Homework for Math 5210 - 002, Spring 2025

A. Treibergs, Instructor

January 16, 2025

Our text is by N. L. Carothers, *Real Analysis,* Cambridge University Press, Cambridge (2000). Please read the relevant sections in the text as well as any cited references. Assignments are due the following Friday, or on April 30, whichever comes first.

Your written work reflects your professionalism. Please copy or paraphrase each question. Make answers complete and self contained, written in good technical English. This means that you should write in complete sentences, provide adequate explanation to help the reader understand the structure of your argument, be thorough in the details, state any theorem that you use and proofread your answer. Homework from Wednesday to Tuesday will be due Friday. Late homework that is up to one week late will receive half credit. Homework that is more than one week late will receive no credit at all. Homework that is placed in my mailbox in JWB 228 before 4 pm Friday afternoon will be considered to be on time.

Please hand in problems A1 – A3 on Friday, January 10.

A1. Equivalence. Please hand in these exercises from Carothers' *Real Analysis.*

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A2. Is the set of all finite subsets of $\mathbb{N}$ countable or uncountable? Give a proof of your assertion. [Strichartz, p. 13]

A3. Let $A_1, A_2, A_3, \ldots$ be countably infinite sets, and let their Cartesian product $A_1 \times A_2 \times A_3 \times \cdots$ be defined as the set of all sequences $(a_1, a_2, a_3, \ldots)$ where a_k is an element of A_k, Prove that the Cartesian product is uncountable. Show that the same conclusion holds if each of the sets $A_1, A_2, \ldots$ has at least two elements. [Strichartz, p. 13]

Please hand in problems B1 on Friday, January 17.

B1. Cantor Set. Please hand in these exercises from Carothers' *Real Analysis.*

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Chapter 2 Problems 18, 21, 26, 34.
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Please hand in problems C1–C4 on Friday, January 24.

C1. Associativity of Reals. Write out the proof of the associative law of addition for the real numbers. [Strichartz, p.48.]

C2. Increasing Representative. Let x be a real number. Show that there is a Cauchy sequence of rationals (b_n) representing x such that $b_n < b_{n+1}$ for every n. [Strichartz, p.49.]

C3. Density of Rationals. Prove that there are infinitely many rational numbers between any two distinct real numbers. [Haaser & Sullivan, p.34.]

C4. Order in Reals. Show that if a real number x can be represented by a Cauchy sequence of positive rationals, then $x \geq 0$. [Strichartz, p.49.]