Math 3080 § 1.	**Hardness Example: Tukey's**	Name: Example
Treibergs	**Multiple Comparisons**	January 25, 2016

This **R**© program explores Tukey's Honest Significant Differences test. This data was taken from "An investigation of the CaCO3-CaF2-K2SiO3-SiO2-Fe Flux System Using the Submerged Arc Welding Process on HSLA-100 and AISI-1081 Steels" by G. Fredrickson, Thesis, Coloroda School of Mines 1992, as quoted by Navidi, Statistics for Engineers and Scientists, 2nd ed., 2008. Five welds are made for each of four fluxes. Brinell Hardness is measured

The Multiple Comparisons is an *aposteriori* test. Once the one-way randomized fixed factor ANOVA rejects the null hypothesis that the means are equal $\mu_1 = \cdots = \mu_I$, it can be used to determine which pairs are significantly different. Thus it simultaneously tests $\mathcal{H}_0$: $\mu_{i_1} = \mu_{i_2}$ versus the alternative $\mathcal{H}_a$: $\mu_{i_1} \neq \mu_{i_2}$ for all pairs $1 \leq i_1 < i_2 \leq I$. It uses Tukey's Studentized Range distribution, that accounts for the maximum differences of I normal variables. One cannot apply a paired t-test at the alpha level because for every pair because with $m = \binom{I}{2}$ pairs to test, it is unlikely that all of them will not have a type-one error.

The means are significantly different if zero is not within the confidence interval. The print or plot of the CI's for differences shows that treatments C and A are significantly different, whereas A, D and B are not significanly different from one another, and D, B and C are not significantly different from one another. The bar pattern for this is

```
A     D     B     C
-------------
      -------------
```

Data Set Used in this Analysis :

```
# Math 3080-1             Weld Hardness Data             Spring 2016
# Treibergs
#
# From "An investigation of the CaCO3-CaF2-K2SiO3-SiO2-Fe Flux System Using
# the Submerged Arc Welding Process on HSLA-100 and AISI-1081 Steels" by
# G. Fredrickson, Thesis, Coloroda School of Mines 1992, as quoted by
# Navidi, Statistics for Engineers and Scientists, 2nd ed., 2008.
#
# Five welds are made using four fluxes. Brinell Hardness is measured
#
"Flux" "Hardness"
A 250
A 264
A 256
A 260
A 239
B 263
B 254
B 267
B 265
B 267
C 257
C 279
C 269
C 273
C 277
```

```
D 253
D 258
D 262
D 264
D 273
```

R Session:

```
R version 2.13.1 (2011-07-08)
Copyright (C) 2011 The R Foundation for Statistical Computing
ISBN 3-900051-07-0
Platform: i386-apple-darwin9.8.0/i386 (32-bit)

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  Natural language support but running in an English locale

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'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

[R.app GUI 1.41 (5874) i386-apple-darwin9.8.0]

[Workspace restored from /Users/andrejstreibergs/.RData]
[History restored from /Users/andrejstreibergs/.Rapp.history]

> tt=read.table("M3083HardnessData.txt",header=T)
> tt
   Flux Hardness
1     A      250
2     A      264
3     A      256
4     A      260
5     A      239
6     B      263
7     B      254
8     B      267
9     B      265
10    B      267
11    C      257
12    C      279
13    C      269
14    C      273
15    C      277
16    D      253
17    D      258
```
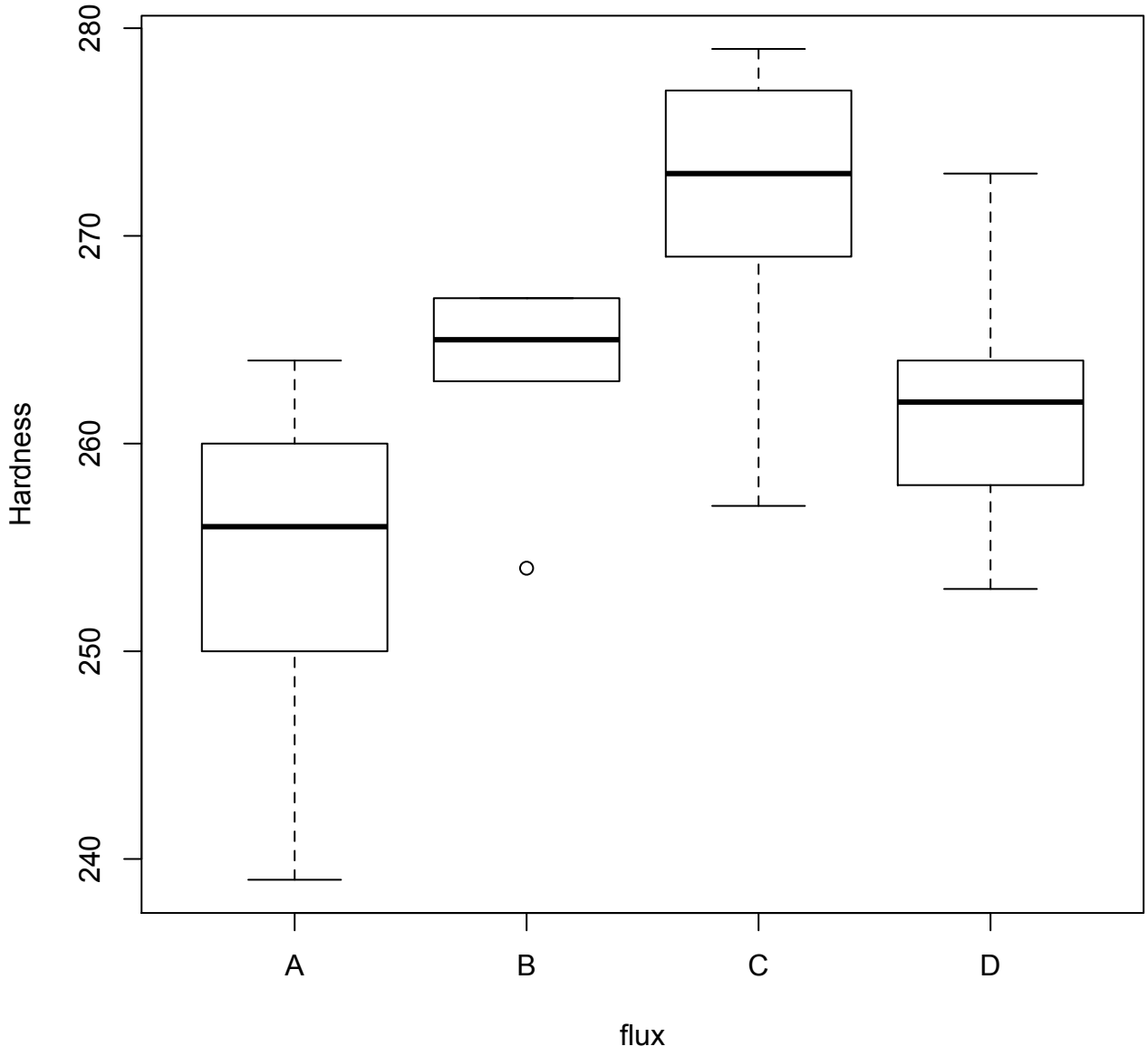
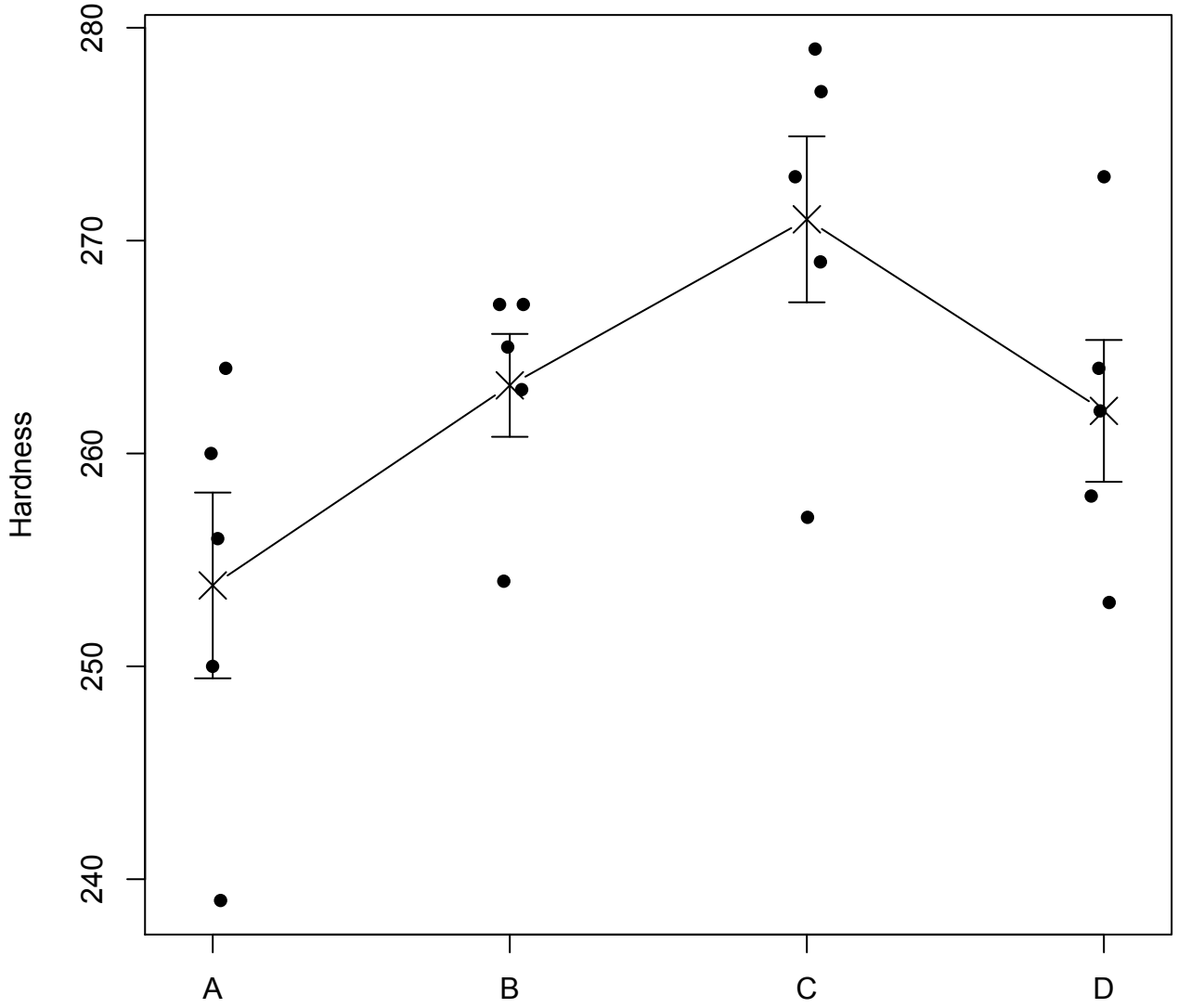
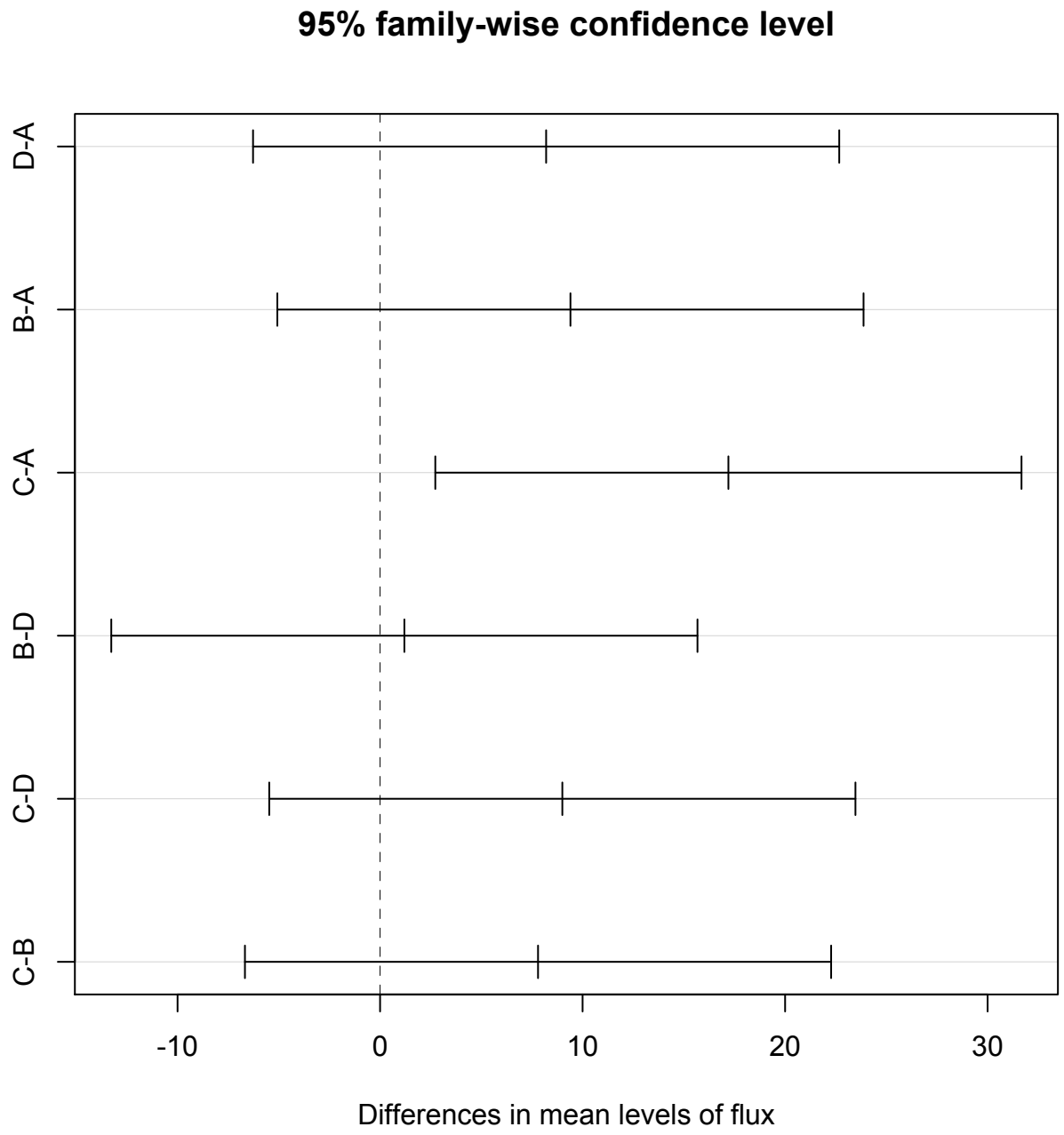

```
18    D      262
19    D      264
20    D      273
> attach(tt)
> summary(tt)
 Flux     Hardness
 A:5   Min.   :239.0
 B:5   1st Qu.:256.8
 C:5   Median :263.5
 D:5   Mean   :262.5
       3rd Qu.:267.5
       Max.   :279.0
> flux=ordered(Flux)
>
> ####### SIDE-BY-SIDE BOX PLOTS FOR VARIOUS TREATMENTS

> plot(Hardness~flux)
> # M3083Hardness1.pdf
>
> ########  COMPUTE MEANS, SD, J  FOR EACH FACTOR LEVEL
>
> xbar=tapply(Hardness,flux,mean)
> s=tapply(Hardness,flux,sd)
> n=tapply(Hardness,flux,length)
> sem=s/sqrt(n)
> n
A B C D
5 5 5 5
> xbar
    A     B     C     D
253.8 263.2 271.0 262.0
> s
       A        B        C        D
9.757049 5.403702 8.717798 7.449832
> v=tapply(Hardness,flux,var);v
   A    B    C    D
95.2 29.2 76.0 55.5
> MSE=mean(v);MSE
[1] 63.975
> I=4;J=5;
> MSTr=J*var(xbar);MSTr
[1] 247.8
>
> ###### ALTERNATIVE PLOT SHOWING OBSERVED POINTS AND 1 S.D.
> ###### FROM P. DALGAARD, "INTRODUCTORY STATISTICS WITH R," 2008.
>
> stripchart(Hardness~flux,method="jitter",jitter=.05,pch=16,vert=T)
> arrows(1:4,xbar+sem,1:4,xbar-sem,angle=90,code=3,length=.1)
> lines(1:4,xbar,pch=4,type="b",cex=2)
>
```

```
> ####### ONE-WAY FIXED EFFECTS ANOVA TO SEE IF MEANS DIFFER
>
> a1=aov(Hardness~flux);summary(a1)
            Df Sum Sq Mean Sq F value  Pr(>F)
flux         3  743.4 247.800  3.8734 0.02944 *
Residuals   16 1023.6  63.975
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

> print(a1)
Call:
   aov(formula = Hardness ~ flux)

Terms:
                   flux Residuals
Sum of Squares   743.4    1023.6
Deg. of Freedom      3        16

Residual standard error: 7.998437
Estimated effects are balanced

>
> ############  MULTIPLE COMPARISONS USING  TukeyHSD
>
> a2=TukeyHSD(a1,ordered=T)

> print(a2)
  Tukey multiple comparisons of means
    95% family-wise confidence level
    factor levels have been ordered

Fit: aov(formula = Hardness ~ flux)

$flux
    diff        lwr      upr     p adj
D-A  8.2  -6.272915 22.67291 0.3953011
B-A  9.4  -5.072915 23.87291 0.2839920
C-A 17.2   2.727085 31.67291 0.0172933
B-D  1.2 -13.272915 15.67291 0.9951084
C-D  9.0  -5.472915 23.47291 0.3185074
C-B  7.8  -6.672915 22.27291 0.4372295

> plot(a2)
>
```

```
> ########## COMPUTE MULTIPLE COMPARISONS "BY HAND"
>
> ########## TABLE A10 GIVES  Q(alpha,I,J(J-1))
> # Studentized range for alpha=.05, nu1=5, nu2=I(J-1)=16  is
> Q=4.33
>
> ######### CANNED STUDENTIZED RANGE
> Q=qtukey(.95,4,16);Q
[1] 4.046093

> sort(xbar)
    A     D     B     C
253.8 262.0 263.2 271.0

> "D-A";c(xbar[4]-xbar[1],xbar[4]-xbar[1]-Q*sqrt(MSE/J),xbar[4]-xbar[1]+Q*sqrt(MSE/J))
[1] "D-A"
        D         D         D
 8.200000 -6.272915 22.672915
> "B-A";c(xbar[2]-xbar[1],xbar[2]-xbar[1]-Q*sqrt(MSE/J),xbar[2]-xbar[1]+Q*sqrt(MSE/J))
[1] "B-A"
        B         B         B
 9.400000 -5.072915 23.872915
> "C-A";c(xbar[3]-xbar[1],xbar[3]-xbar[1]-Q*sqrt(MSE/J),xbar[3]-xbar[1]+Q*sqrt(MSE/J))
[1] "C-A"
        C         C         C
17.200000  2.727085 31.672915
> "B-D";c(xbar[2]-xbar[4],xbar[2]-xbar[4]-Q*sqrt(MSE/J),xbar[2]-xbar[4]+Q*sqrt(MSE/J))
[1] "B-D"
        B         B         B
  1.20000 -13.27291  15.67291
> "C-D";c(xbar[3]-xbar[4],xbar[3]-xbar[4]-Q*sqrt(MSE/J),xbar[3]-xbar[4]+Q*sqrt(MSE/J))
[1] "C-D"
        C         C         C
 9.000000 -5.472915 23.472915
> "C-B";c(xbar[3]-xbar[2],xbar[3]-xbar[2]-Q*sqrt(MSE/J),xbar[3]-xbar[2]+Q*sqrt(MSE/J))
[1] "C-B"
        C         C         C
 7.800000 -6.672915 22.272915
```


Hardness
280
270
260
250
240
A
B
C
D
flux


Hardness
280
270
260
250
240
A
B
C
D


95% family-wise confidence level
D-A
B-A
C-A
B-D
C-D
C-B
-10
0
10
20
30
Differences in mean levels of flux