: Properties of Real Numbers

Objectives:

- \* Identify and use the properties of real numbers.
- \* Develop additional properties of real numbers.

Which properties are used?

$$a(b+c) = ab + ac = ba + ca = ca + ba = (c+b)a$$

## PROPERTIES OF REAL NUMBERS

Commutative Property: *order does not matter*

① addition:  $a + b = b + a$  *ex*  $2 + 3 = 3 + 2$

② multiplication:  $ab = ba$  *ex*  $3(5) = 5(3)$

Associative Property: *grouping does not matter*

① addition  $(a + b) + c = a + (b + c)$

② multiplication  $(ab)c = a(bc)$

Distributive Property of multiplication over addition/subtraction

$$a(b + c) = ab + ac$$

*ex*  $5(2 + 1) = 5(2) + 5(1) = 15$

Additive Identity Property:

$$a + 0 = a$$
  
$$= 0 + a$$
  $0 = \text{additive identity}$

Multiplicative Identity Property:

$$a \cdot 1 = a = 1 \cdot a$$
  $1 = \text{mult. identity}$

Additive Inverse Property:

$$a + -a = 0$$

*ex*  $5 + -5 = 0$

 $5$  is add. inverse of  $-5$  $-a$  is add. inverse of  $a$ 

Multiplicative Inverse Property:

$$a\left(\frac{1}{a}\right) = 1$$

*ex*  $-3\left(\frac{-1}{3}\right) = 1$

 $\frac{1}{a}$  is mult. inverse of  $a$   
↳ reciprocal

opposite

PROPERTIES OF EQUALITY (true for equations)

Addition Property of Equality: add same quantity to both sides of eqn.

$$3x - 1 = 4$$

Multiplication Property of Equality:

multiply by same quantity on both sides of eqn (not zero)

$$3x - 1 + 1 = 4 + 1$$

Cancellation Property of Addition:

add inverse to both sides of eqn.

$$5x + 2 = 0 \quad + -2 \quad + -2 \quad 5x = -2$$

Cancellation Property of Multiplication:

multiply both sides of eqn by mult. inverse

$$\frac{1}{5} \cdot 5x = -2 \cdot \frac{1}{5}$$

$$x = -\frac{2}{5}$$

### PROPERTIES OF ZERO

zero = additive  
identity

Multiplication Property of Zero:

if  $ab = 0$ , then  $a = 0$  or  $b = 0$ .

Division Property of Zero:

$$\frac{0}{a} = 0, a \neq 0$$

ex  $\frac{0}{3} = 0 \iff 0 = 3 \cdot 0$

Division by Zero is Undefined:

case 1

$$\frac{5}{0} = ?$$

$$\iff 5 = ? \cdot 0$$

nothing works

$\Rightarrow \frac{5}{0}$  undefined

case 2

$$\frac{0}{0} = ?$$

$$\iff 0 = ? \cdot 0$$

everything works

$\Rightarrow$  undefined

## PROPERTIES OF NEGATION

Multiplication by -1:

$$-1 \cdot a = -a$$

ex  $-1 \cdot -5 = 5$

↑  
opposite

Placement of Negative Signs:

in a fraction

ex  $-\frac{2}{5} = \frac{2}{-5} = -\frac{2}{5}$

Product of Two Opposites:

$$-a \cdot -b = (-1 \cdot -1)ab = ab$$

① EXAMPLE: Simplify and state the properties used.

a)  $10(2x) = (10 \cdot 2)x = 20x$       Associativity of multiplication

b)  $17 - 17 = 17 + -17 = 0$       additive inverse

c)  $(-3/4)(-4/3) = \frac{-3}{4} \cdot \frac{-4}{3} = 1$       mult. inverse

d)  $6(x+3) = 6x + 18$       Distributivity

e) The additive inverse of -16 is 16, the multiplicative inverse is  $-\frac{1}{16}$ .

f) The additive inverse of  $-\frac{2}{3}$  is  $\frac{2}{3}$ , the multiplicative inverse is  $-\frac{3}{2}$ .

g)  $12(\$19.95) = 12(20 - 0.05) = 12(20) - 12(0.05)$

h) BEWARE  $(a+b)^2 =$   
 $= 240 - 0.60$   
 $= \$239.40$

$$(a+b)^2 \neq a^2 + b^2$$

Distributivity

$$\begin{aligned} (a+b)^2 &= (a+b)(a+b) \\ &= a(a+b) + b(a+b) \\ &= a^2 + ab + ab + b^2 \\ &= a^2 + 2ab + b^2 \end{aligned}$$