Math 3070 § 1. **Gas Mileage Example: To Pool or Not to Pool in a Two Sample t-Test.** Name: Example
Treibergs June 22, 2011

Data File Used in this Analysis:

```
# Math 3074 - 1     Gas Data          June 22, 2011
# Treibergs
#
# Taken from Elliott, "Learning SAS in the computer Lab, 2nd ed.,"
# Brooks/Cole 2000. (data set "gas")
#
# From a study by Montgomery & Friedman, "Prediction Using Regression Models with
# Multicollinear Predictor Variables," IIE Transactions, May 1993.
# Test whether the mean of MPG for cars with manual transmission is significantly
# higher than the mean of MPG for cars with automatic transmission. Is a pooled test
# appropriate?
#
# Variables
#
# Disp            Displacement in cubic inches
# HP              Horsepower in ft-lbs
# Torque          in ft-lbs
# ComprRatio
# RearAxleRatio
# NBar            Number of carburetor barresl
# NSpeeds             Number of transmission speeds
# Len             Car Length in inches
# Width           Car width in inches
# Wt              Car weight in pounds
# Tran            Transmission type       0=manual, 1=automatic
# MPG             Gas mileage in miles per gallon
#
Disp HP  Torque ComprRatio RearAxleRatio NBar NSpeeds  Len Width Wt Tran MPG
318.0 140 255 8.5 2.7 2 3 215.3 76.3 4370 1 19.7
440.0 215 330 8.2 2.9 4 3 184.5 69.0 4215 1 11.2
351.0 143 255 8.0 3.0 2 3 199.9 74.0 3890 1 18.3
360.0 180 290 8.4 2.5 2 3 214.2 76.3 4250 1 21.5
140.0  83 109 8.4 3.4 2 4 168.8 69.4 2700 0 20.3
 85.3  80  83 8.5 3.9 2 4 160.6 62.2 2009 0 36.5
350.0 165 260 8.0 2.6 4 3 200.3 69.9 3910 1 18.9
 96.9  75  83 9.0 4.3 2 5 162.5 65.0 2320 0 30.4
351.0 148 243 8.0 3.3 2 3 215.5 78.5 4540 1 13.9
440.0 215 330 8.2 2.7 4 3 231.0 79.7 5185 1 14.9
171.0 109 146 8.2 3.2 2 4 170.4 66.9 2655 1 21.5
302.0 129 220 8.0 3.0 2 3 199.9 74.0 3890 1 17.8
350.0 155 250 8.5 3.1 4 3 196.7 72.2 3910 1 17.8
318.0 145 255 8.5 2.5 2 3 197.6 71.0 3666 1 16.4
231.0 110 175 8.0 2.6 2 3 179.3 65.4 3050 1 23.5
 96.9  75  83 9.0 4.3 2 5 165.2 61.8 2275 0 31.9
500.0 190 360 8.5 2.7 4 3 224.1 79.8 5290 1 14.4
231.0 110 175 8.0 2.6 2 3 179.3 65.4 3020 1 22.1
350.0 170 275 8.5 2.6 4 3 199.6 72.9 3860 1 17.0
```

```
250.0 105 185 8.3 2.7 1 3 196.7 72.2 3510 1 20.0
225.0  95 170 8.4 2.8 1 3 194.0 71.8 3365 0 20.1
 89.7  70  81 8.2 3.9 2 4 155.7 64.0 1905 0 34.7
350.0 155 250 8.5 3.1 4 3 195.4 74.4 3885 1 16.5
258.0 110 195 8.0 3.1 1 3 171.5 77.0 3375 1 19.7
460.0 223 366 8.0 3.0 4 3 228.0 79.8 5430 1 13.3
360.0 195 295 8.3 3.2 4 3 209.3 77.4 4215 1 13.8
262.0 110 200 8.5 2.6 2 3 179.3 65.4 3180 1 21.5
350.0 165 255 8.5 2.7 4 3 185.2 69.0 3660 1 16.5
351.0 148 243 8.0 3.3 2 3 216.1 78.5 4715 1 13.3
133.6  96 120 8.4 3.9 2 5 171.5 63.4 2535 0 23.9
```

R Session:

```
R version 2.11.1 (2010-05-31)
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'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

[R.app GUI 1.34 (5589) i386-apple-darwin9.8.0]

[Workspace restored from /home/1004/ma/treibergs/.RData]


> tt <-read.table("M3074GasData.txt",header=TRUE)
```
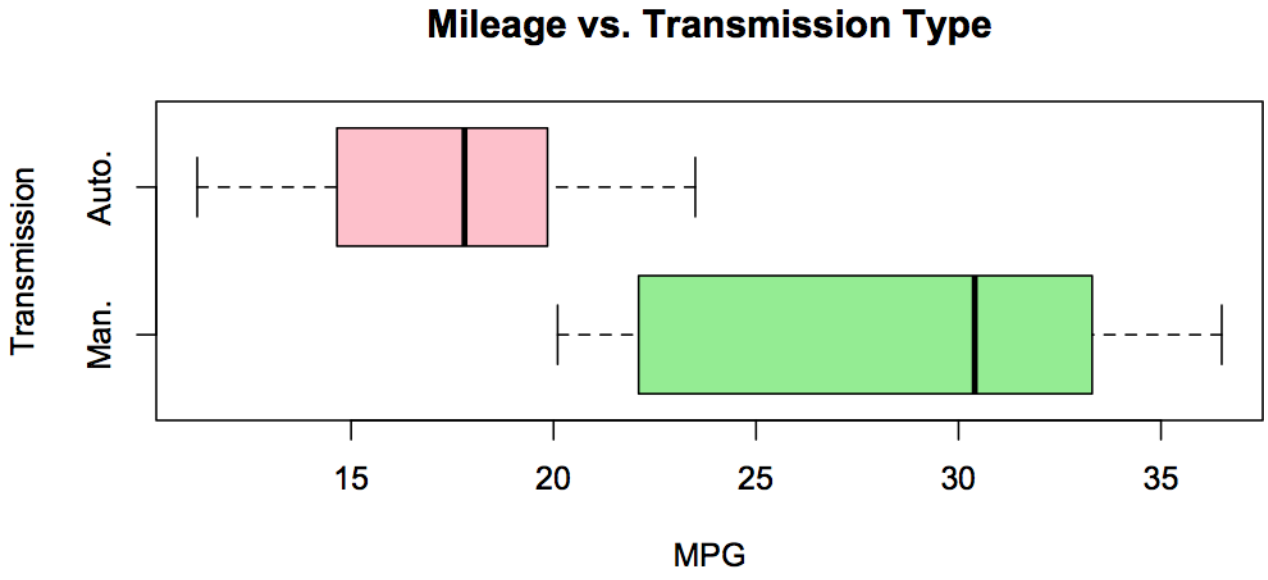
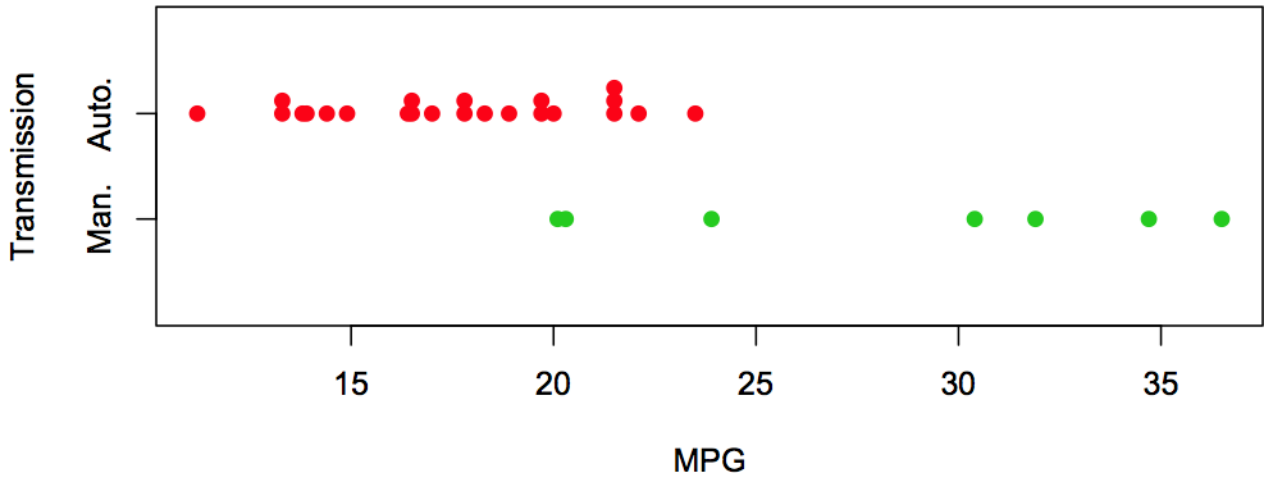
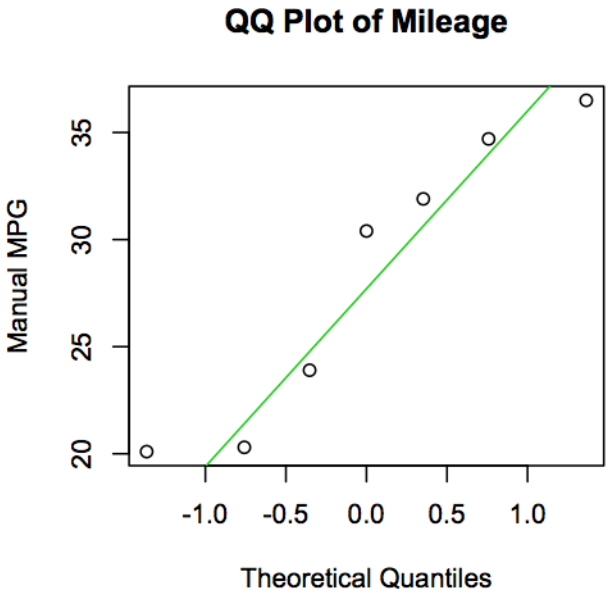
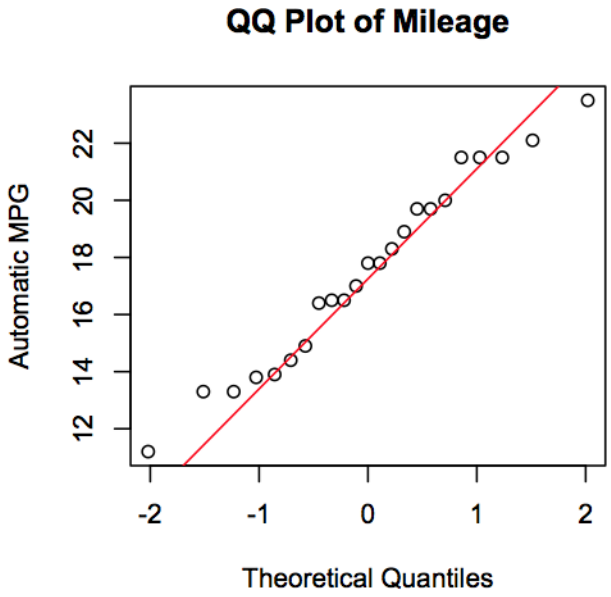

```
> tt
    Disp  HP Torque ComprRatio RearAxleRatio NBar NSpeeds   Len Width   Wt Tran  MPG
1  318.0 140    255        8.5           2.7    2       3 215.3  76.3 4370    1 19.7
2  440.0 215    330        8.2           2.9    4       3 184.5  69.0 4215    1 11.2
3  351.0 143    255        8.0           3.0    2       3 199.9  74.0 3890    1 18.3
4  360.0 180    290        8.4           2.5    2       3 214.2  76.3 4250    1 21.5
5  140.0  83    109        8.4           3.4    2       4 168.8  69.4 2700    0 20.3
6   85.3  80     83        8.5           3.9    2       4 160.6  62.2 2009    0 36.5
7  350.0 165    260        8.0           2.6    4       3 200.3  69.9 3910    1 18.9
8   96.9  75     83        9.0           4.3    2       5 162.5  65.0 2320    0 30.4
9  351.0 148    243        8.0           3.3    2       3 215.5  78.5 4540    1 13.9
10 440.0 215    330        8.2           2.7    4       3 231.0  79.7 5185    1 14.9
11 171.0 109    146        8.2           3.2    2       4 170.4  66.9 2655    1 21.5
12 302.0 129    220        8.0           3.0    2       3 199.9  74.0 3890    1 17.8
13 350.0 155    250        8.5           3.1    4       3 196.7  72.2 3910    1 17.8
14 318.0 145    255        8.5           2.5    2       3 197.6  71.0 3666    1 16.4
15 231.0 110    175        8.0           2.6    2       3 179.3  65.4 3050    1 23.5
16  96.9  75     83        9.0           4.3    2       5 165.2  61.8 2275    0 31.9
17 500.0 190    360        8.5           2.7    4       3 224.1  79.8 5290    1 14.4
18 231.0 110    175        8.0           2.6    2       3 179.3  65.4 3020    1 22.1
19 350.0 170    275        8.5           2.6    4       3 199.6  72.9 3860    1 17.0
20 250.0 105    185        8.3           2.7    1       3 196.7  72.2 3510    1 20.0
21 225.0  95    170        8.4           2.8    1       3 194.0  71.8 3365    0 20.1
22  89.7  70     81        8.2           3.9    2       4 155.7  64.0 1905    0 34.7
23 350.0 155    250        8.5           3.1    4       3 195.4  74.4 3885    1 16.5
24 258.0 110    195        8.0           3.1    1       3 171.5  77.0 3375    1 19.7
25 460.0 223    366        8.0           3.0    4       3 228.0  79.8 5430    1 13.3
26 360.0 195    295        8.3           3.2    4       3 209.3  77.4 4215    1 13.8
27 262.0 110    200        8.5           2.6    2       3 179.3  65.4 3180    1 21.5
28 350.0 165    255        8.5           2.7    4       3 185.2  69.0 3660    1 16.5
29 351.0 148    243        8.0           3.3    2       3 216.1  78.5 4715    1 13.3
30 133.6  96    120        8.4           3.9    2       5 171.5  63.4 2535    0 23.9

> ###################### PICK OFF MPG VS TRAN ########################################
> attach(tt)
> ManualMPG <- MPG[Tran == 0]
> AutomaticMPG <- MPG[Tran == 1]
> Tran <- factor(Tran)
>
> layout(matrix(1:2,ncol=1))
>
> plot(MPG ~ Tran, horizontal = TRUE, xlab = "Transmission",
+ col=c("lightgreen","pink"), main = "Mileage vs. Transmission Type")
> stripchart(MPG ~ Tran, col = c(3,2), method="stack",
+ ylim = c(0.1,2.9), pch = 19, ylab = "Transmission")
> # M3074GasEg.pdf
>
> # The means look different. I'm not sure about if I can assume
> # that the variances are equal.
```


Mileage vs. Transmission Type
Transmission
Auto.
Man.
15
20
25
30
35
MPG


Transmission
Auto.
Man.
15
20
25
30
35
MPG

```
> ################## RUN TEST FOR EQUALITY OF VARIANCES ###################
>
> mean(ManualMPG)
[1] 28.25714
> mean(AutomaticMPG)
[1] 17.54348
> var(ManualMPG)
[1] 46.02619
> var(AutomaticMPG)
[1] 11.14893
> var.test(manualMPG,AutomaticMPG)
Error in var.test(manualMPG, AutomaticMPG) : object 'manualMPG' not found
> var.test(ManualMPG,AutomaticMPG)

F test to compare two variances

data:  ManualMPG and AutomaticMPG
F = 4.1283, num df = 6, denom df = 22, p-value = 0.01264
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
  1.351487 21.221801
sample estimates:
ratio of variances
          4.128305

> # Variances are significantly different. So pooled t-test not appropriate.
>
>
> ###################### QQ PLOTS TO CHECK NORMALITY ASSUMPTION ###############
> layout(matrix(c(1,3,2,4),ncol=2))

> qqnorm(ManualMPG,main="QQ Plot of Mileage",ylab="Manual MPG")
> qqline(ManualMPG,col=3)
> qqnorm(AutomaticMPG,main="QQ Plot of Mileage",ylab="Automatic MPG")
> qqline(AutomaticMPG,col=2)
> # M3074Gas2.pdf
>
> # Small data sets but neither qq plot showsw strong reason to doubt normality.
```






```
> ###################### RUN 2-SAMPLE T-TEST ##############################
> t.test(ManualMPG,AutomaticMPG,alternative = "greater")

Welch Two Sample t-test

data:  ManualMPG and AutomaticMPG
t = 4.0322, df = 6.907, p-value = 0.002561
alternative hypothesis: true difference in means is greater than 0
95 percent confidence interval:
 5.669466      Inf
sample estimates:
mean of x mean of y
 28.25714  17.54348

> # Data significantly indicates that MPG for manual transmission is
> # greater than MPG for automatic.
>
> ######## WELCH-SATTERTHWAITE TEST BY HAND ############################
>
> ManBar <- mean(ManualMPG)
> ManV <- var(ManualMPG)
> ManN <- length(ManualMPG)
> AutBar <- mean(AutomaticMPG)
> AutV <- var(AutomaticMPG)
> AutN <- length(AutomaticMPG)
> nu <- (ManV/ManN + AutV/AutN)^2/((ManV/ManN)^2/(ManN-1) + (AutV/AutN)^2/(AutN-1))
> nu
[1] 6.907039
> T <- (ManBar-AutBar)/sqrt(ManV/ManN + AutV/AutN)
> T
[1] 4.032168
> pval <- pt(T,nu,lower.tail=FALSE); pval
[1] 0.002561244
```

```
> alpha<- .05
> se <- sqrt(ManV/ManN + AutV/AutN)
> talpha <- qt(alpha,nu,lower.tail=FALSE)
> LowCI <- ManBar-AutBar-talpha*se
> LowCI
[1] 5.669466
>
> cat("\n\n   Welch-Satterthwaite Two Sample t-test BY HAND\n\n",
+ "data: ManualMPG and AutomaticMPG\n t =", T, ", df =", nu, ", p-value =",
+ pval ,"\n alternative hypothesis: true difference in means > 0\n",
+ "95 percent confidence interval:\n", c(LowCI,Inf),
+ "\n sample estimates:\n mean of manualMPG,  mean of AutomaticMPG\n",
+ c(ManBar,AutBar),"\n\n")


   Welch-Satterthwaite Two Sample t-test BY HAND

 data: ManualMPG and AutomaticMPG
 t = 4.032168 , df = 6.907039 , p-value = 0.002561244
 alternative hypothesis: true difference in means > 0
 95 percent confidence interval:
 5.669466 Inf
 sample estimates:
 mean of manualMPG,  mean of AutomaticMPG
 28.25714 17.54348

>
```