Math 521 § 1.	**First Midterm Exam**	Name:____________________
Treibergs		Nov. 1, 1995

This is a closed book exam No books, papers, calculators, tablets, laptops, phones or other messaging devices are permitted. Give complete solutions. Be clear about your logic and definitions and justify any theorems that you use. There are [60] total points.

1.	/12
2.	/12
3.	/12
4.	/12
5.	/12
Total	/60

1. [12] Let $S = \{(a_i) : a_i \in \mathbb{Z} \text{ for all } i \in \mathbb{N}.\}$ be the set of all integer sequences . Show that S is not countable.

2. [12] Suppose A, B are nonempty subsets of $\mathbb{R}$ such that $A \cap B = \emptyset$, $A \cup B = \mathbb{R}$ and if whenever $a \in A$ and $b \in B$, then $a < b$. Show that there is a real number $x \in \mathbb{R}$ such that every element $a \in A$ satisfies $a \le x$, and every element $b \in B$ satisfies $b \ge x$.

3. [12] Suppose (a_i) is a Cauchy sequences of rational numbers ($a_i \in \mathbb{Q}$ for all $i \in \mathbb{N}$). Assume that (a_i) has infinitely many positive terms and infinitely many negative terms. Show that
$$0 = \lim_{n\to\infty} a_n.$$

4. (a) [4] Let F be a field. Define what is meant by an ordering on F ?
   (b) [8] Let
   $$\mathcal{C} = \{(a_i) : a_i \in \mathbb{Q} \text{ for all } i \in \mathbb{N} \text{ and } (a_i) \text{ is Cauchy } \}$$
   be the set of Cauchy sequences of rational numbers and
   $$\mathcal{N} = \{(a_i) : a_i \in \mathbb{Q} \text{ for all } i \in \mathbb{N}, a_i \to 0 \text{ as } i \to \infty\}$$
   be the set of null rational sequences. Consider the quotient space $\mathcal{R} = \mathcal{C}/\mathcal{N}$ where the Cauchy sequences (a_i) and (b_i) are equivalent if $(a_i - b_i) \in \mathcal{N}$. Assuming only that $\mathcal{R}$ is a field, explain how an ordering is placed on $\mathcal{R}$. (Prove that your condition is well defined. You do not have to prove the properties of an ordering.)

5. [12] Suppose that V, W are real vector spaces such that the dimension of V is a smaller cardinal number than the dimension of W. Show that there exists a one-to-one linear transformation $T : V \to W$.