5010 solutions, Assignment 6. Chapter 4: 18, 25, 27, 29.

18. $E[X^{-1}] = \sum_1^\infty k^{-1}q^{k-1}p = (p/q)\sum_1^\infty k^{-1}q^k = -(p/q)\ln p$ from 5(iii). The book gives $(q/p)\ln p$, which is certainly wrong because it is negative.

25. Let N be the number of eggs and X be the number that develop. Then $P(X=k) = \sum_{n\ge k} P(N=n)P(X=k \mid N=n) = \sum_{n\ge k}(e^{-\mu}\mu^n/n!)\binom{n}{k}p^k(1-p)^{n-k} = (e^{-\mu}(\mu p)^k/k!)\sum_{n\ge k}(\mu(1-p))^{n-k}/(n-k)! = (e^{-\mu}(\mu p)^k/k!)e^{\mu(1-p)} = e^{-\mu p}(\mu p)^k/k!$, which is Poisson$(\mu p)$.

27. (a) nth tomato is kth defective is a negative binomial probability, namely $\binom{n-1}{k-1}p^k(1-p)^{n-k}$.

(b) Same as (a) with pr in place of p.

(c) $P(X=k) = P(k$ defectives among first $n \mid n+1$ is first rejected$)$. Write this as the ratio of two probabilities. The numerator probability is $P(k$ defectives among first n, no rejected among first n, and $n+1$ is rejected$) = \binom{n}{0,k,n-k}(pr)^0(p(1-r))^k(1-p)^{n-k}pr$. Denominator probability is geometric, $(1-pr)^n pr$. Result is

$$\binom{n}{k}\left(\frac{p(1-r)}{1-pr}\right)^k\left(\frac{1-p}{1-pr}\right)^{n-k},$$

which is binomial$(n, p(1-r)/(1-pr))$.

29. (a) p^n.

(b) $p_n = p^{n-1}(1-p)$.

(c) Mean of a geometric: $1/(1-p)$.