Math 3010 Exam, Spring 2016

Write up your solutions neatly on a separate sheet of paper. Show your work, but also put a box around your final answer for each question.

(1) (a) (5pts) Use Euclid's algorithm to compute the greatest common divisor of 369 and 120. (If you compute the greatest common divisor by another method, you will not receive full credit.)

(b) (5pts) Find an integer n such that $120n \equiv 3 \pmod{369}$.

(2) (a) (3pts) What does it mean for there to be a *bijection* between two sets A and B? Give a precise definition.

(b) (3pts) If A is a set with 100 elements, exhibit a bijection between the following two sets:

$$\{\text{subsets of } A \text{ having 80 elements}\} \overset{???}{\longleftrightarrow} \{\text{subsets of A having 20 elements}\}$$

(c) (3pts) State a version of the Epimenides/Liar Paradox (or, if you prefer, of Russell's Paradox).

(d) (6pts) Exhibit a bijection between the positive integers and the positive rational numbers.

(3) (a) (3pts) List (any) three of the five regular convex polyhedra ("Platonic solids").

(b) (9pts) For each of the three polyhedra listed in part (a), give the number of vertices (V), edges (E), and faces (F).

(c) (3pts) What is Euler's formula for $V - E + F$ (for any regular convex polyhedron)?

(4) Consider the cubic equation $x^3 + d = bx^2$, where b and d are positive real numbers.

(a) (5pts) Show that the solution(s) to this cubic are given by intersecting the two conics with equations $xy = d$ and $y^2 + dx = db$.

(b) (5pts) When $b = d = 1$, how many (real) points of intersection do the conics have? Graph the two conics on the same set of axes.

(c) (5pts) Still when $b = d = 1$, graph the function $f(x) = x^3 - bx^2 + d$. In what regions is $f(x)$ increasing/decreasing? Make sure your graph reflects this.

(d) (Bonus, 5pts) Express by radicals a real root of $x^3 + 1 = x^2$.