University of Utah, Department of Mathematics
August 2024, Algebra II Qualifying Exam

There are five problems on the exam. You may attempt as many problems as you wish; three correct solutions count as a high pass. Show all your work and provide reasonable justification for your answers.

1. Prove that the alternating group A_5 is the only simple group of order 60.

2. Prove that if p is prime, then each group with p^d elements has a non-trivial center.

3. Let K be an arbitrary field. Prove that the polynomial ring $K[x]$ has infinitely many prime ideals.

4. Show that if L/K is Galois and $f \in K[x]$ is monic irreducible, then every irreducible factor of f in $L[x]$ has the same degree.

5. Find a complete list of irreducible representations of the alternating group A_4.

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