University of Utah, Department of Mathematics
Algebra 2 Qualifying Exam
August 2021

There are five problems on this exam. You may attempt as many problems as you wish; two correct solutions count as a *pass*, and three correct as a *high pass*. Show all of your work and provide reasonable justification for your answers.

1. Find a degree 5 monic polynomial with integer coefficients that is irreducible in $\mathbb{Q}[x]$ but does not satisfy the Eisenstein criterion for any prime p.

2. Find a non-abelian finite subgroup of $\mathrm{GL}_2(\mathbb{C})$ that is not conjugate to a finite subgroup of $\mathrm{GL}_2(\mathbb{Z})$. Here for a ring R, $\mathrm{GL}_2(R)$ denotes the group of invertible 2×2 matrices with entries in R.

3. Find a complex number η such that $\mathbb{Q}(\eta)/\mathbb{Q}$ is Galois with Galois group isomorphic to the dihedral group of order 8.

4. For p a prime number, show that $x^{p^n}-x \in \mathbb{F}_p[x]$ is the product of all irreducible polynomials of degree dividing n in $\mathbb{F}_p[x]$.

5. Is every group of order 480 solvable?