University of Utah, Department of Mathematics
May 2020, Algebra Qualifying Exam

There are five problems on the exam. You may attempt as many problems as you wish; three correct solutions count as a high pass. 2 correct solutions count as a pass. Show all your work, and provide reasonable justification for your answers.

1. Show there is no simple group of order $2304 = 2^8 \cdot 3^2 = 256 \cdot 9$.
2. Let $K = \mathbb{Q}[\sqrt{2}, \sqrt{3}, \sqrt{5}]$. Show that K is Galois over $\mathbb{Q}$ and identify $\mathrm{Gal}(K/\mathbb{Q})$ up to isomorphism.
3. Classify the groups of order 28 up to isomorphism.
4. Let F be a field of characteristic zero and let E/F be a finite Galois extension. Suppose that $E = F[\alpha]$. Suppose that there exists $\sigma \in G = \mathrm{Gal}(E/F)$ with $\sigma(\alpha) = -\alpha$. Prove that $E \neq F[\alpha^2]$.
5. Let L be the splitting field of $(x^3 + 2x + 1)(x^3 + x^2 + 2)(x^2 + 1)$ over $\mathbb{F}_3$. How many proper subfields does L have?