UNIVERSITY OF UTAH DEPARTMENT OF MATHEMATICS
Ph.D. Preliminary Examination in Differentiable Manifolds
August, 2021.

Instructions. Answer as many questions as you can. Each question is worth 10 points. For a high pass you need to solve *completely* at least three problems and score at least 30 points. For a pass you need to solve *completely* at least two problems and score at least 25 points.

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1. Identify $\mathcal{M}(2)$, the set of two-by-two matrices, with $\mathbb{R}^4$. Let $SL(2,\mathbb{R}) \subset \mathcal{M}(2)$ be the set of matrices with determinant one. Show that $SL(2,\mathbb{R})$ is a smooth submanifold and calculate the tangent space $T_{id}SL(2,\mathbb{R})$ as a subspace of $T_{id}\mathcal{M}(2) = \mathbb{R}^4$.

2. Let $N = \{(x,y,z,w) \in \mathbb{R}^4 : x^2 + y^2 = z^2 + w^2 = 1\}$ and $f(p,q,r) : \mathbb{R}^3 \to \mathbb{R}^4$ be given by $x = p+r$, $y = q-r$, $z = p-r$, $w = p+q$. Show that $f^{-1}(N)$ an embedded submanifold of $\mathbb{R}^3$. What is its dimension?

3. Let M be a smooth manifold and V a vector field on M such that flow for V is defined for all time. Let W be another vector field on M such that V and W agree outside of a compact set in M. Show that W is defined for all time.

4. Let X and Y be closed sub-manifolds of $\mathbb{R}^n$. Show that for almost all $a \in \mathbb{R}^n$ the translate $X+a = \{X+a | x \in X\}$ intersects Y transversally.

5. Let M be a closed, compact manifold and show that if ω is a nowhere zero 1-form on M then the de Rham cohomology group $H^1_{dR}(M)$ is non-trivial. Use this to show that if there is a submersion from M to the circle then $H^1_{dR}(M)$ is non-trivial.

6. Let $\phi\colon G \to H$ be a Lie group homomorphism between Lie groups G and H. Let X and Y be ϕ-related vectors fields on G and H, respectively, and assume that Y is left invariant. Is X left invariant? Either prove this or provide a counterexample.