

Homework for Math 3080 §1, Spring 2010

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Please read the relevant sections in the text *Probability and Statistics for Science, Engineering and Finance*, by Walter A. Rozenkrantz, CRC Press (2009). Make sure that your papers are **self-contained**. You should copy or paraphrase the question, give complete explanation of your solution, state the results that you quote and supply adequate labels to any computer output.

Please hand in problems A1 – A3 on Wednesday, January 20.

A1. Type I and type II errors. (From Math 3070 final given 4-30-01.) Two candidates AA and GW are running for office in a certain state. It is believed that 0.5 of the voters favor AA. Suppose you conduct a poll to test the alternative hypothesis that more than 0.5 of the voters favor AA. In a random sample of 15 voters, let X denote the number who favor AA. Suppose that the rejection region for the null hypothesis is $X \geq 9$.

1. What is the probability of making a Type I error, that is, of rejecting \mathcal{H}_0 even though it is true?
2. What is the probability of a Type II error, that is of accepting \mathcal{H}_0 even though the alternative is true, that actually 0.7 of the voters favor AA?
3. How many voters should you poll in order to be sure that the probability of both errors be at most .05?

A2. Testing a hypothesis about the mean. (From Math 3070 final given 4-30-01.) The desired percentage of SiO_2 in a certain type of cement is 7.85. To test whether the true average percentage is 7.85 for a particular production facility, 25 independent samples are analyzed. Suppose that the percentage of SiO_2 is approximately normally distributed with $\sigma = .500$ and that $\bar{x} = 8.02$.

1. Does this indicate conclusively that the true average differs from 7.85? State the null and alternative hypotheses. State the test statistic and why it is appropriate. State the rejection region for the null hypothesis. Compute the P -value and draw a conclusion.
2. If the true average percentage is $\mu = 8.10$ and a level $\alpha = .05$ test based on a test with $n = 25$ is used, what is the probability of detecting this departure from \mathcal{H}_0 ?
3. What value of n is required to satisfy $\alpha = .05$ and $\beta(8.10) \leq .05$?

A3. Test for a single proportion. A plan for an executive traveller's club has been developed by Useless Airlines on the premise that 5% of its current customers would qualify for a membership. Of a random sample of 500 customers, 39 were found to qualify.

1. With this data, test at the .05 level of significance the null hypothesis that 5% is correct against the alternative that 5% is not correct.

2. What is the probability, that when the test in part (1.) were used, the company's premise will be judged correct when in fact 10% of all current customers qualify?
3. How large a random sample is required to be sure that the probability is at most 5% that when the test in part (1.) were used, the company's premise will be judged correct when in fact 10% of all current customers qualify?

Please hand in problems B on Wednesday, January 27.

B. Problems from the text:

384[9.4, 9.8, 9.10, 9.18]

Please hand in problems C on Wednesday, February 3.

C. Problems from the text:

427[10.3, 10.5, 10.6, 10.7, 10.13]

Deadline extended due to test and holiday.
Please hand in problems D on Wednesday, February 17.

D. Problems from the text:

427[10.17, 10.20, 10.23]

Please hand in problems E on Wednesday, February 24.

E1. Problems from the text:

427[10.29, 10.35]

E2. Additional problem. (from Devore, *Probability and Statistics for Engineering and the Sciences* 4th ed., 1995, p. 538)

The accompanying data resulted from an experiment to investigate the relationship between applied stress x and the time to rupture y for notched cold-rolled brass specimens exposed to a cracking solution ("On initiation and Growth of Stress Corrosion Cracks in Tarnished Brass," *J. Electrochemical Soc.*, 1995, pp. 131–138):

x	22.5	25.0	28.0	30.5	38.0	40.5	42.5	48.0	54.5	55.0	70.0
y	44.0	42.0	33.5	28.0	18.0	13.6	15.0	10.3	9.0	6.3	4.0

1. Run a simple regression on the data. Make at least two different variable transformations and regress in the new variables.
2. Do model checking on each of your models. Plot the points in the transformed variables with the corresponding regression line. Plot residuals vs. independent variables. Plot residuals vs. predicted values. Plot studentized residuals v. predicted values. Make a normalized probability plot (qq-plot) of the residuals.
3. Decide which model best fits the data. Explain why.
4. Transform the regression lines back to the original variables and superimpose the fitted curves from each of your models.

Computer output alone is not sufficient to answer this problem. You must include an explanation and reasoning behind your conclusions. Clearly label references to your output.

Please hand in problems F on Wednesday, March 3.

F. Problems from the text:

465[11.7, 11.9(a-d), 11.12(a)]

Please hand in problems G1 and G2 on Wednesday, March 10.

G1. Problems from the text:

466[11.10, 11.11]

G2. Here is data on length (in inches) and weight (in pounds) of alligators captured in central Florida. Discuss how well this model predicts the weight in terms of the length. Fit a cubic polynomial to this data and plot the fitted polynomial on top of the scatter plot. Check the assumptions on this model.

Weight	Length	Weight	Length
130	94	83	86
51	74	70	88
640	147	61	72
28	58	54	74
80	86	44	61
110	94	106	90
33	63	84	89
90	86	39	68
36	69	42	76
38	72	197	114
366	128	102	90
84	85	57	78
80	82		

Please hand in problems H on Wednesday, March 17.

H. Problems from the text:

490[12.5, 12.6, 12.8]

Please hand in problems I on Wednesday, March 31.

I. Problems from the text:

492[12.9, 12.14]

Please hand in problems J on Wednesday, April 7.

J. Problems from the text:

492[12.22], 541[13.3]

Please hand in problems K on Wednesday, April 14.

K. Problems from the text:

541[13.6, 13.15]

Please hand in problems L on Wednesday, April 21.

L. Problems from the text:

541[13.17, 13.18, 13.19]

Please hand in problems M1, M2 and M3 on Wednesday, April 28.

M1. Problems from the text:

546[13.21, 13.22]

M2. In the text by Navidi, *Principles of Statistics for Engineers and Scientists*, (McGraw Hill, 2010), reports a study “Efficient Pyruvate Production by a Multi-Vitamin . . .,” (*Applied Microbiology and Bacteriology*, 2001) of the effect of three factors nicotinic *A*, thiamine *B* and biotin *C* on glucose consumption (g/L) by a cell culture. Compute the estimates for all effects and interactions. Assume from past experience that it is known that the additive model holds. Add the sum squares of interactions to get a proxy for the error sum square. test the hypotheses that the main effects are zero.

Treatment Combination	Glucose Consumption
1	68.0
<i>a</i>	77.5
<i>b</i>	98.0
<i>ab</i>	98.0
<i>c</i>	74.0
<i>ac</i>	77.0
<i>bc</i>	97.0
<i>abc</i>	98.0

M3. In the text by Walpole, Myers, Myers, Ye, *Probability and Statistics for Engineers and Scientists, 7th ed.*, (Prentice Hall, 2002), reports a an experiment to study the effect of sealing temperature A , cooling bar temperature B , percent polyethylene additive C and pressure D on seal strength (gm/in) of bread-wrapper stock. A $1/2$ fractional 2^4 experiment was run with defining contrast $ABCD$. Perform an analysis of variance on the main effects and two factor interactions, assuming that all three factor and higher interactions are negligible. Use $\alpha = 0.05$.

Treatment Combination	Seal Strength
1	6.6
<i>ad</i>	6.9
<i>bd</i>	7.9
<i>ab</i>	6.1
<i>cd</i>	9.2
<i>ac</i>	6.8
<i>bc</i>	10.4
<i>abcd</i>	7.3