Math 3010 § 1. **Final Exam** Name:______________

Treibergs $\sigma\!\tau$ May 1, 2024

1.	____/18
2.	____/15
3.	____/14
4.	____/14
5.	____/14
6.	____/14
7.	____/14
8.	____/14
9.	____/14
Total	____/100

This is a closed book test except that you are allowed three "cheat sheets," 8.5" x 11" pieces of paper with notes on both sides, one for the final plus the two from the midterms. No other notes, books, papers, calculators, tablets, phones or messaging devices are permitted. Define terms, give complete solutions and explain your logic. There are [100] total points. **Do SEVEN of nine problems.** If you do more than seven problems, only the first seven will be graded. Cross out the problems you don't wish to be graded.

1. [18] Match the contribution with the mathematician.

Contributions.

1. Developed logarithms. First to publish extensive tables of logarithms of numbers and logarithms of sines.
2. Developed the new field of perspective geometry, a theory of perspective drawing. It extends the Euclidean plane by adding pints at infinity.
3. Used coordinates to redo Apollonius where an equation determines a curve. Found slopes using adequality. Studied number theory and probability.
4. Geometrical approach to calculus using differentials. Transmutation formula was used to square the circle. Found closed form solutions of differential equations involving logarithms and exponentials. Converted Newton's geometric arguments to differential equations. Contributed to mathematical logic and combinatorics.
5. Developed complex sine and cosine series, total differential and multiple integrals. Used these with reduction of order methods to solve differential equations. Introduced the zeta function to connect analysis to prime numbers.
6. Reduced problems of mechanics and physics to differential equations. Developed algebraic methods of calculus not involving infinitesimally small quantities or limits. Proved a fundamental theorem of calculus, error estimate. Understood that the cubic / quartic polynomials have three / four complex roots.
7. Quadratic reciprocity, fundamental theorem of algebra and linear systems. Perturbation series to compute the orbit of planetoids. Convergence of series, elliptic functions, non-Euclidean geometry and error functions. Theory and calculation of curvature of surfaces.
8. Emphasized rigor. Used limits to define the derivative, continuity and sum the integral. Proved the existence theorem for ODE's. Systemized groups. Found integral theorems in complex analysis. Also studied quadratic forms, rigidity and elasticity.
9. Made a new theory of integration. Proved integral formulas for complex functions and the conformal mapping theorem. Introduced topology in the study of complex analysis. Built a new theory of curvature and calculation on manifolds

Mathematician			Contribution
Augustin Louis Cauchy	1789 - 1857	Paris	________
Gerard Desargues	1591 - 1661	Lyons	________
Leonhard Euler	1707 - 1783	St. Petersburg	________
Pierre de Fermat	1601 - 1665	Toulouse	________
Karl Friedrich Gauss	1777 - 1855	Gottingen	________
Joseph Louis Lagrange	1736 - 1813	Paris	________
Gottfried Wilhelm Leibnitz	1646 - 1716	Hanover	________
John Napier	1550 - 1617	Merchison	________
Bernhard Riemann	1826 - 1866	Gottingen	________

2. [15] **Short Answer.** Here a list of areas where mathematics developed. For each, identify an important mathematician of that region, state an important discovery by this mathematician, and state a feature of the mathematics of that region that distinguishes it from other regions. [Seven or fewer words!]

Region	Mathematician	Their Discovery	Distinguishing Feature
China			
India			
Islam			
Italy			
France			

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3. Determine whether the following statements are true or false.

   (a) [2] Cardano gave the solution of a general cubic equation such as $x^3 = 3x + 4$.

TRUE: ◯	FALSE: ◯

   (b) [2] The Euler Characteristic of every polyhedron equals two. .

TRUE: ◯	FALSE: ◯

   (c) [2] D'Alembert's solution of $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$ is $u(t, x) = f(x + t) + g(x - t)$ where f and g are functions.

TRUE: ◯	FALSE: ◯

   (d) [2] There are geometries in which all of Euclid's axioms hold, except that the "Fifth Postulate" is replaced by the "Postulate of the acute angle."

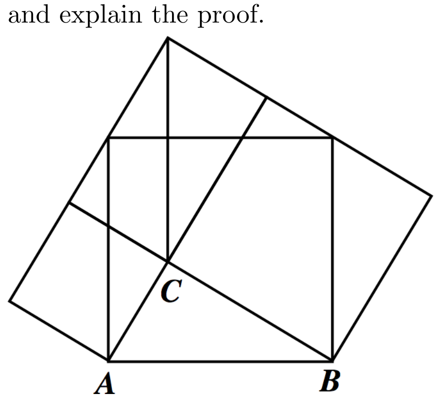
TRUE: ◯	FALSE: ◯

   [6] Give a detailed explanantion of ONE of your answers (a)–(d) above.

4. (a) [7] Use Euler's method to find $S = 1 + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \dfrac{1}{7^2} + \cdots$.

Hint: $\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \cdots = \left(1 - \dfrac{4x^2}{\pi^2}\right)\left(1 - \dfrac{4x^2}{3^2\pi^2}\right)\left(1 - \dfrac{4x^2}{5^2\pi^2}\right)\cdots$

(b) [7] Thabit ibn Qurrah 836 – 901 drew this diagram for his argument. State the theorem and explain the proof.



5. (a) [7] Find all integer solutions of $28x + 65y = 1$.

(b) [7] Find an integer solution of $13x^2 + 1 = y^2$.
[Hint: Start by noting that $(x, y, k) = (5, 18, -1)$ is a solution of $13x^2 + k = y^2$.]

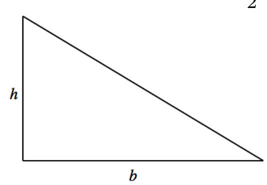
6. [14] Find all integers x that simultaneously satisfy the congruences.

$$\begin{aligned} x &\equiv 4 \mod 5 \\ x &\equiv 5 \mod 6 \\ x &\equiv 6 \mod 7 \end{aligned}$$

7. (a) [7] Recall the Fibonacci Sequence $\{F_1, F_2, F_3, F_4, F_5, \ldots\} = \{1, 1, 2, 3, 5, \ldots\}$. What is the recursion formula for generating the Fibonacci sequence? Prove for all n that the sum of the first n Fibonacci numbers equals

$$F_1 + F_2 + F_3 + \cdots + F_n = F_{n+2} - 1.$$

(b) [7] Let T be a triangle with vertices $(0,0)$, $(b,0)$ and $(0,h)$, with base b and height h. Prove the area $A = \frac{1}{2}bh$ using Eudoxus' Method of Exhaustion.



8. [14] Find the derivative of $f(x)$ for $x > 0$ using one of three early methods: Fermat's method of ad-equality, Newton's method of fluxions or Lagrange's algebraic method. [Other methods will receive no credit.]

$$f(x) = x + \frac{1}{x}$$

9. (a) [7] **Short Answer.** Describe the model of our solar system advocated by these mathematicians. Explain how each new theory was an improvement over the previous one.

Claudius Ptolemy 100–170
Nicolaus Copernicus 1473–1543
Johannes Kepler 1571–1630
Isaac Newton 1642–1727

(b) [7] Following Newton, find the first four nonvanishing terms of the power series for $f(x) = \left(1 - x^4\right)^{\frac{1}{3}}$.