# Qualifying Exam
# Analysis of Numerical Methods II, August 2023

**Instructions: This exam is closed book, no notes, and no electronic devices or calculators are allowed. You have two hours and you will be graded on work for only 3 out of the 4 questions below. All questions have equal weight and a cumulative score of 65% or more on your 3 graded questions is considered a pass. A cumulative score of 80% or greater is considered a high pass. Indicate clearly the work and the 3 questions that you wish to be graded.**

Problem 1(**Numerical Integration**).
Determine the coefficients $A_0, A_1$, and $A_2$ that make the formula

$$\int_0^2 f(x)dx \approx A_0 f(0) + A_1 f(1) + A_2 f(2)$$

exact for all polynomials of degree 3.

Problem 2(**Unstable Multistep Method**).
Consider the numerical method

$$y_{k+1} = 3y_k - 2y_{k-1} + \frac{h}{2}(f(x_k, y_k) - 3f(x_{k-1}, y_{k-1})), \quad k \geq 1$$

Illustrate with an example of a simple initial value problem that the above scheme is unstable.

Problem 3(**Linear Multistep Methods**).
a) Define linear multistep method (give formula). Give definition of the region of absolute stability.
b) Show that the region of absolute stability for the trapezoidal method is the set of all complex $h\lambda$ with Real($\lambda$)$< 0$.

Problem 4( **Heat Equation and Stability of the Scheme**).
Consider the implicit in time, Backward Euler method for the solution of the heat equation:

$$\frac{u_m^{n+1} - u_m^n}{\Delta t} - 10\frac{u_{m+1}^{n+1} - 2u_m^{n+1} + u_{m-1}^{n+1}}{h^2} = f_m^{n+1},$$

$$u_m^0 = g_m, \quad m = 0, \pm 1, \pm 2, ..., \quad n = 0, 1, ..., [T/\Delta t] - 1,$$

and investigate the stability of the scheme using the von Neumann analysis.