Math 3080 § 1.	**Air Bag Example:**	Name: Example
Treibergs	**Three-Factor Fixed Effects ANOVA**	Jan. 22, 2014

Today's example was motivated from problem 11.56 of Devore's *Probability and Statistics for Engineering and the Sciences, 8th ed.*, Brooks Cole 2012. It is a balanced three fixed factor model with replication. Thus we may estimate two way and three way interactions.

The diagnostic plots show a problem with data points 3 and 4. These points are inconsistent with the rest of the data and may be due to clerical error or experimental error.The experimentalist will need to know and perhaps order further tests for that cell. The QQ-normal plot and residuals vs fitted are swamped by these two points. Dealing with them requires a deeper consideration which is not given here. Analyzing the residuals after removing those two points shows that it is plausible that assumptions hold. The plot of residuals vs. fitted values shows that the variances for the three treatments are similar. The QQ-normal plot of the standardized residuals follows the 45° line nicely indicating that there is no evidence that normality assumption is violated.

Data Set Used in this Analysis :

```
# M3082     Air Bag Data       2-2-14
# Treibergs
#
# From 1996 article in Journal of the Textile Institute   as quoted by
# Devore   Probability and Statistics for Engineering and the Sciences
# 5th ed., Duxbury (Brooks/Cole)   2000. Permeability of airbag material is
# measured for three factors at three levels.
# A (Temperature) 8C   50C   75C
# B (Fabric denier) 420D   630D   840D
# C (Air Pressure) 17.2kPa 34.4kPa 103.4kPa
#
# Analyze as fixed factor experiment
"Response"  "Pressure"  "Temp"  "Fabric"
73  "Pressure17.2"  "8 degrees"  "420-D"
80  "Pressure17.2"  "8 degrees"  "420-D"
35  "Pressure17.2"  "8 degrees"  "630-D"
433  "Pressure17.2"  "8 degrees"  "630-D"
125  "Pressure17.2"  "8 degrees"  "840-D"
111  "Pressure17.2"  "8 degrees"  "840-D"
157  "Pressure34.4"  "8 degrees"  "420-D"
155  "Pressure34.4"  "8 degrees"  "420-D"
91  "Pressure34.4"  "8 degrees"  "630-D"
98  "Pressure34.4"  "8 degrees"  "630-D"
234  "Pressure34.4"  "8 degrees"  "840-D"
233  "Pressure34.4"  "8 degrees"  "840-D"
332  "Pressure 103.4"  "8 degrees"  "420-D"
322  "Pressure 103.4"  "8 degrees"  "420-D"
288  "Pressure 103.4"  "8 degrees"  "630-D"
271  "Pressure 103.4"  "8 degrees"  "630-D"
477  "Pressure 103.4"  "8 degrees"  "840-D"
464  "Pressure 103.4"  "8 degrees"  "840-D"
52  "Pressure17.2"  "50 degrees"  "420-D"
51  "Pressure17.2"  "50 degrees"  "420-D"
16  "Pressure17.2"  "50 degrees"  "630-D"
```

```
12  "Pressure17.2"  "50 degrees"  "630-D"
96  "Pressure17.2"  "50 degrees"  "840-D"
100  "Pressure17.2"  "50 degrees"  "840-D"
125  "Pressure34.4"  "50 degrees"  "420-D"
118  "Pressure34.4"  "50 degrees"  "420-D"
72  "Pressure34.4"  "50 degrees"  "630-D"
78  "Pressure34.4"  "50 degrees"  "630-D"
149  "Pressure34.4"  "50 degrees"  "840-D"
155  "Pressure34.4"  "50 degrees"  "840-D"
281  "Pressure 103.4"  "50 degrees"  "420-D"
264  "Pressure 103.4"  "50 degrees"  "420-D"
169  "Pressure 103.4"  "50 degrees"  "630-D"
173  "Pressure 103.4"  "50 degrees"  "630-D"
338  "Pressure 103.4"  "50 degrees"  "840-D"
350  "Pressure 103.4"  "50 degrees"  "840-D"
37  "Pressure17.2"  "75 degrees"  "420-D"
31  "Pressure17.2"  "75 degrees"  "420-D"
30  "Pressure17.2"  "75 degrees"  "630-D"
41  "Pressure17.2"  "75 degrees"  "630-D"
102  "Pressure17.2"  "75 degrees"  "840-D"
98  "Pressure17.2"  "75 degrees"  "840-D"
95  "Pressure34.4"  "75 degrees"  "420-D"
106  "Pressure34.4"  "75 degrees"  "420-D"
91  "Pressure34.4"  "75 degrees"  "630-D"
100  "Pressure34.4"  "75 degrees"  "630-D"
170  "Pressure34.4"  "75 degrees"  "840-D"
160  "Pressure34.4"  "75 degrees"  "840-D"
276  "Pressure 103.4"  "75 degrees"  "420-D"
281  "Pressure 103.4"  "75 degrees"  "420-D"
213  "Pressure 103.4"  "75 degrees"  "630-D"
211  "Pressure 103.4"  "75 degrees"  "630-D"
307  "Pressure 103.4"  "75 degrees"  "840-D"
311  "Pressure 103.4"  "75 degrees"  "840-D"
```

R Session:

```
R version 2.14.0 (2011-10-31)
Copyright (C) 2011 The R Foundation for Statistical Computing
ISBN 3-900051-07-0
Platform: i386-apple-darwin9.8.0/i386 (32-bit)

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  Natural language support but running in an English locale

R is a collaborative project with many contributors.
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'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or
```
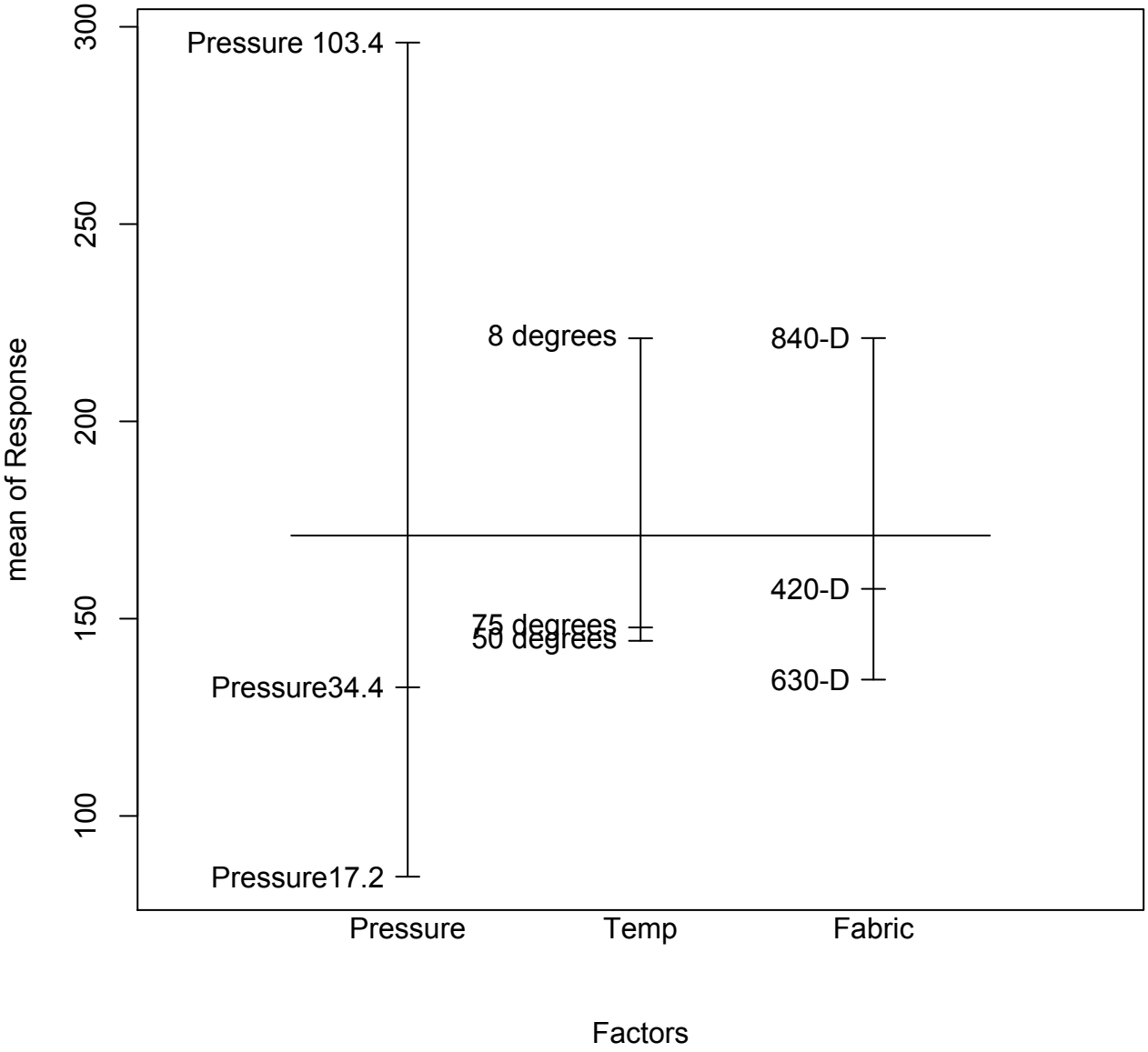
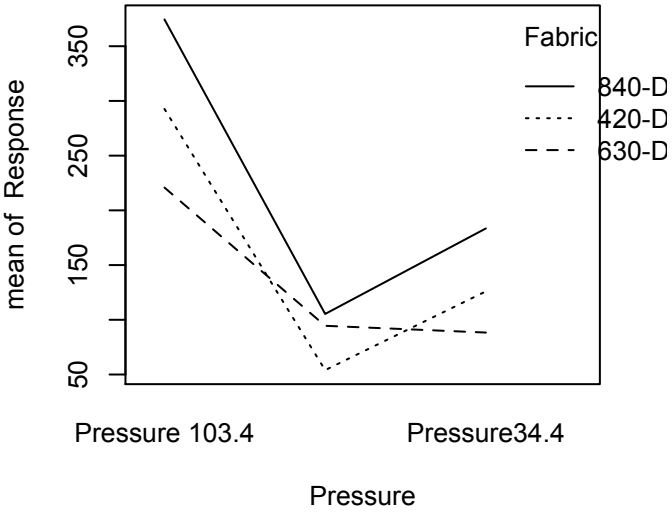

```
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

[R.app GUI 1.42 (5933) i386-apple-darwin9.8.0]

[Workspace restored from /home/1004/ma/treibergs/.RData]
[History restored from /home/1004/ma/treibergs/.Rhistory]

> tt=read.table("M3082DataAirBag.txt",header=T)
> attach(tt)
>
> ######################## ANOVA #######################################
> a1=aov(Response~Pressure*Temp*Fabric)
> summary(a1)
                    Df Sum Sq Mean Sq F value   Pr(>F)
Pressure             2 442111  221056  74.428 1.03e-11 ***
Temp                 2  67553   33777  11.372 0.000261 ***
Fabric               2  72361   36181  12.182 0.000170 ***
Pressure:Temp        4   6213    1553   0.523 0.719657
Pressure:Fabric      4  34928    8732   2.940 0.038677 *
Temp:Fabric          4   9696    2424   0.816 0.526111
Pressure:Temp:Fabric 8  33487    4186   1.409 0.237470
Residuals           27  80192    2970
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

> ################### DESIGN AND INTERACTION PLOTS ########################
> plot.design(tt)
> interaction.plot(Pressure,Fabric,Response)
> layout(matrix(c(1,3,2,4),ncol=2))
> interaction.plot(Pressure,Fabric,Response)
> interaction.plot(Pressure,Temp,Response)
> interaction.plot(Fabric,Temp,Response)
> interaction.plot(Fabric,Pressure,Response)

> ################## DIAGNOSTIC PLOTS ##########################################
> plot(a1)
> ### OBSERVATIONS 3 AND 4 ARE WAY OFF, PLOT DIAGNOSTICS OMITTING OBS. 3,4 ####
> r=residuals(a1);f=fitted(a1);y=Response
> rs=(r[-4:-3]-mean(r[-4:-3]))/sd(r[-4:-3])
> plot(rs~f[-4:-3],main="Std. Resid. vs. Fitted (Omit 3,4)",ylab="Standardized Residuals")
> qqnorm(rs,main="QQ Normal Plot (Omit 3,4)");abline(0,1,lty=4)
> plot(Response[-4:-3]~f[-4:-3],main="Response vs. Fitted (Omit 3,4)",ylab="Response")
> plot(Response~Pressure)
```

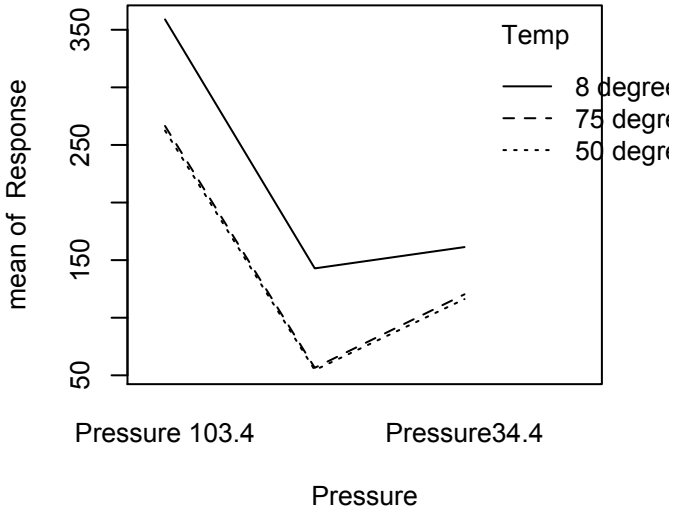
```
> ###################### MODEL OF ESTIMATED EFFECTS AND MEANS #################
> model.tables(a1)
Tables of effects

 Pressure
Pressure
Pressure 103.4   Pressure17.2   Pressure34.4
        124.93         -86.46         -38.46

 Temp
Temp
50 degrees 75 degrees  8 degrees
    -26.69     -23.30      49.98

 Fabric
Fabric
 420-D  630-D  840-D
-13.52 -36.52  50.04

 Pressure:Temp
               Temp
Pressure         50 degrees 75 degrees 8 degrees
  Pressure 103.4  -6.815     -6.204     13.019
  Pressure17.2    -3.426     -4.815      8.241
  Pressure34.4    10.241     11.019    -21.259

 Pressure:Fabric
               Fabric
Pressure         420-D  630-D  840-D
  Pressure 103.4  10.19 -38.65  28.46
  Pressure17.2   -17.09  46.41 -29.31
  Pressure34.4     6.91  -7.76   0.85

 Temp:Fabric
            Fabric
Temp         420-D   630-D   840-D
  50 degrees  17.630 -21.204   3.574
  75 degrees   3.407   3.074  -6.481
  8 degrees  -21.037  18.130   2.907
```

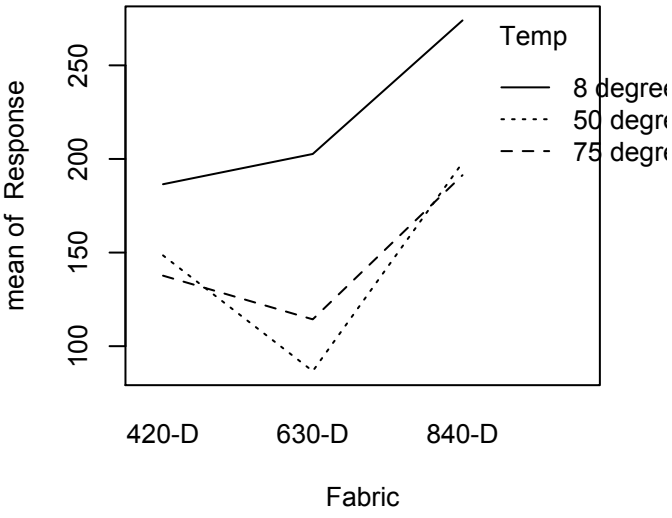
```
 Pressure:Temp:Fabric
, , Fabric = 420-D

               Temp
Pressure         50 degrees 75 degrees 8 degrees
  Pressure 103.4  -4.30      11.93      -7.63
  Pressure17.2     9.98       4.70     -14.69
  Pressure34.4    -5.69     -16.63      22.31

, , Fabric = 630-D

               Temp
Pressure         50 degrees 75 degrees 8 degrees
  Pressure 103.4   4.87      17.59     -22.46
  Pressure17.2   -29.19     -33.96      63.15
  Pressure34.4    24.31      16.37     -40.69

, , Fabric = 840-D

               Temp
Pressure         50 degrees 75 degrees 8 degrees
  Pressure 103.4  -0.57     -29.52      30.09
  Pressure17.2    19.20      29.26     -48.46
  Pressure34.4   -18.63       0.26      18.37

> model.tables(a1,"means")
Tables of means
Grand mean

171.0741

 Pressure
Pressure
Pressure 103.4   Pressure17.2   Pressure34.4
        296.00          84.61         132.61

 Temp
Temp
50 degrees 75 degrees  8 degrees
    144.39     147.78     221.06

 Fabric
Fabric
 420-D  630-D  840-D
157.56 134.56 221.11
```

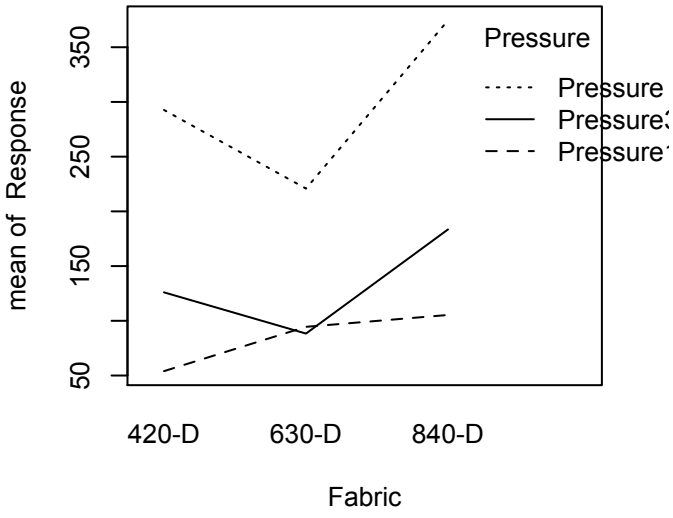
```
 Pressure:Temp
                Temp
Pressure         50 degrees 75 degrees 8 degrees
  Pressure 103.4 262.5      266.5      359.0
  Pressure17.2    54.5       56.5      142.8
  Pressure34.4   116.2      120.3      161.3

 Pressure:Fabric
                Fabric
Pressure         420-D 630-D 840-D
  Pressure 103.4 292.7 220.8 374.5
  Pressure17.2    54.0  94.5 105.3
  Pressure34.4   126.0  88.3 183.5

 Temp:Fabric
            Fabric
Temp         420-D  630-D  840-D
  50 degrees 148.50  86.67 198.00
  75 degrees 137.67 114.33 191.33
  8 degrees  186.50 202.67 274.00

 Pressure:Temp:Fabric
, , Fabric = 420-D

                Temp
Pressure         50 degrees 75 degrees 8 degrees
  Pressure 103.4 272.5      278.5      327.0
  Pressure17.2    51.5       34.0       76.5
  Pressure34.4   121.5      100.5      156.0

, , Fabric = 630-D

                Temp
Pressure         50 degrees 75 degrees 8 degrees
  Pressure 103.4 171.0      212.0      279.5
  Pressure17.2    14.0       35.5      234.0
  Pressure34.4    75.0       95.5       94.5

, , Fabric = 840-D

                Temp
Pressure         50 degrees 75 degrees 8 degrees
  Pressure 103.4 344.0      309.0      470.5
  Pressure17.2    98.0      100.0      118.0
  Pressure34.4   152.0      165.0      233.5
```

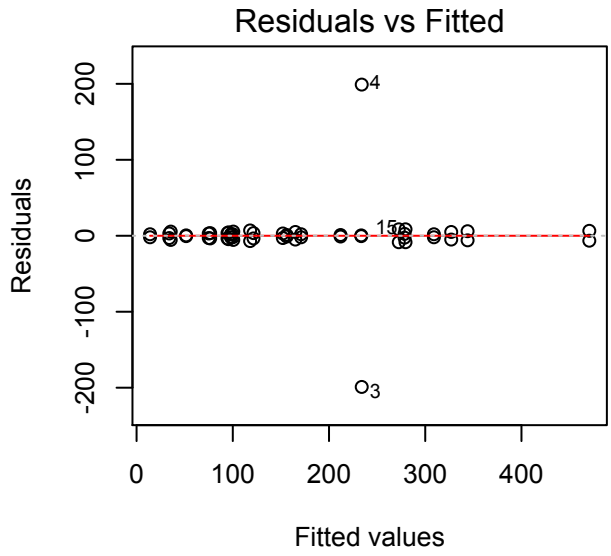

mean of Response
300
250
200
150
100
Pressure 103.4
Pressure34.4
Pressure17.2
8 degrees
75 degrees
50 degrees
840-D
420-D
630-D
Pressure
Temp
Fabric
Factors

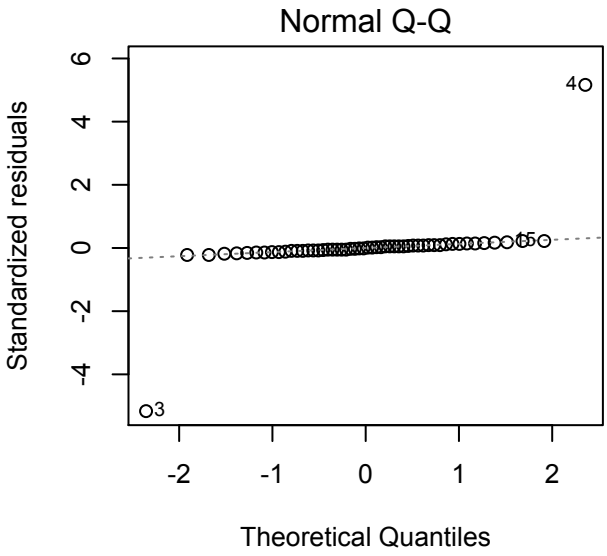

mean of Response
350
250
150
50
Fabric
840-D
420-D
630-D
Pressure 103.4
Pressure34.4
Pressure

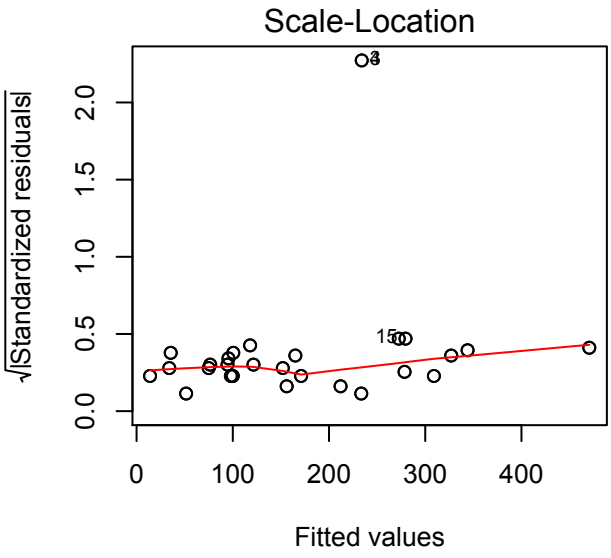

mean of Response
350
250
150
50
Temp
8 degre
75 degr
50 degr
Pressure 103.4
Pressure34.4
Pressure

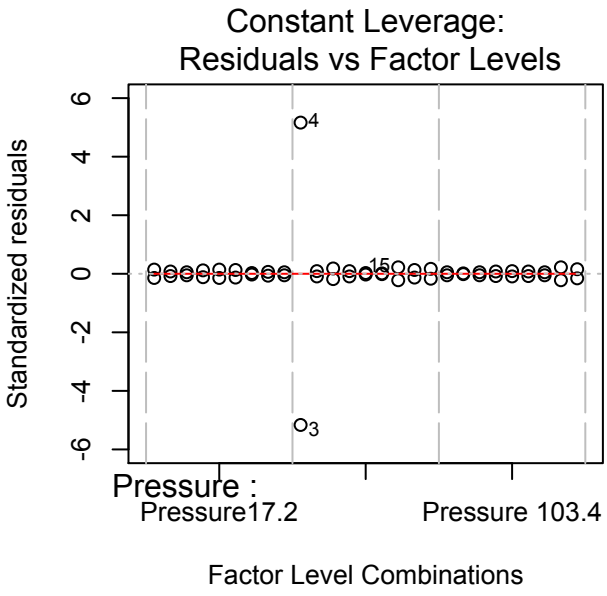

mean of Response
250
200
150
100
Temp
8 degre
50 degr
75 degr
420-D
630-D
840-D
Fabric

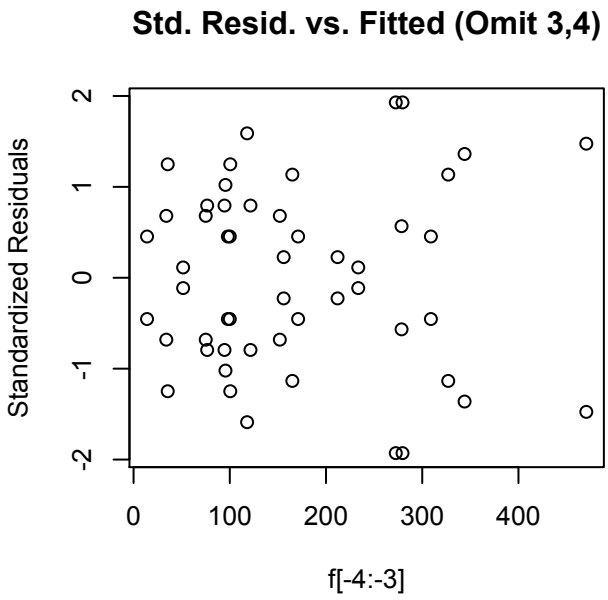

mean of Response
350
250
150
50
Pressure
Pressure
Pressure3
Pressure1
420-D
630-D
840-D
Fabric

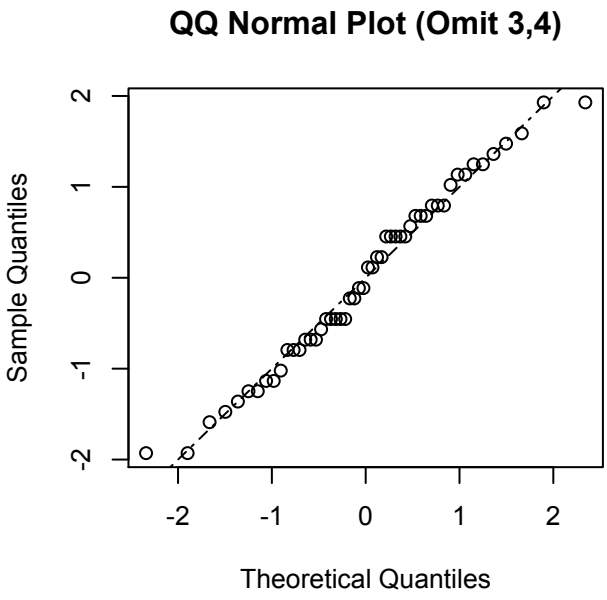

Residuals vs Fitted
Residuals
Fitted values
4
3
15

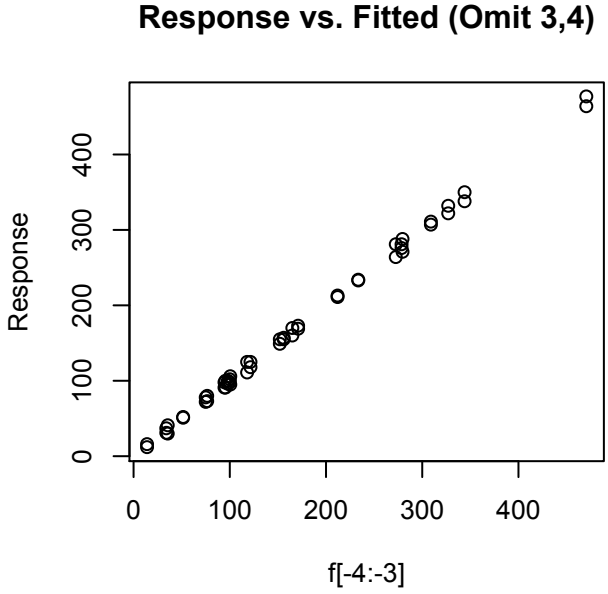

Normal Q-Q
Standardized residuals
Theoretical Quantiles
4
3
15

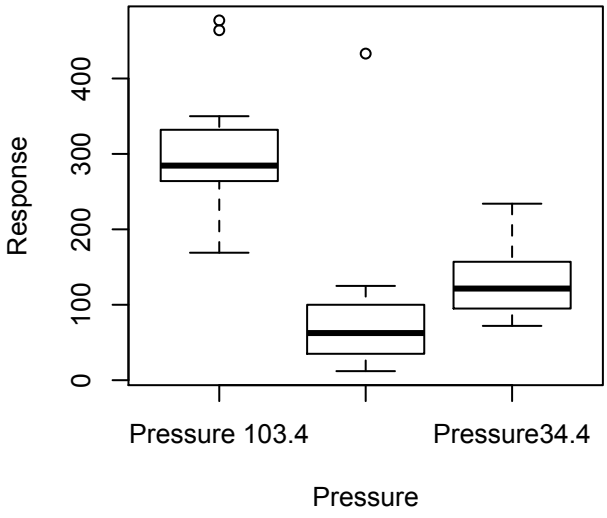

Scale-Location
√|Standardized residuals|
Fitted values
15


Constant Leverage:
Residuals vs Factor Levels
Standardized residuals
Pressure :
Pressure17.2
Pressure 103.4
Factor Level Combinations
4
3
15


Std. Resid. vs. Fitted (Omit 3,4)
Standardized Residuals
f[-4:-3]


QQ Normal Plot (Omit 3,4)
Sample Quantiles
Theoretical Quantiles


Response vs. Fitted (Omit 3,4)
Response
f[-4:-3]


Response
Pressure 103.4
Pressure34.4
Pressure