Mathematics 3100 Final Exam Part 1 **Name** ______________________

1. Prove the Pythagorean theorem and its converse. Be thorough.

2. Prove that the bisector of an angle is the set of all points equidistant from both sides of the angle.

3. Using any method that you like, find an expression for the length of a diagonal in a regular pentagon with side length 1.

4. Prove that a the image of a line *l* rotated 90 degrees about a point *O* is perpendicular to the original line.

5. Prove that the slopes of perpendicular lines are negative reciprocals.

6. Draw a counterexample to the “SSA theorem”.

7. Given a line, a point on the line, and a point not on the line, construct a circle through both points, tangent to the line.

8. Name a fundamental theorem of Euclid whose proof is not fully justified by Euclid's five postulates. Explain how we know this.

9. Without using the parallel postulate (or its consequences -- which include the angle sum formula and the alternate interior angles theorem), prove: In a quadrilateral ABCD, if A and B are right angles, and AD and BC are equal in length, then the angles at C and D are congruent.

10. Compute from first principles the sine and cosine of 30°.

Mathematics 3100 Fall 2011 Final Exam Part 2. **Name:** ____________________________

1. The area of a triangle in hyperbolic geometry is pi minus the sum of the angles. Find and justify a formula in hyperbolic geometry for the area of an arbitrary *n*-gon.

2. Prove: any two great circles on the sphere intersect.

3. Construct a regular hexagon.

4. Describe, with justification, the composition of two rotations.

5. Prove that in a convex quadrilateral, the point P in the interior such that the sum of the distances from P to the vertices is minimum is at the intersection of the diagonals.

6. Compute the volume of a regular octahedron of side length 1.