Math 544 § 1.	**First Midterm Exam**	Name:________________
Treibergs		Oct. 26, 1994

This is a closed book test. No books, papers, calculators.
There are [60] total points.

1.______	/15
2.______	/15
3.______	/15
5.______	/15
Total______	/60

1. Solve the equation $2yu_x + u_y = 1$ with the condition $u(x, 0) = e^x$.

2. Show that disturbances propagate at finite speed in the wave equation. Let $u(x, t) \in \mathcal{C}^2\big((-\infty, \infty) \times [0, \infty)\big)$ be a solution of

$$\begin{aligned} &\text{(P.D.E.)} \qquad u_{tt} - c^2 u_{xx} = 0, && \text{for } -\infty < x < \infty,\ 0 \le y; \\ &\text{(I. C.)} \qquad u(x,0) = f(x), \qquad u_t(x,0) = g(x), && \text{for } -\infty < x < \infty; \end{aligned}$$

Find the solution to this problem. Assuming $f(x) = g(x) = 0$ if $|x| > 1$, show that $u(x, t) = 0$ for $|x| > 1 + ct$.

3. Let c, h be positive constants and $f(x)$ and $g(x)$ be $\mathcal{C}^2$ functions of compact support. Consider the initial value problem for the telegrapher's equation

$$\begin{aligned} &\text{(P.D.E.)} \qquad u_{tt} = c^2 u_{xx} - hu, && \text{for } -\infty < x < \infty,\ 0 \le t; \\ &\text{(I. C.)} \qquad u(x,0) = f(x), \qquad u_t(x,0) = g(x), && \text{for } -\infty < x < \infty. \end{aligned}$$

Show that a solution $u(x, t) \in \mathcal{C}^2\big((-\infty, \infty) \times [0, \infty)\big)$ is unique. Hint: Consider the function

$$E(t) = \int_{-\infty}^{\infty} u_t^2(s,t) + c^2 u_x^2(s,t) + hu^2(s,t)\, ds.$$

4. Consider the problem of finding $u(x, t) \in \mathcal{C}^2\big((-\infty, \infty) \times [0, \infty)\big)$ so that

$$\begin{aligned} &\text{(P.D.E.)} \qquad u_{xx} + u_{xy} = 0, && \text{for } -\infty < x < \infty,\ 0 \le y; \\ &\text{(I. C.)} \qquad u(x,0) = f(x), \qquad u_y(x,0) = g(x), && \text{for } -\infty < x < \infty. \end{aligned}$$

   (a) What type is this partial differential equation?
   (b) Find a change of variables to reduce this equation to normal form.
   (c) State what it means for this problem to be well posed.
   (d) Determine whether this problem is well posed. Assume that $f(x) \in \mathcal{C}^2$ and $g(x) \in \mathcal{C}^1$ but are otherwise arbitrary functions.