

# Quiz 3

## Introduction to partial differential equations (5440)

Name and Unid: \_\_\_\_\_

1. Exercise 1 (properties of functions)

1) For each of the following functions, state whether it is even or odd or periodic. If periodic, what is the smallest period:

- $\sinh(x^2)$

- $\sin(3x)$

- $\arctan(x)$

2) Find the even part  $P_e$  and the odd part  $P_o$  of the polynomial:

$$P(x) = x^2 - x + 1.$$

2. Exercise 2 (convergence of a series of function) Let  $f_n$  be the function

$$f_n(x) = \left(\frac{x}{3}\right)^n, \quad \forall x \in [-3, 3], \quad \forall n \in \mathbb{N}.$$

a) Compute  $\sum_{n=0}^N f_n(x)$  on  $(-3, 3)$ .

b) Does the series  $\sum_{n=0}^{\infty} f_n$  converge pointwise on  $(-3, 3)$ ?

c) Does the series  $\sum_{n=0}^{\infty} f_n$  converge uniformly on  $[-3, 3]$ , on  $[-1, 1]$ ?

d) Does the series  $\sum_{n=0}^{\infty} f_n$  converge in the  $L^2$  sense on  $(-1, 1)$ ?

3. Exercise 3 (Fourier Series) Let  $f(x) = \exp(|x|)$  on  $[-1, 1]$  and its full Fourier be defined by Does the full Fourier series

$$\frac{A_0}{2} + \sum_{n=1}^{\infty} (A_n \cos(n\pi x) + B_n \sin(n\pi x))$$

with

$$A_n = \int_{-1}^1 f(x) \cos(n\pi x) dx \quad \forall n \in \mathbb{N} \quad \text{and} \quad B_n = \int_{-1}^1 f(x) \sin(n\pi x) dx \quad \forall n \in \mathbb{N}^*.$$

- a) What is the value of the  $B_n$  coefficients  $\forall n \in \mathbb{N}^*$ ?
- b) (bonus question) Does the full Fourier series of  $f$  converge uniformly to  $f$  on  $[-1, 1]$ ?  
pointwise to  $f$  on  $[-1, 1]$ ? in the  $L^2$  sense to  $f$  on  $(-1, 1)$ ?