Math 3070 § 1. **Naphthalene Data: Two-Sample t-Test.** Name: Example
Treibergs December 9, 2010

Data File Used in this Analysis:

```
# M 3070 - 1           Naphthalene Data       Nov. 27, 2010
# Treibergs
#
# From Navidi, Statistics for Engineers and Scientists, 2nd ed
# McGraw Hill, 2008
#
# Taken from T. Ebihara & P. Bishop, "Influence of supplemental acetate on
# bioremediation of dissolved polycyclic aromatic hydrocarbons," J. Environ.
# Engineering, 2002.
#
# Experiment measured naphthalene concentrations in mg/L (PAH) in water measured
# in a sand column at various depths and after as number of days
#
# Assume all measurements are independent.
#
# We test whether there is strong evidence that PAH has decreased more than
# 1.2 mg/L from 45 days to 90 days.
#
Depth Days PAH
0 25 11.15
0 25 11.39
0 25 11.36
0 45 9.28
0 45 8.15
0 45 8.59
0 90 7.68
0 90 7.59
0 90 7.41
5 25 14.4
5 25 11.78
5 25 11.92
5 45 9.44
5 45 9.34
5 45 9.33
5 90 7.53
5 90 7.92
5 90 7.12
15 25 11.51
15 25 11.01
15 25 11.09
15 45 9.34
15 45 9.11
15 45 8.94
15 90 7.43
15 90 7.47
15 90 7.53
30 25 12.77
```

```
30 25 12.18
30 25 11.65
30 45 9.37
30 45 9.27
30 45 9.05
30 90 7.6
30 90 7.48
30 90 7.84
50 25 11.71
50 25 11.29
50 25 11.2
50 45 9.25
50 45 8.97
50 45 9.29
50 90 7.76
50 90 7.84
50 90 7.68
75 25 11.18
75 25 11.45
75 25 11.27
75 45 9.09
75 45 8.86
75 45 8.78
75 90 7.72
75 90 7.61
75 90 7.74
```

R Session:

```
R version 2.10.1 (2009-12-14)
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Type 'q()' to quit R.

[R.app GUI 1.31 (5538) powerpc-apple-darwin8.11.1]

[Workspace restored from /Users/andrejstreibergs/.RData]
```
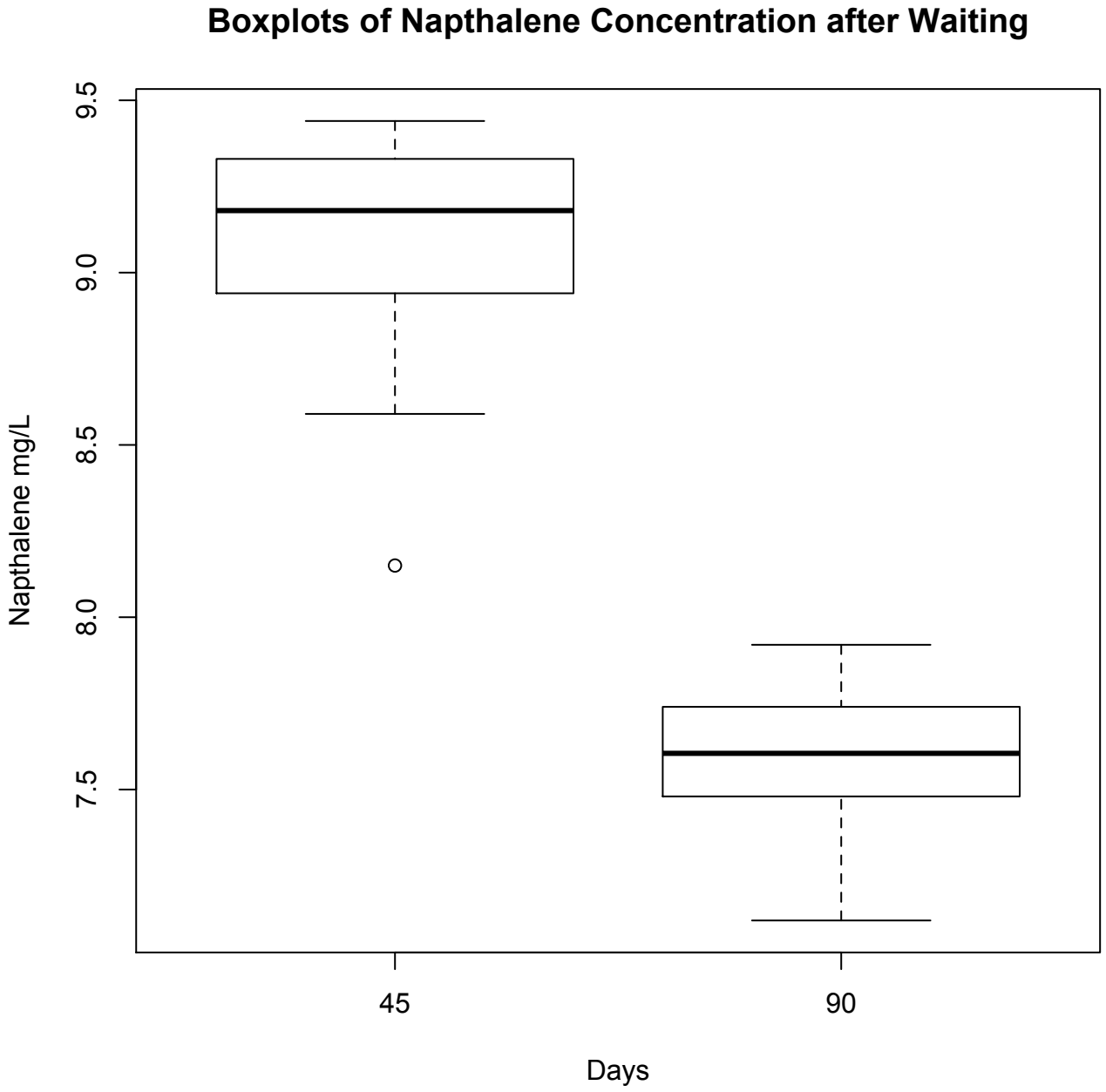

```
> tt <- read.table("M3073NaphthaleneData.txt",header=TRUE)
> tt
   Depth Days   PAH
1      0   25 11.15
2      0   25 11.39
3      0   25 11.36
4      0   45  9.28
5      0   45  8.15
6      0   45  8.59
7      0   90  7.68
8      0   90  7.59
9      0   90  7.41
10     5   25 14.40
11     5   25 11.78
12     5   25 11.92
13     5   45  9.44
14     5   45  9.34
15     5   45  9.33
16     5   90  7.53
17     5   90  7.92
18     5   90  7.12
19    15   25 11.51
20    15   25 11.01
21    15   25 11.09
22    15   45  9.34
23    15   45  9.11
24    15   45  8.94
25    15   90  7.43
26    15   90  7.47
27    15   90  7.53
28    30   25 12.77
29    30   25 12.18
30    30   25 11.65
31    30   45  9.37
32    30   45  9.27
33    30   45  9.05
34    30   90  7.60
35    30   90  7.48
36    30   90  7.84
37    50   25 11.71
38    50   25 11.29
39    50   25 11.20
40    50   45  9.25
41    50   45  8.97
42    50   45  9.29
43    50   90  7.76
44    50   90  7.84
45    50   90  7.68
46    75   25 11.18
47    75   25 11.45
48    75   25 11.27
49    75   45  9.09
```

```
50    75   45  8.86
51    75   45  8.78
52    75   90  7.72
53    75   90  7.61
54    75   90  7.74
```

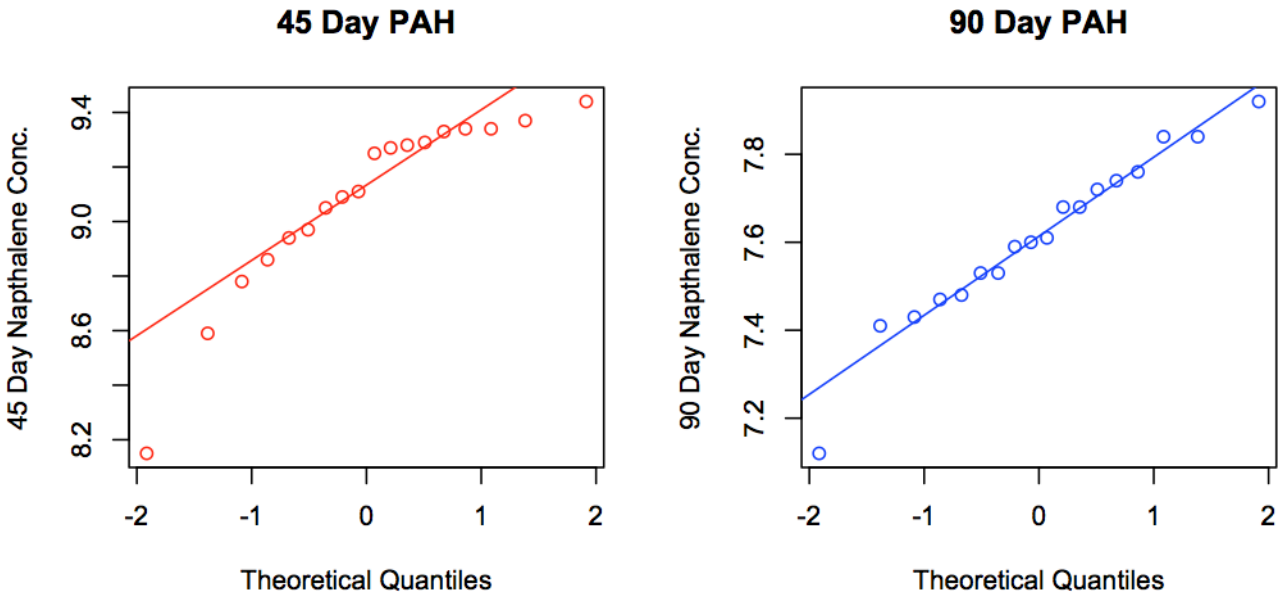
```
> # For this study, we exclude the data for 25 Days. Take  only tt with Days>25
```

```
> tt2 <- subset(tt, Days>25)
> attach(tt2)
> tt2
   Depth Days  PAH
4      0   45 9.28
5      0   45 8.15
6      0   45 8.59
7      0   90 7.68
8      0   90 7.59
9      0   90 7.41
13     5   45 9.44
14     5   45 9.34
15     5   45 9.33
16     5   90 7.53
17     5   90 7.92
18     5   90 7.12
22    15   45 9.34
23    15   45 9.11
24    15   45 8.94
25    15   90 7.43
26    15   90 7.47
27    15   90 7.53
31    30   45 9.37
32    30   45 9.27
33    30   45 9.05
34    30   90 7.60
35    30   90 7.48
36    30   90 7.84
40    50   45 9.25
41    50   45 8.97
42    50   45 9.29
43    50   90 7.76
44    50   90 7.84
45    50   90 7.68
49    75   45 9.09
50    75   45 8.86
51    75   45 8.78
52    75   90 7.72
53    75   90 7.61
54    75   90 7.74
```

```
> # LET  x  BE THE  45  DAY CONCENTRATIONS
> day <- factor(Days)
> x <- subset(PAH,day==45)

> # LET  y  BE THE  90  DAY CONCENTRATIONS
> y <- subset(PAH,day==90)

> # BOXPLOTS OF THE TWO DATA SETS.
> layout(1)
> boxplot(PAH~day,xlab="Days",ylab="Napthalene mg/L",
  main="Boxplots of Napthalene Concentration after Waiting")
```


Boxplots of Napthalene Concentration after Waiting
Napthalene mg/L
9.5
9.0
8.5
8.0
7.5
45
90
Days

```
> # QQ - PLOTS OF THE TWO DATA SETS.  NORMALITY WORSE IN 45 DAY BUT NOT BAD.
> layout(matrix(c(1,3,2,4),ncol=2))

> qqnorm(x,col=2,main="45 Day PAH",ylab="45 Day Napthalene Conc.");qqline(x,col=2)
> qqnorm(y,col=4,main="90 Day PAH",ylab="90 Day Napthalene Conc.");qqline(y,col=4)
```




```
> #   TEST THE HYPOTHESES
> #      H0:    mu_x - mu_y = Delta_0 = 1.2     VS
> #      Ha:    mu_x - mu_y > Delta_0 = 1.2
> #
> ########## DO THE COMPUTATION "BY HAND" ##########################################
> #
> ###### SAMPLE MEAN, SD, N FOR THE TWO SAMPLES ####################################

> m <- length(x); mx <- mean(x); sx <- sd(x); c(m,mx,sx)
[1] 18.0000000  9.0805556  0.3287673

> n <- length(y); my <- mean(y); sy <- sd(y); c(n,my,sy)
[1] 18.0000000  7.6083333  0.1918103

> ########## STANDART ERROR OF XBAR - YBAR, T-STATISTIC ############################

> ss <- sqrt(sx^2/m + sy^2/n); ss
[1] 0.08971532

> t=(mx-my-1.2)/ss; t
[1] 3.034289
```

```
> # THIS IS THE COMPUTATION OF THE APPROXIMATE d.f. GIVEN IN THE TEXT WHEN THE
> # TWO VARIANCES CANNOT BE ASSUMED TO BE EQUAL.
> # IT IS KNOWN AS THE WELCH-SATTERTHWAITHE TEST.

> nu <- (sx^2/m+sy^2/n)^2/((sx^2/m)^2/(m-1)+(sy^2/m)^2/(n-1)); nu
[1] 27.37136

> ###### P-VALUE = AREA OF TAIL TO RIGHT OF  T IN T-DIST WITH nu DEGREES OF FREEDOM ##
>  pt(t,nu, lower.tail=F)
[1] 0.002617539
> #   P-VALUE IS SIGNIFICANT.  WE REJECT  H0  IN FAVOR OF  Ha.


> # ONE-SIDED  95 %  CONFIDENCE BOUND: mux - muy IS LESS THAN
> mx-my+qt(.05,nu,lower.tail=FALSE)*ss
[1] 1.624960
>

> #  RUN THE SAME TEST USING THE CANNED ROUTINE.

> t.test(x,y,alternative="greater",mu=1.2,paired=FALSE,var.equal=FALSE)

Welch Two Sample t-test

data:  x and y
t = 3.0343, df = 27.371, p-value = 0.002618
alternative hypothesis: true difference in means is greater than 1.2
95 percent confidence interval:
 1.319485      Inf
sample estimates:
mean of x mean of y
 9.080556  7.608333


> #  NOTE THAT  t.test  GIVES THE LOWER CONFIDENCE BOUND. HOWEVER, RECALL THAT "UPPER" AND
> #  "LOWER" SWAP WHEN PERFORMING A ONE-SIDED TEST OF HYPOTHESIS:
> #  WE REJECT  H0  AT THE  alpha  LEVEL IF
> #
> #                    mx - my - Delta_0 >= ss * tcrit(alpha,nu)
> #
> #  BUT THIS IS THE SAME AS THE TEST VALUE BEING OUTSIDE THE LOWER  1-alpha  CI FOR
> #  mu_x - mu_y:
> #
> #                    mx - my -  ss * tcrit(alpha,nu) >= Delta_0
> #
> mx-my-qt(.05,nu,lower.tail=FALSE)*ss
[1] 1.319485

> # THUS  Delta_0 = 1.2  IS OUTSIDE THE LOWER CI FOR  mu_x - mu_y  SO WE REJECT  H0.
```