Karlsruhe Institute of Technology

The Hall effect in composites

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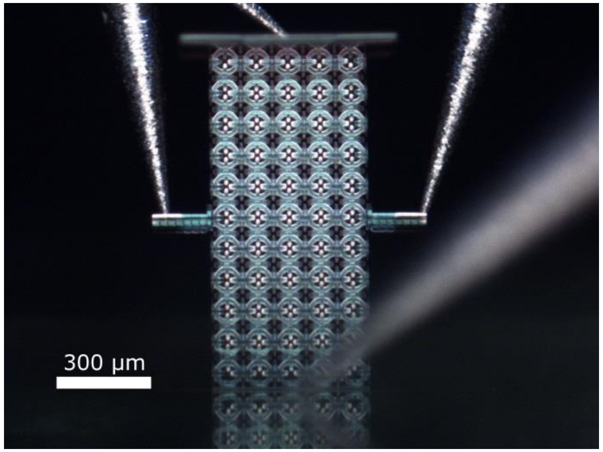
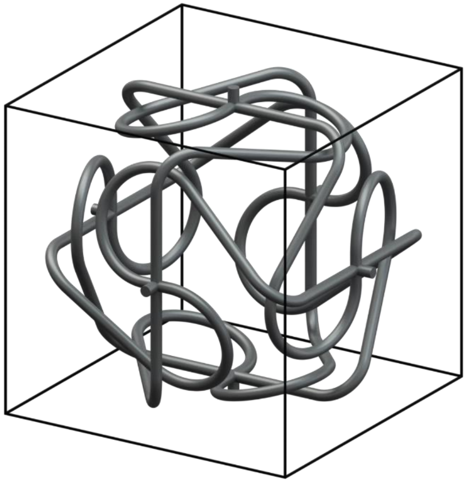
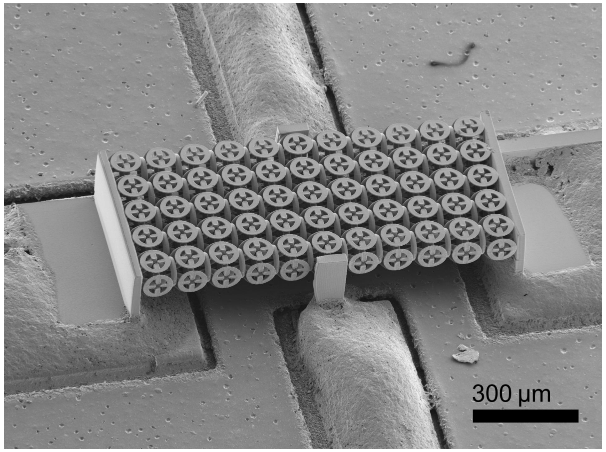




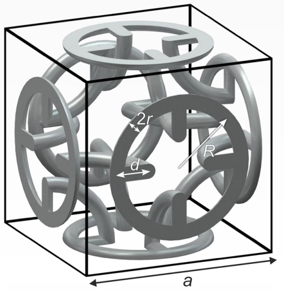


Outline

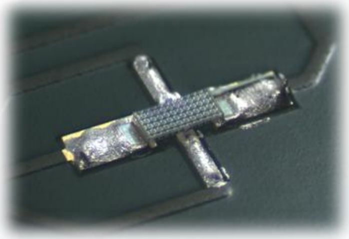
- Theory and homogenization of the Hall effect

$$\mathrm{Cof}\,(\boldsymbol{\sigma}_0^*)\,\boldsymbol{A}_\mathrm{H}^* = \langle \mathrm{Cof}\,(\boldsymbol{\sigma}_0 \nabla \boldsymbol{\Phi})^\intercal\,\boldsymbol{A}_\mathrm{H} \rangle$$

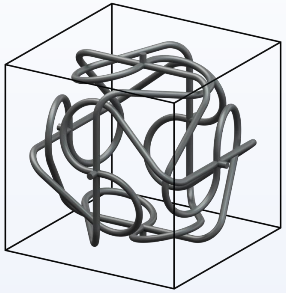
- Sign-inversion of the effective Hall coefficient in chainmail-inspired composites



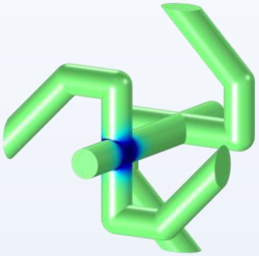
- Symmetry considerations

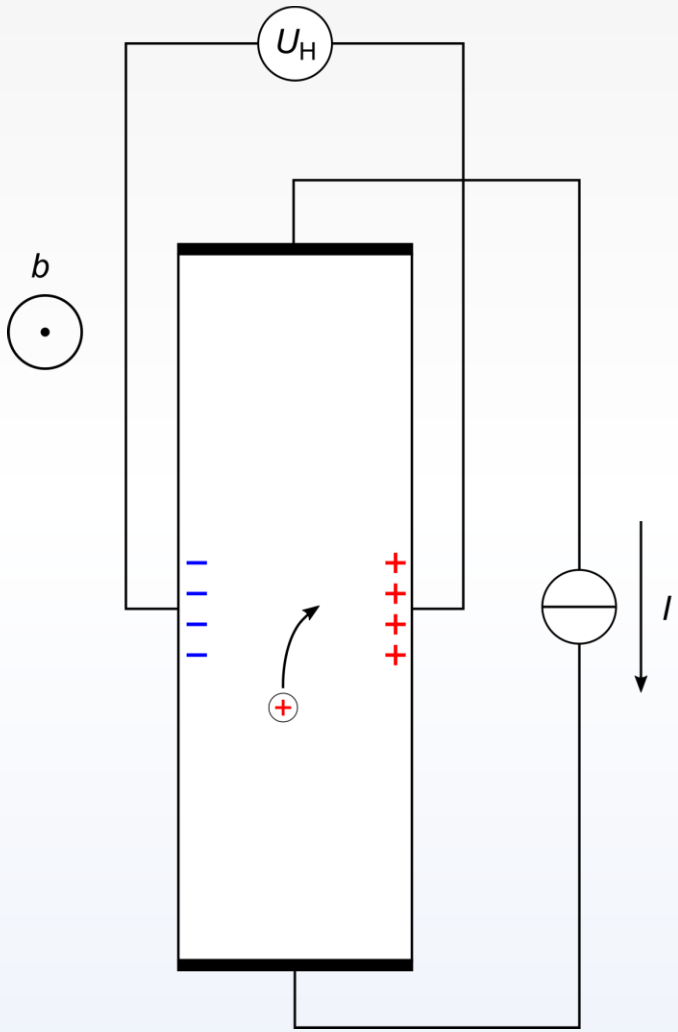


- Current state of experiments
- Novel structures showing a sign-inversion of the effective Hall coefficient



- Anisotropic structures

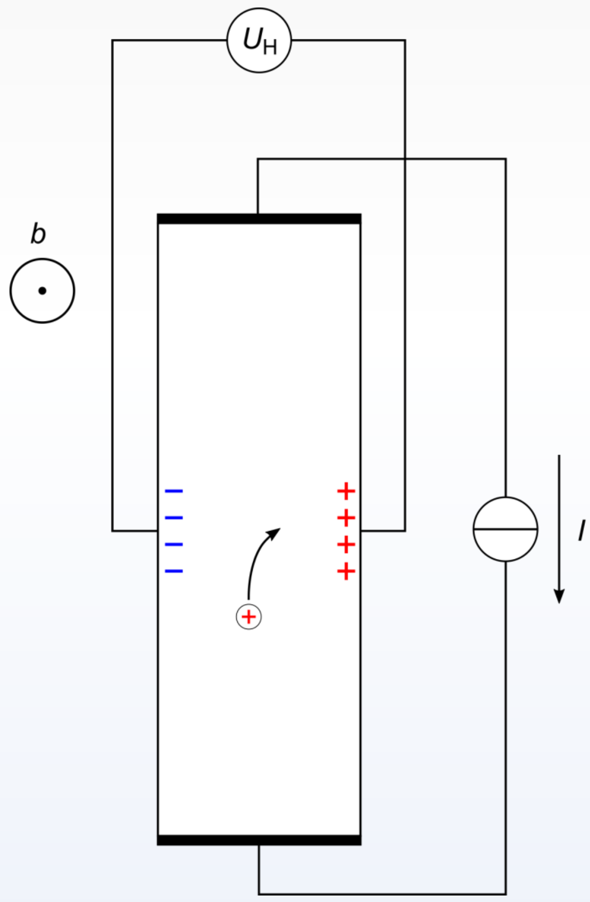




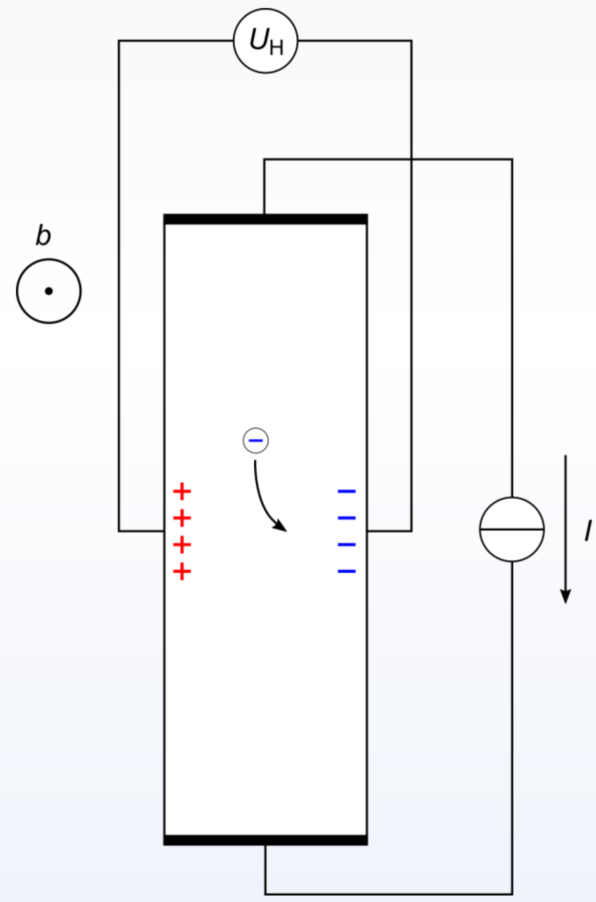
$$U_{\mathrm{H}} = \frac{1}{nq}\frac{Ib}{h} = A_{\mathrm{H}}\frac{Ib}{h}$$

negative charge carriers (e.g. n-type semiconductor) ⟷ negative A_{H}

positive charge carriers (e.g. p-type semiconductor) ⟷ positive A_{H}



positive Hall coefficient



negative Hall coefficient

Conductivity problem $\nabla \cdot \boldsymbol{j} = 0, \quad \boldsymbol{j} = \boldsymbol{\sigma}(\boldsymbol{b})\,\boldsymbol{e}, \quad \nabla \times \boldsymbol{e} = 0$

From Onsager's principle: $\boldsymbol{\sigma}(\boldsymbol{b}) = \boldsymbol{\sigma}(-\boldsymbol{b})^{\mathsf{T}}$

Small magnetic fields

Expansion in orders of the magnetic flux density:

$$\boldsymbol{\sigma}(\boldsymbol{b}) = \boldsymbol{\sigma}_0 + \boldsymbol{\sigma}_1(\boldsymbol{b})$$

$$\boldsymbol{\sigma}_0^{\mathsf{T}} = \boldsymbol{\sigma}_0$$

$$\boldsymbol{\sigma}_1(\boldsymbol{b})^{\mathsf{T}} = -\boldsymbol{\sigma}_1(\boldsymbol{b})$$

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_0 + \mathscr{E}(\boldsymbol{S}\boldsymbol{b})$$

$$\boldsymbol{S} = -\mathrm{Cof}(\boldsymbol{\sigma}_0)\,\boldsymbol{A}_{\mathrm{H}}$$

$$\boldsymbol{\rho} = \boldsymbol{\rho}_0 + \mathscr{E}(\boldsymbol{A}_{\mathrm{H}}\boldsymbol{b})$$

$$\mathrm{Cof}(\boldsymbol{A})_{12} = (-1)^{1+2}\det\begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix} = -(A_{21}A_{33} - A_{31}A_{23})$$

L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media*, 1960

M. Briane and G.W. Milton, Arch. Ration. Mech. Anal. **193**, 715 (2009)

Isotropic materials

$$\boldsymbol{\sigma}_0 = \sigma_0 \boldsymbol{I} \qquad \boldsymbol{A}_\mathrm{H} = A_\mathrm{H} \boldsymbol{I}$$

b-field along $\hat{\boldsymbol{z}}$:

$$\boldsymbol{\sigma}(\boldsymbol{b}) = \begin{pmatrix} \sigma_0 & \sigma_0 A_\mathrm{H} b_z & 0 \\ -\sigma_0 A_\mathrm{H} b_z & \sigma_0 & 0 \\ 0 & 0 & \sigma_0 \end{pmatrix}$$

Effective Hall tensor

$$\boldsymbol{A}_{\mathrm{H}}(\boldsymbol{x}) \longrightarrow \boldsymbol{A}_{\mathrm{H}}^{*}$$

Small magnetic fields

$\boldsymbol{A}_{\mathrm{H}}^{*}$ can be obtained from a solution of the zero magnetic-field problem

$$\mathrm{Cof}\left(\boldsymbol{\sigma}_0^{*}\right)\boldsymbol{A}_{\mathrm{H}}^{*} = \left\langle \mathrm{Cof}\left(\boldsymbol{\sigma}_0 \nabla \boldsymbol{\Phi}\right)^{\mathsf{T}} \boldsymbol{A}_{\mathrm{H}} \right\rangle$$

$$\boldsymbol{\Phi} = (\phi_1, \phi_2, \phi_3)^{\mathsf{T}} \qquad \nabla \cdot (\boldsymbol{\sigma} \nabla \boldsymbol{\Phi}) = 0$$

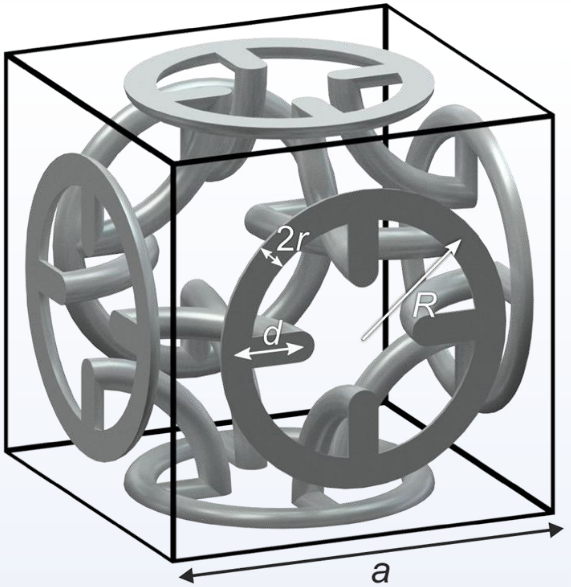
Isotropic case first considered by D. Bergman

$$A_{\mathrm{H}}^{*} = \left\langle \left(\tilde{J}_{11}\tilde{J}_{22} - \tilde{J}_{21}\tilde{J}_{12}\right) A_{\mathrm{H}}(\boldsymbol{x}) \right\rangle$$

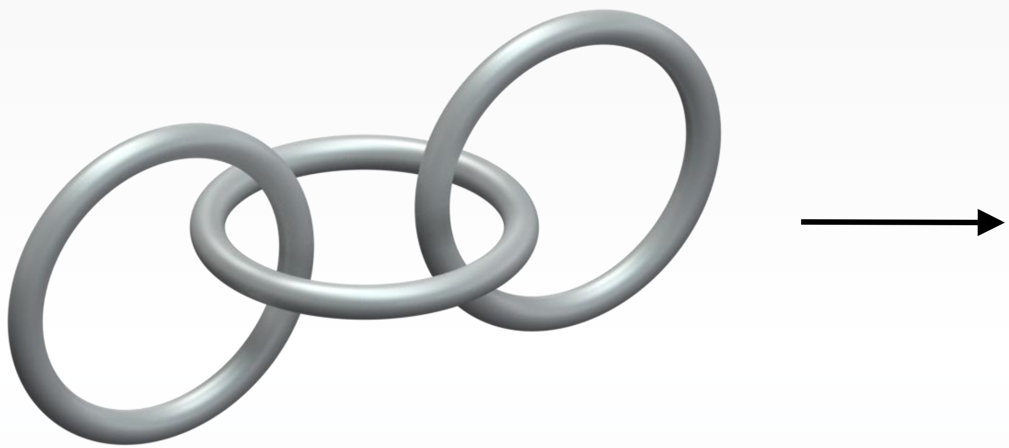
M. Briane and G.W. Milton, Arch. Ration. Mech. Anal. **193**, 715 (2009)

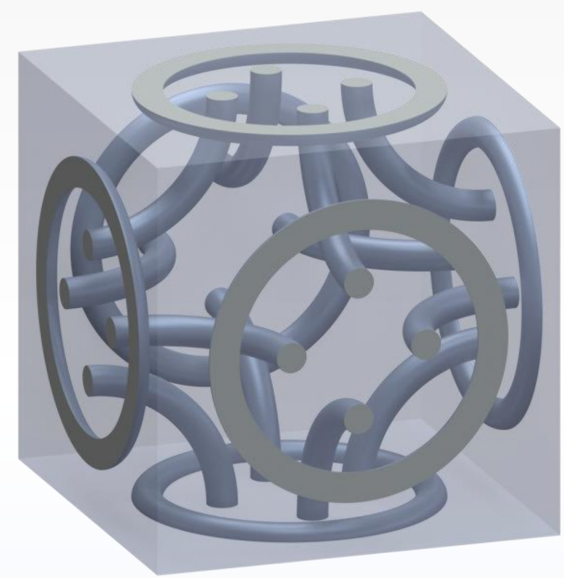
D. Bergman, in *Percolation Structures and Processes,* eds. G. Deutscher, R. Zallen, and J. Adler, 1983, pp. 297

Sign-inversion of the effective Hall coefficient



Change of sign of the determinant

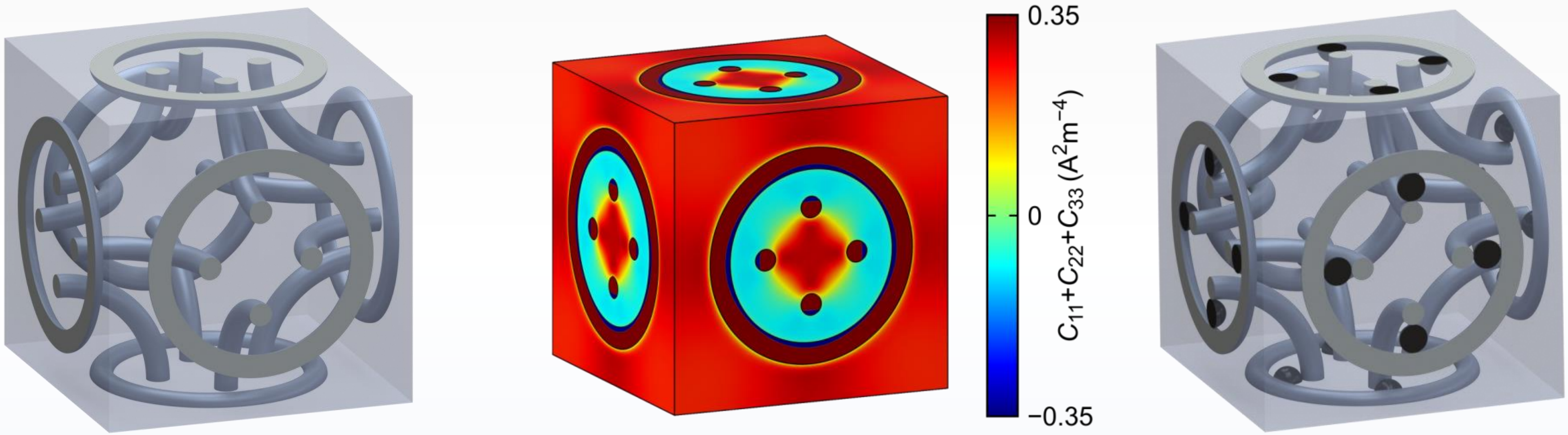




$$\mathrm{Cof}\,(\boldsymbol{\sigma}_0^*)\,\boldsymbol{A}_{\mathrm{H}}^* = \langle \mathrm{Cof}\,(\boldsymbol{\sigma}_0 \nabla \boldsymbol{\Phi})^{\mathsf{T}}\,\boldsymbol{A}_{\mathrm{H}} \rangle$$

M. Briane, G. W. Milton, and V. Nesi, Arch. Ration. Mech. Anal. **173**, 133 (2004)

3D structure suggested by chainmail-artist Dylon Whyte



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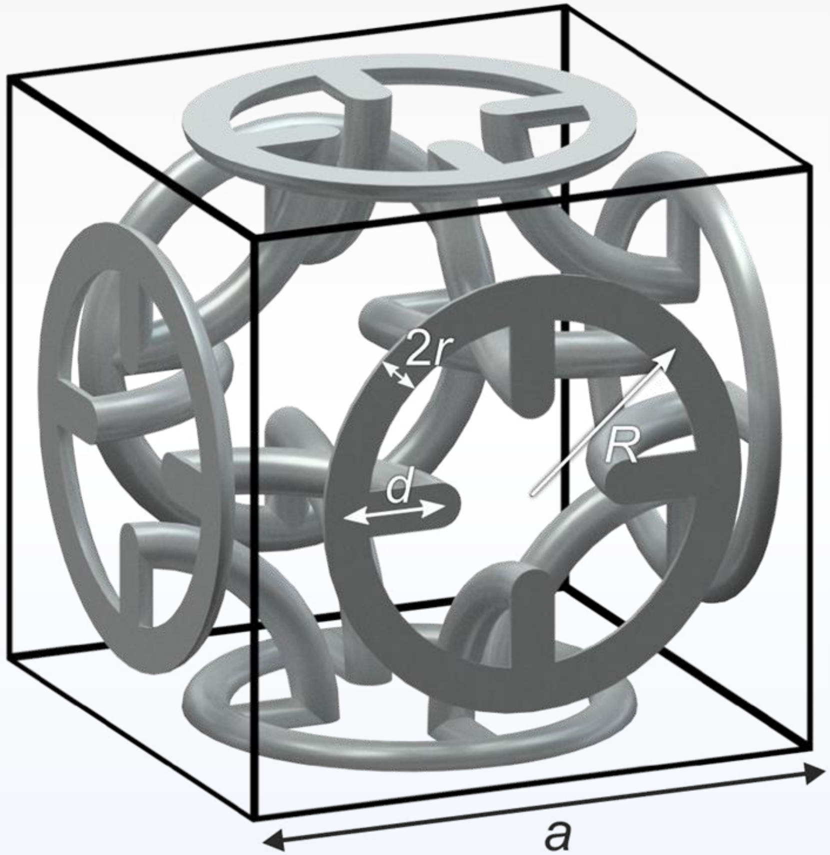
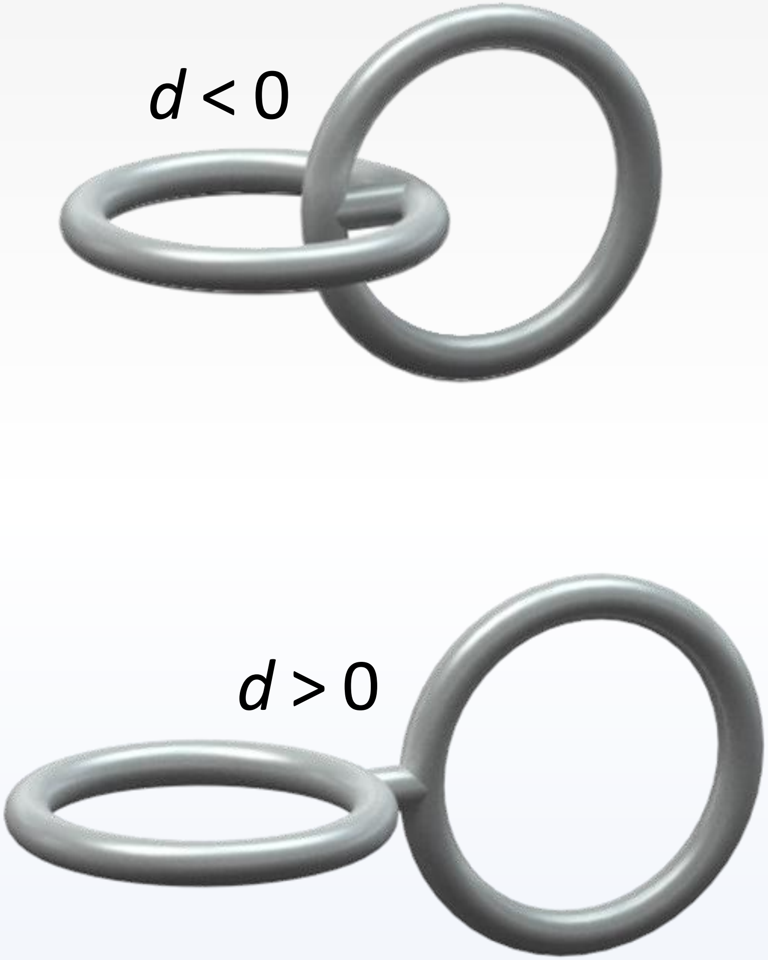
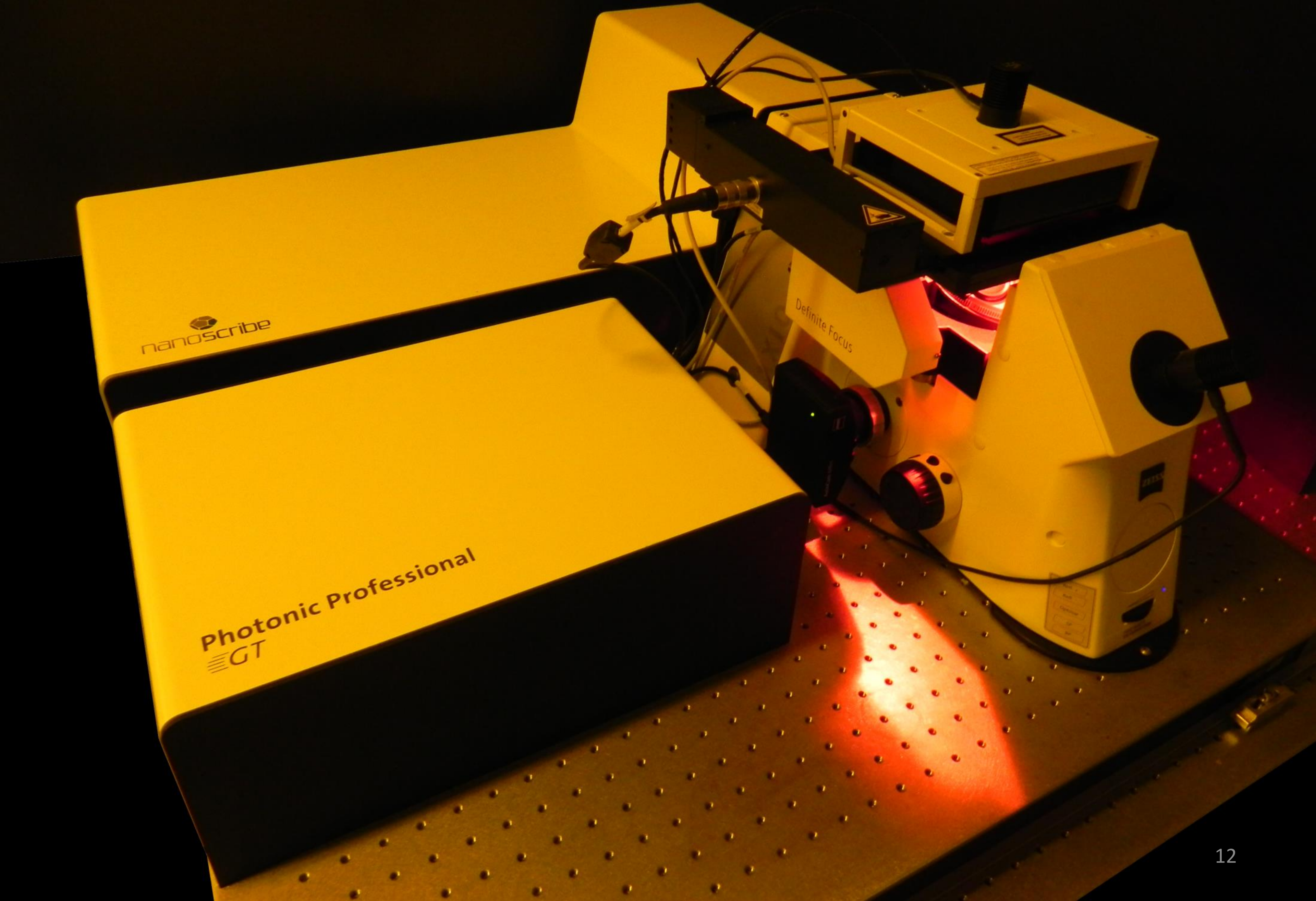
M. Briane and G. W. Milton, Arch. Ration. Mech. Anal. **193**, 715 (2009)

C. Kern *et al.*, arXiv:1806.04914 [cond-mat.mes-hall]





M. Kadic *et al.*, Phys. Rev. X **5**, 021030 (2015)


nanoscribe
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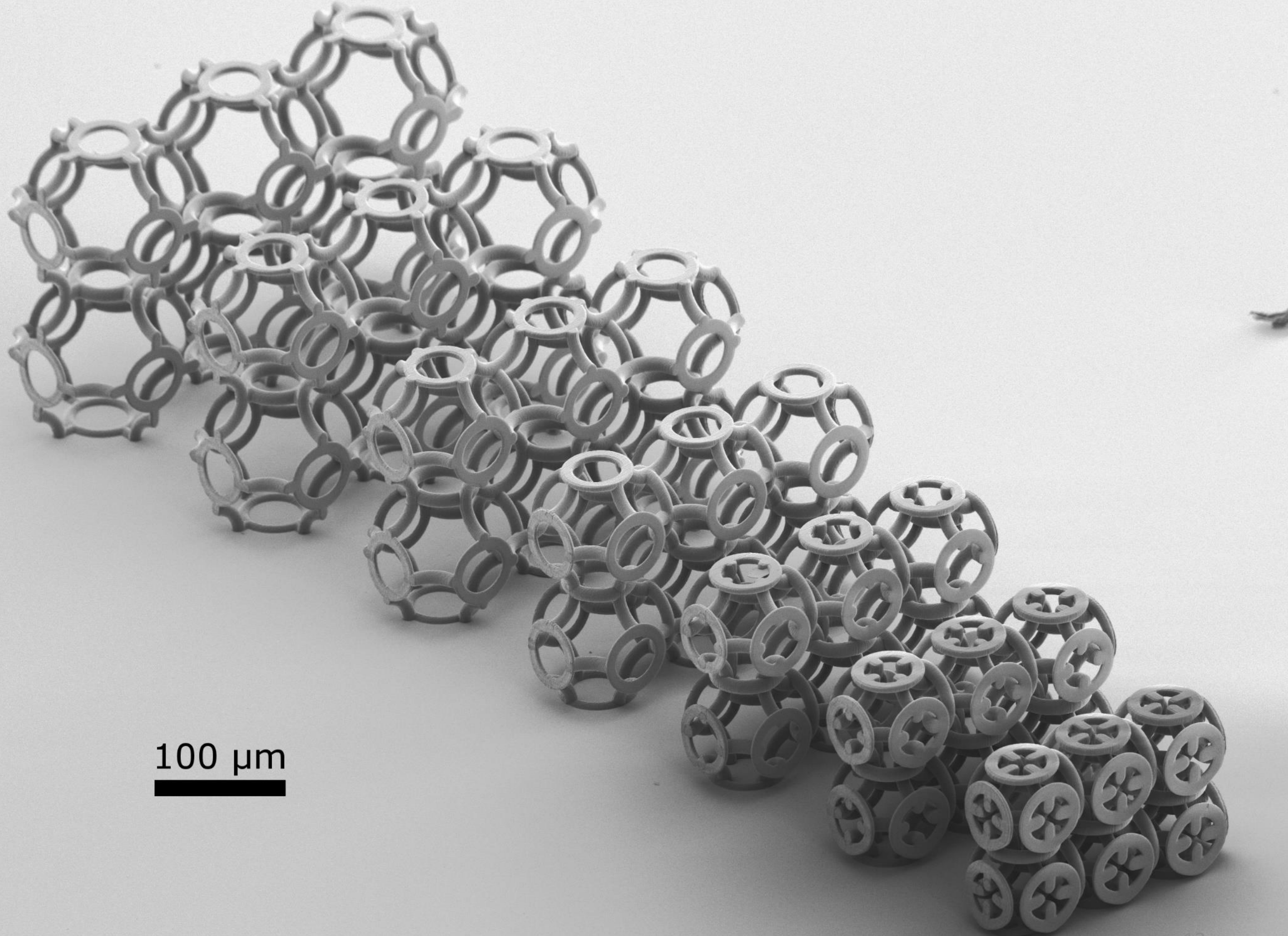
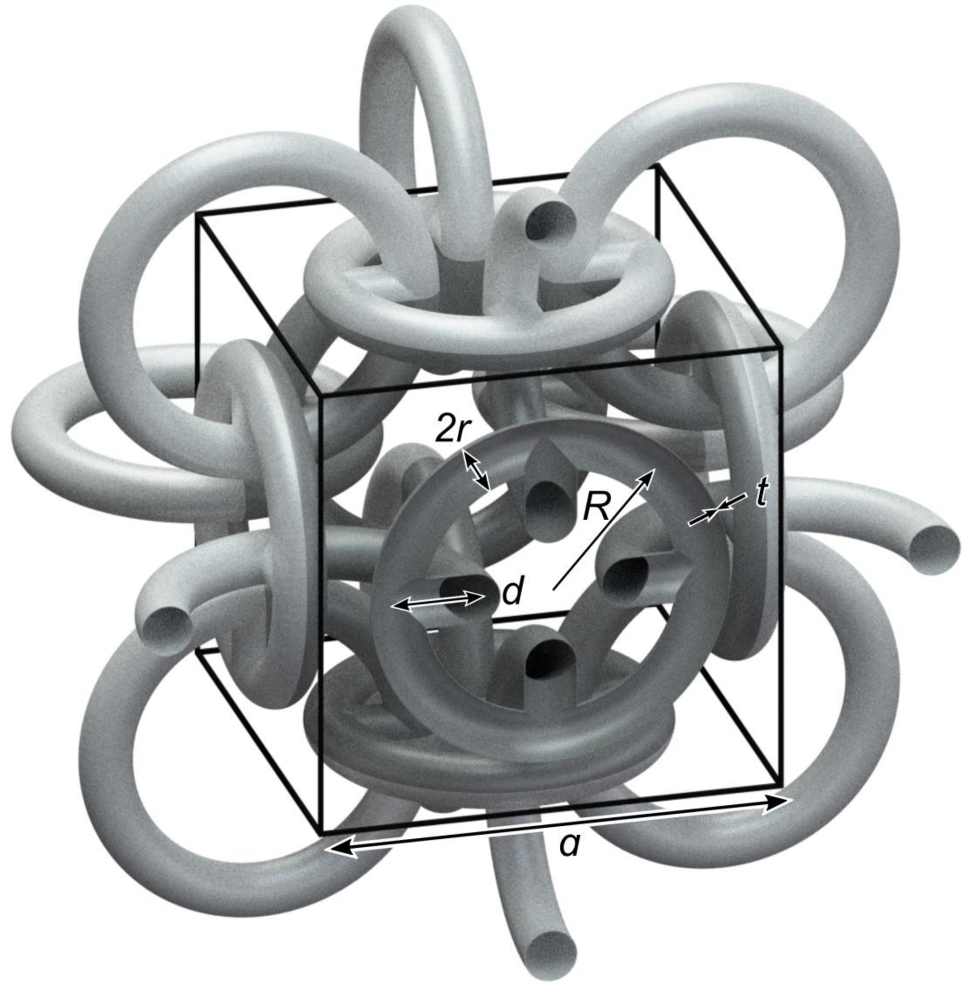
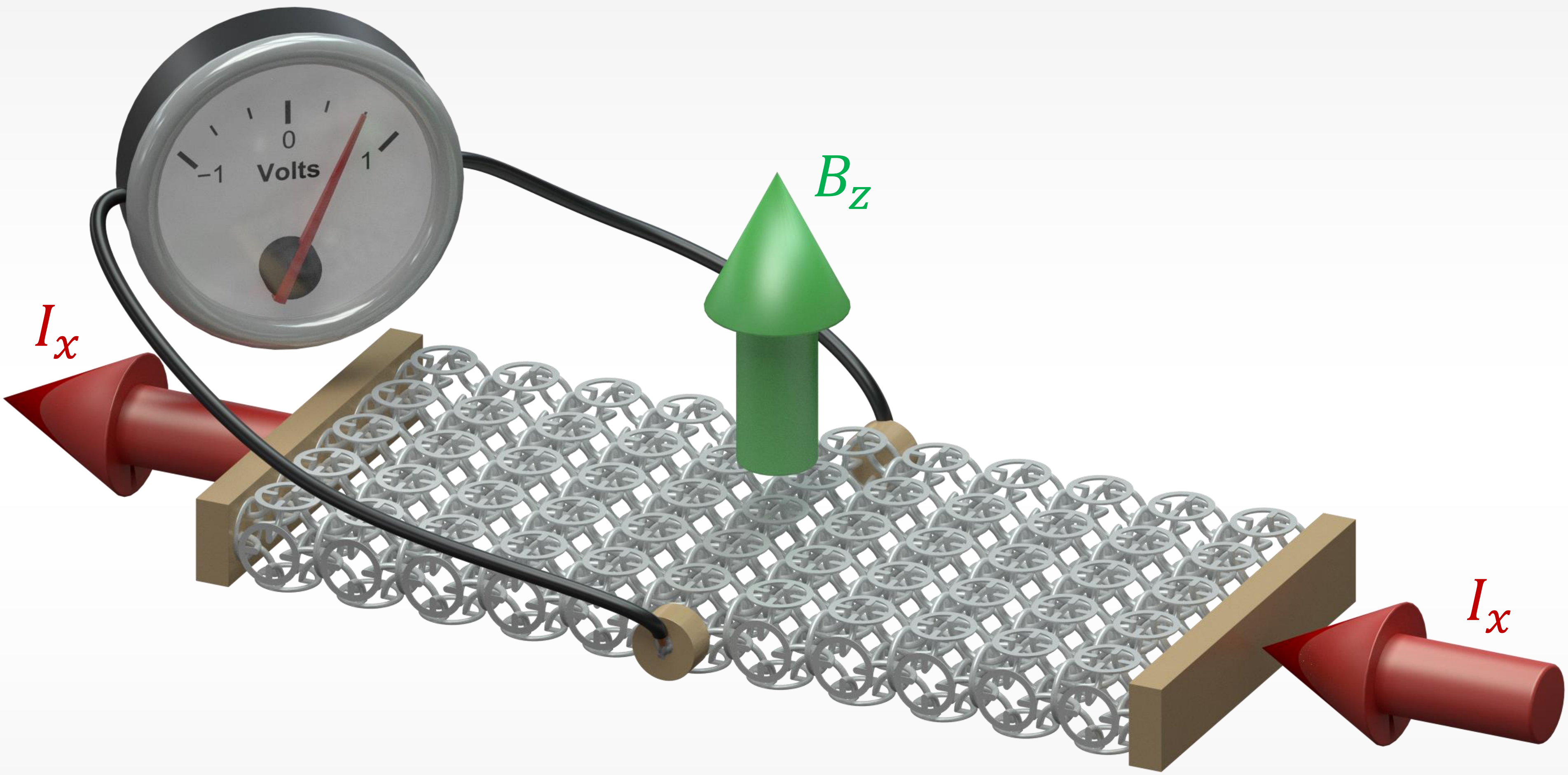


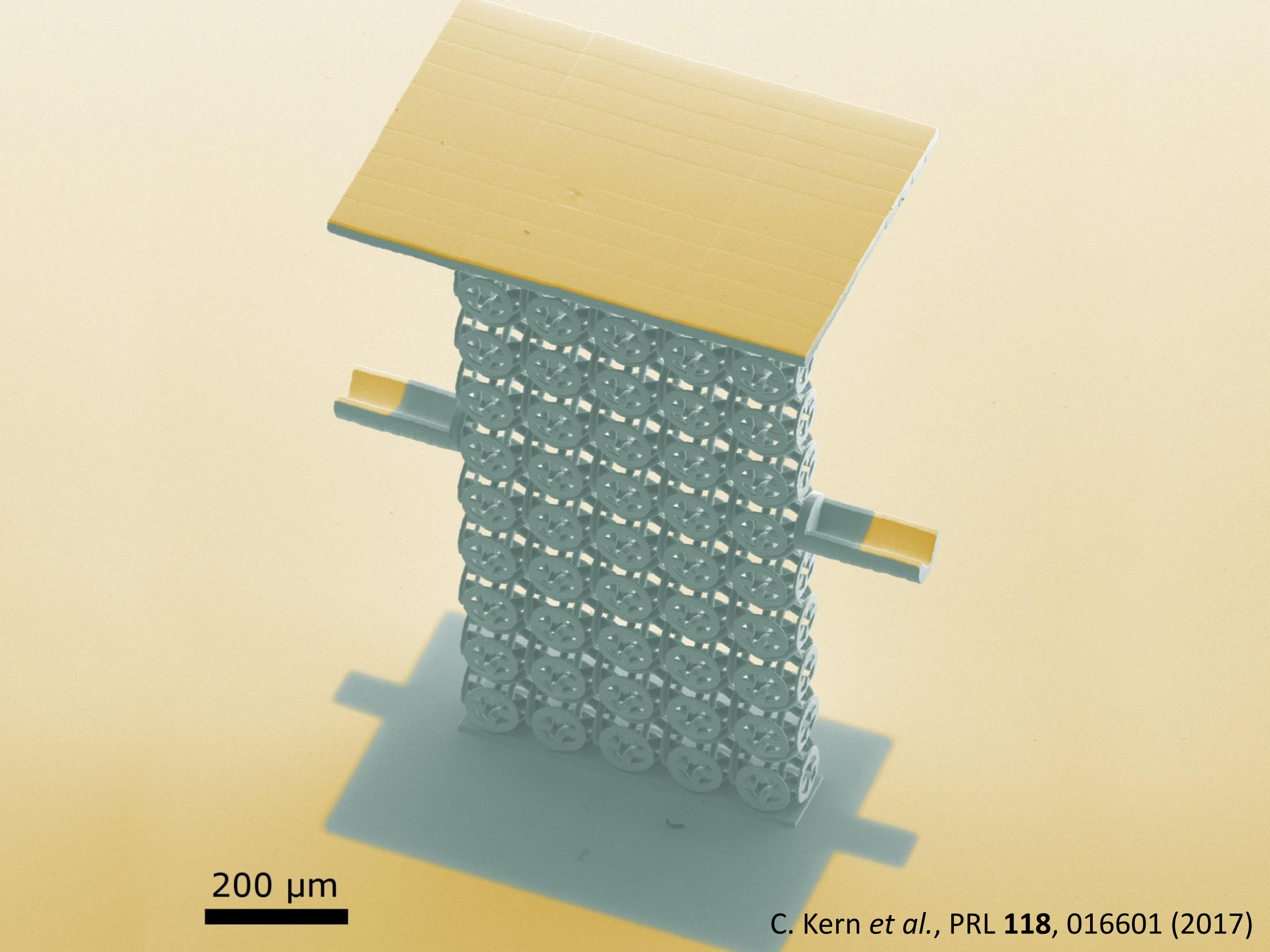
C. Kern *et al.*, PRL **118**, 016601 (2017)



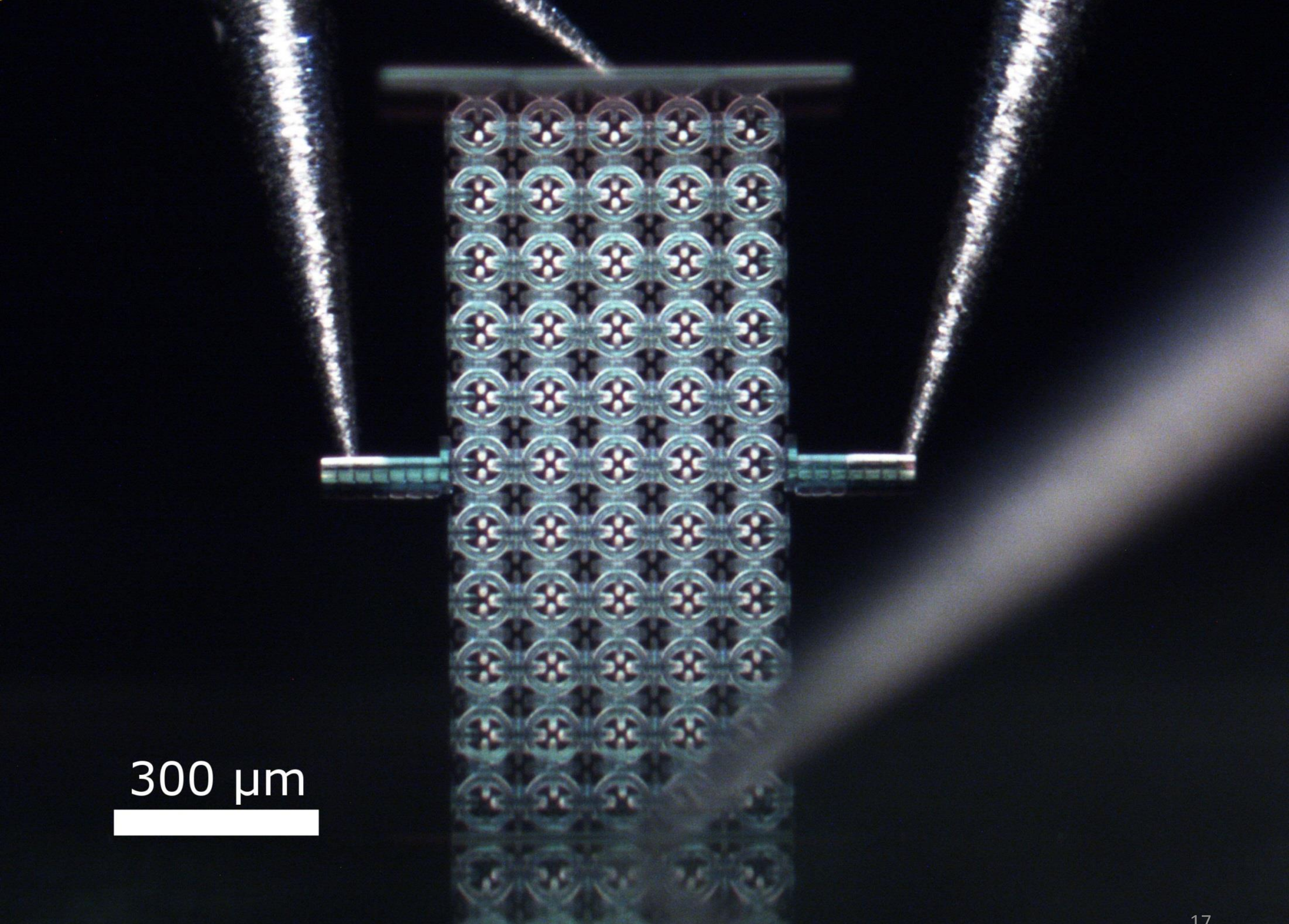
M. Kadic *et al.*, Phys. Rev. X **5**, 021030 (2015)



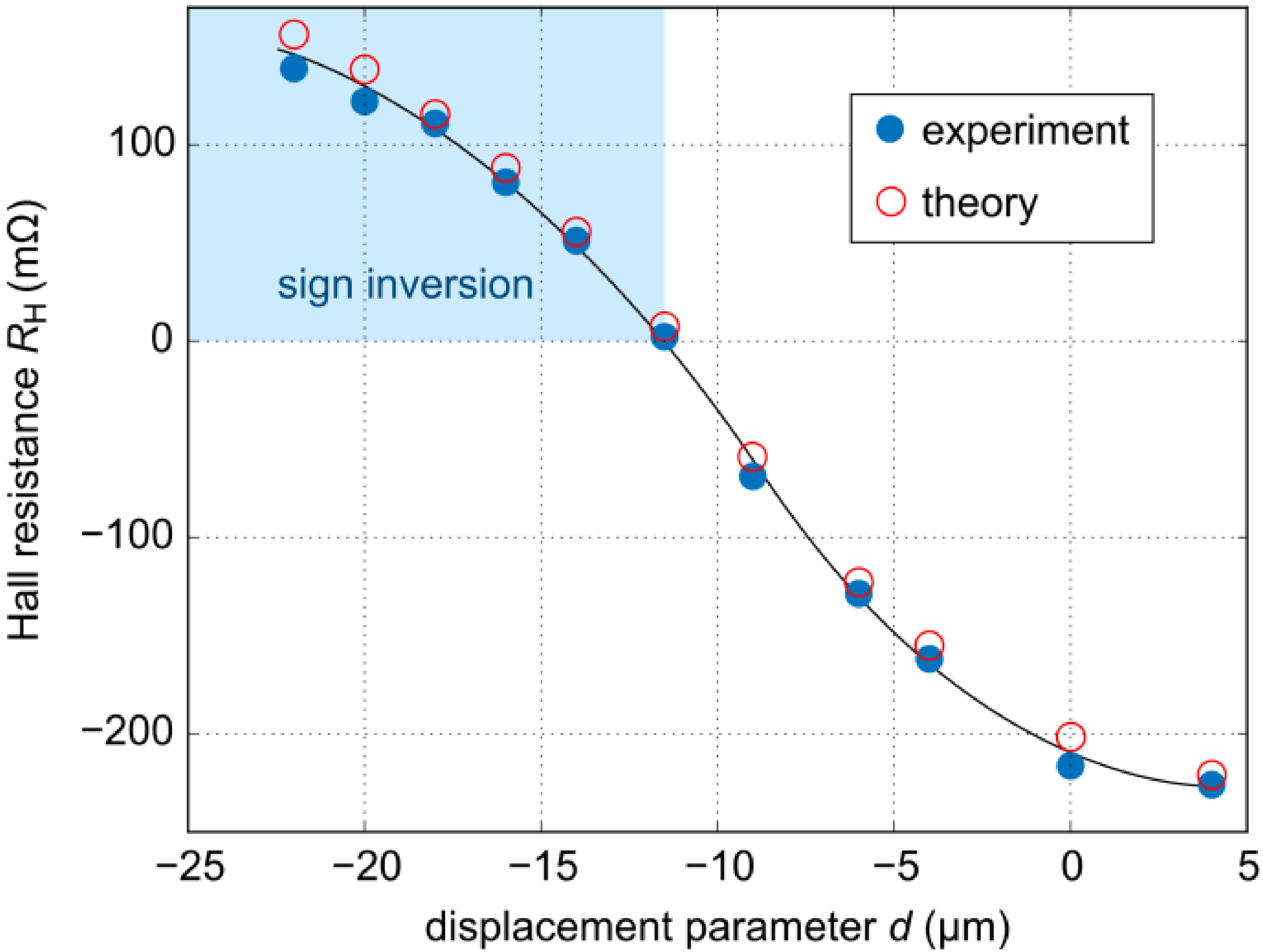
M. Kadic *et al.*, Phys. Rev. X **5**, 021030 (2015)



C. Kern *et al.*, PRL **118**, 016601 (2017)

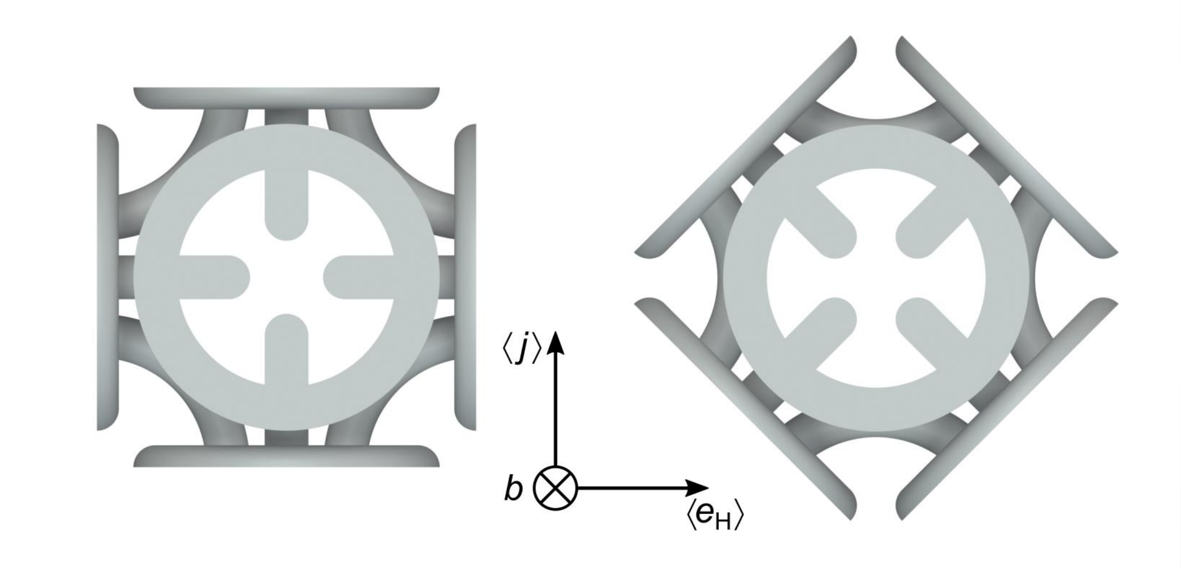


C. Kern *et al.*, PRL **118**, 016601 (2017)



C. Kern *et al.*, PRL **118**, 016601 (2017)

What happens for other unit cell orientations?



Symmetry considerations

Neumann's principle

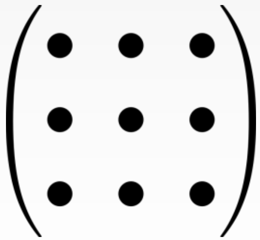
If a system has a certain group of symmetry operations then any physical observable of that system must also possess these symmetry operations.

⟶ Symmetry of the microstructure gives us the form of the effective tensors

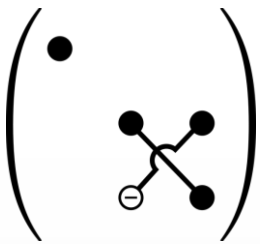
J. F. Nye, *Physical Properties of Crystals*, 1957

Symmetry considerations

Triclinic



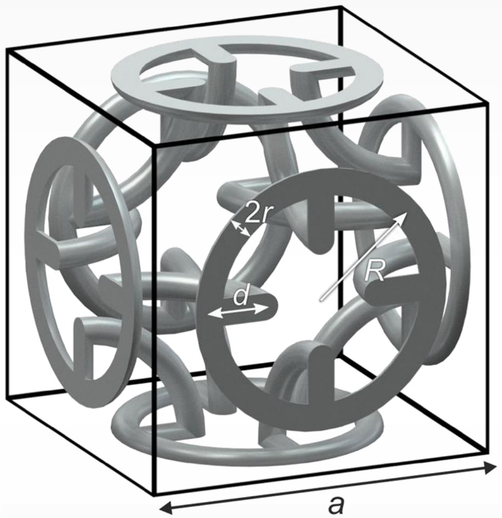
Tetragonal
One four-fold rotational axis



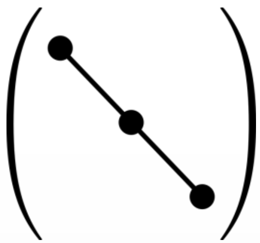
Cubic
Four three-fold rotational axes

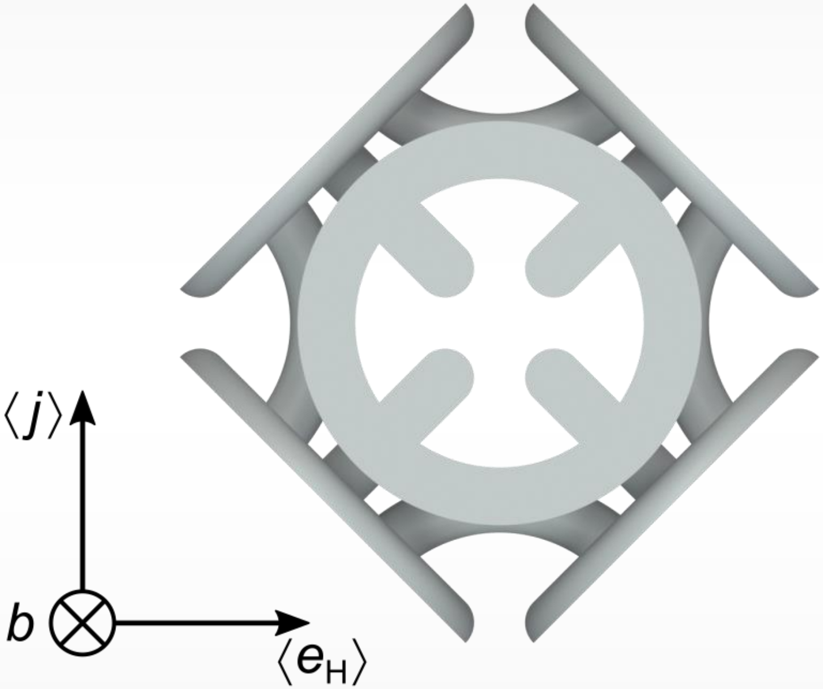
J. F. Nye, *Physical Properties of Crystals*, 1957

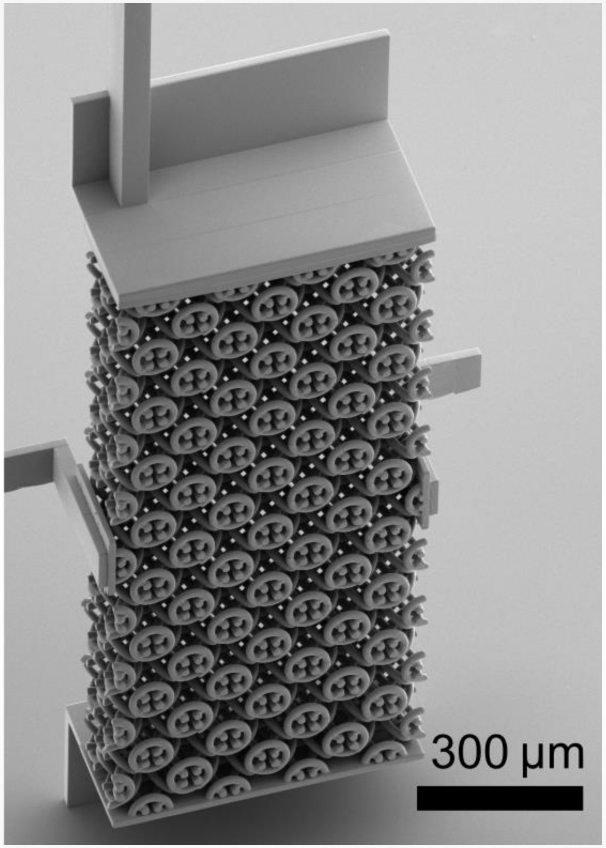
Symmetry considerations



Cubic
One four-fold rotational axis

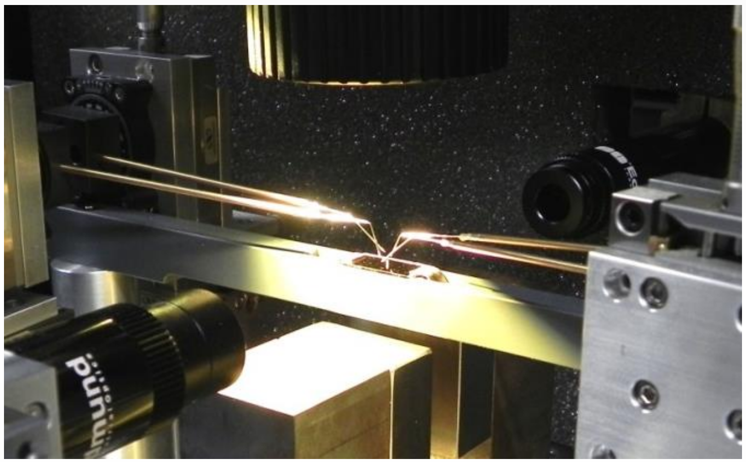



⟨j⟩
b
⟨e_H⟩

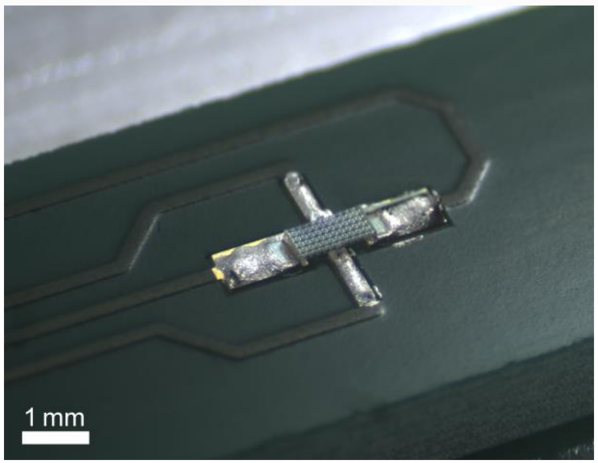

300 µm

Current state of experiments

Probe station based

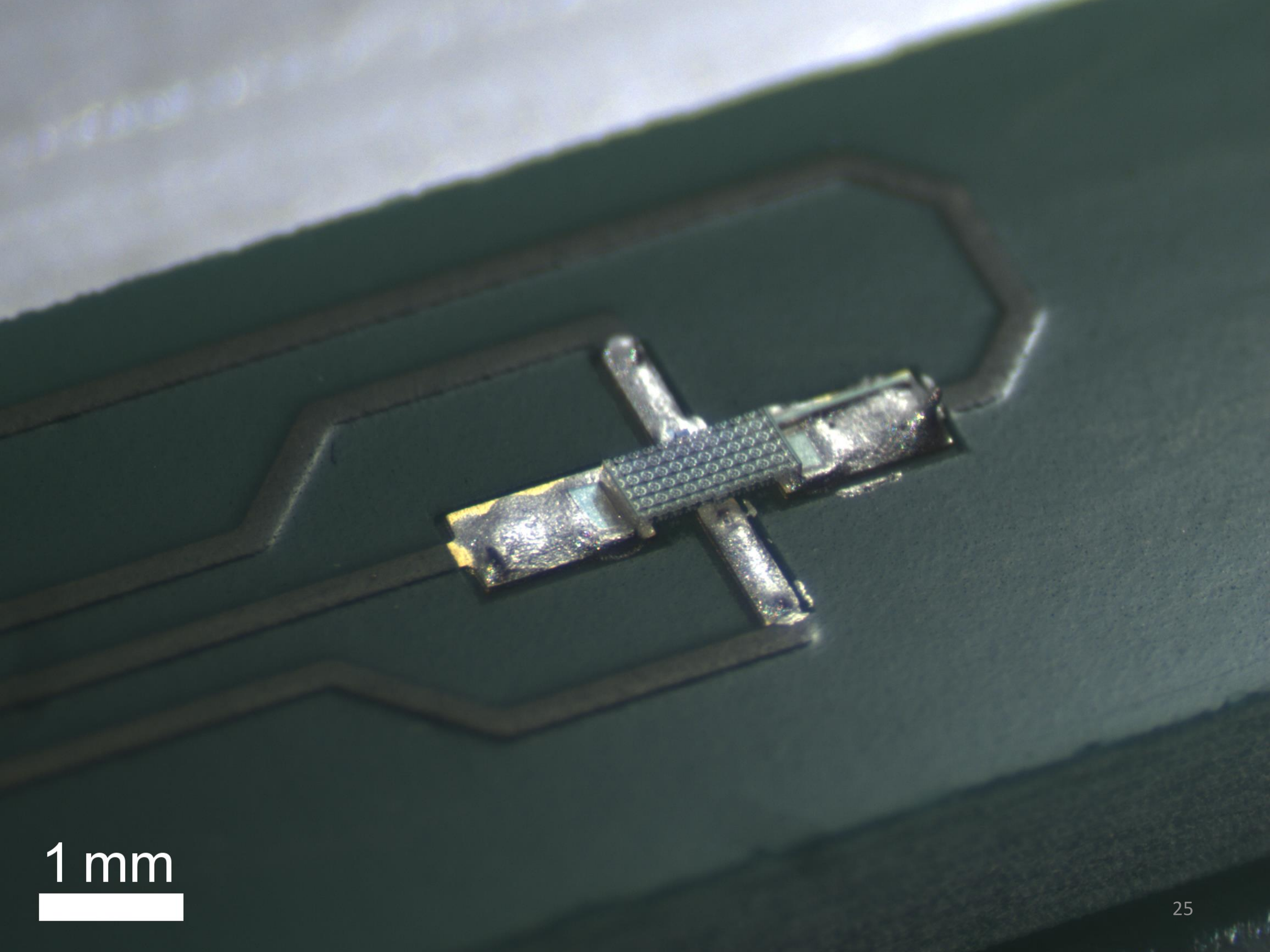


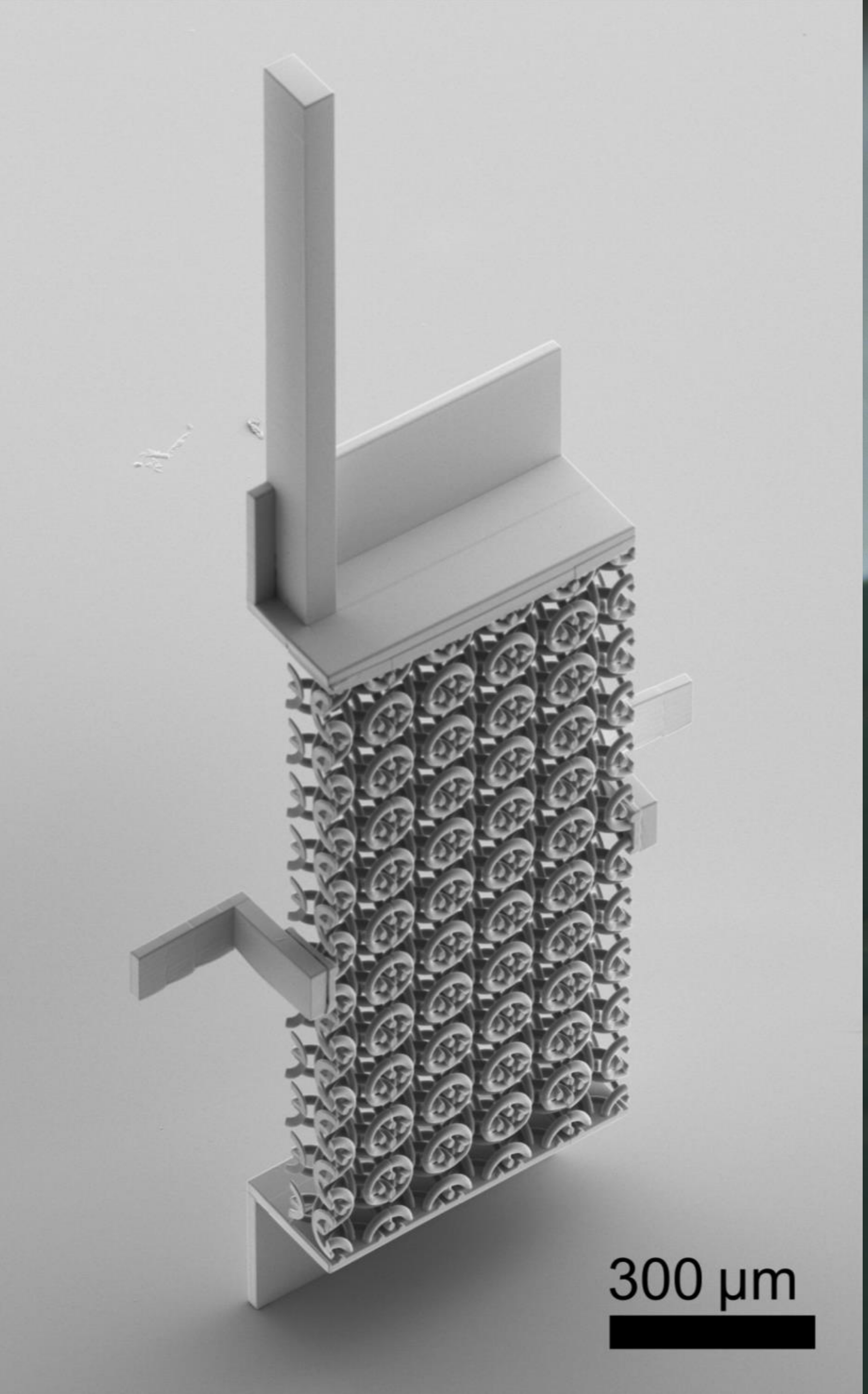
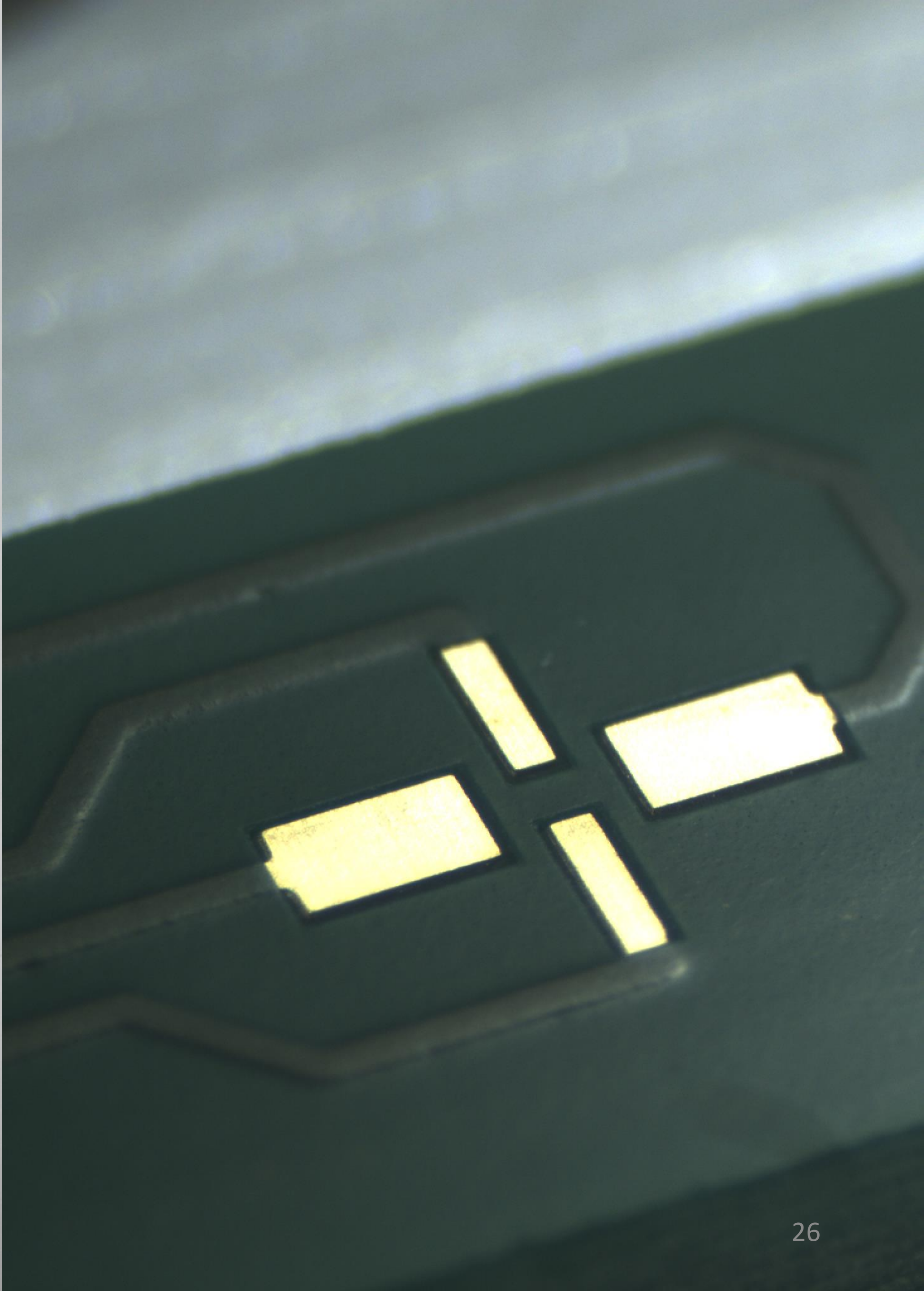
Integrated device

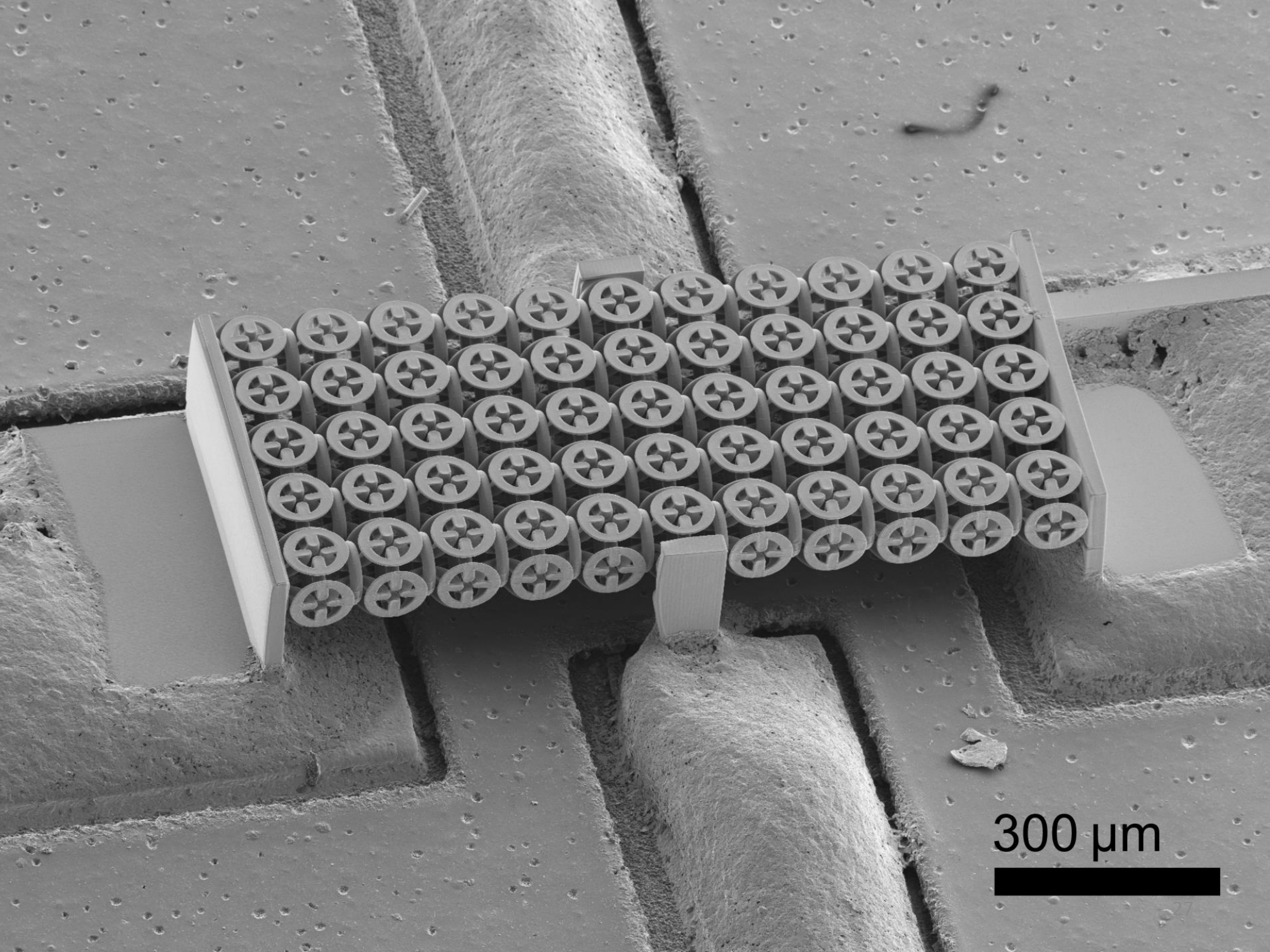


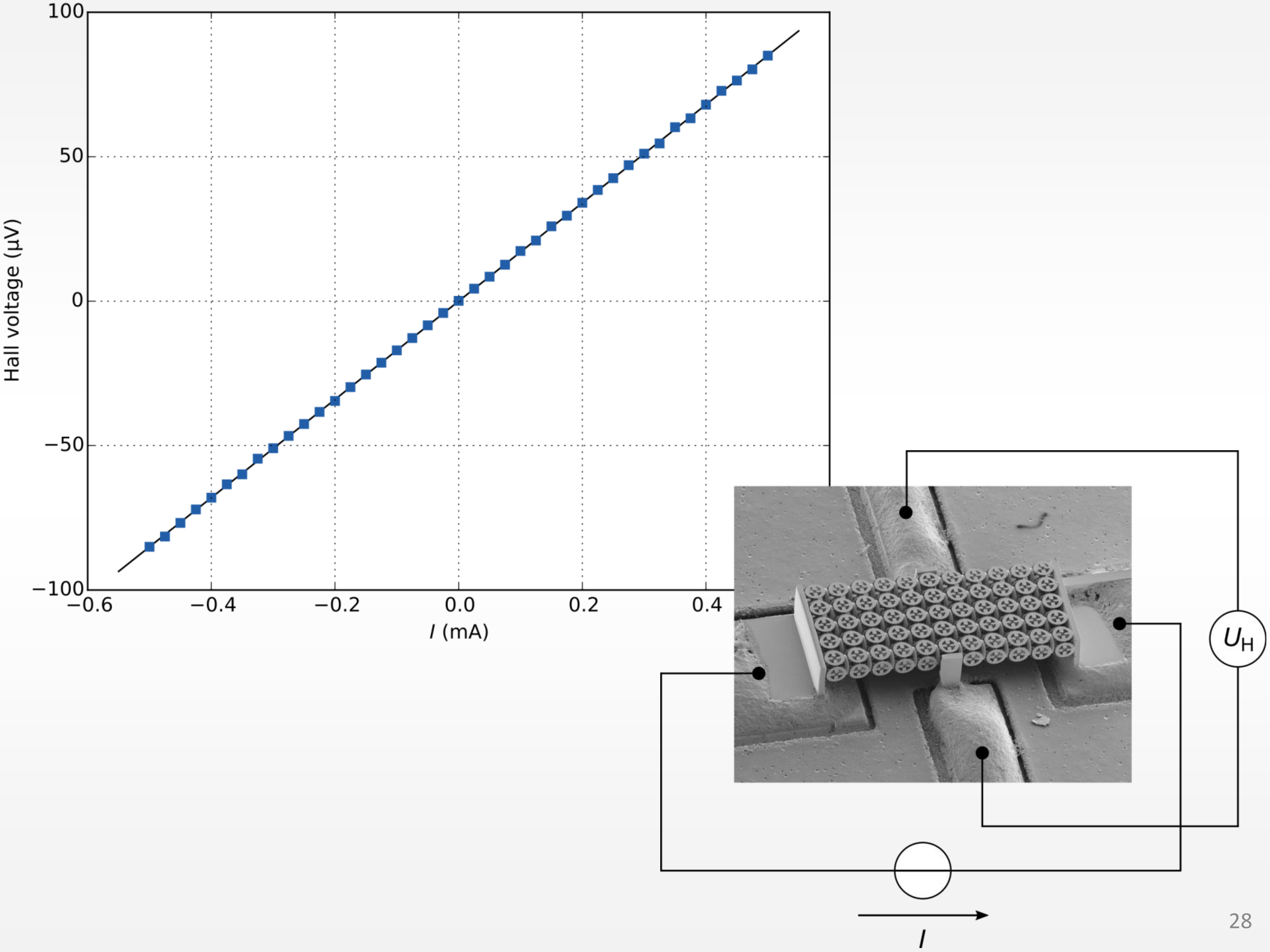
- "Less demanding" fabrication

- Facilitates measurements
- Prerequisite for applications


1 mm

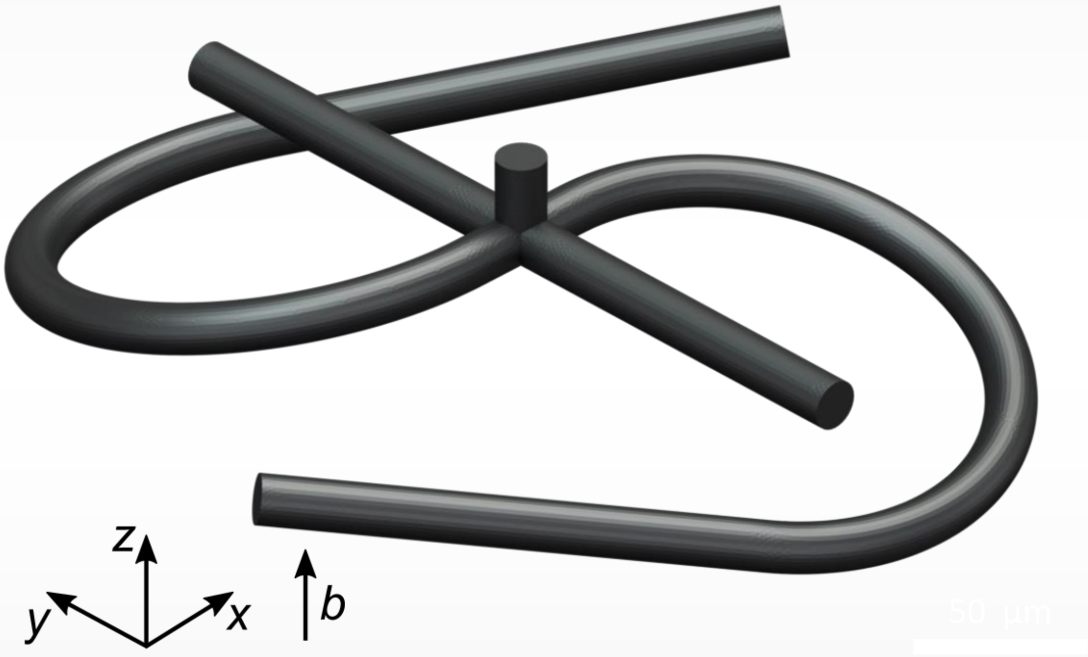

300 µm



300 µm

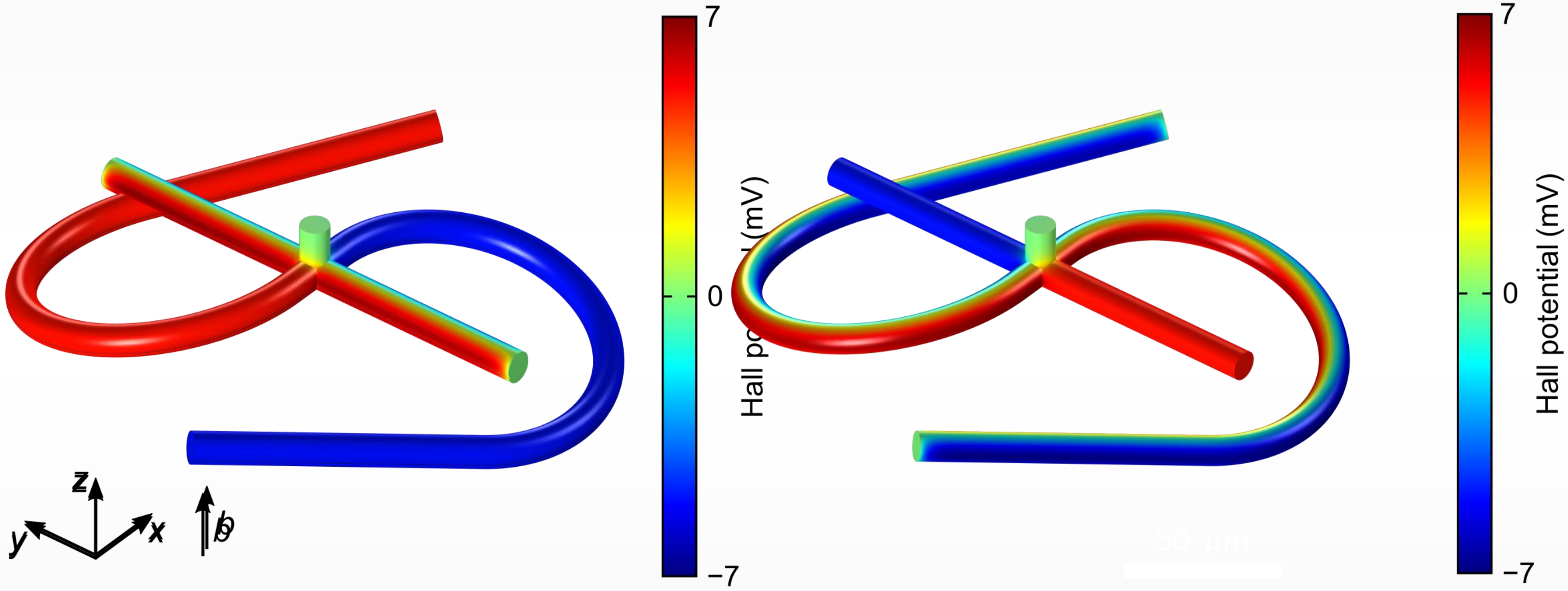


Hall voltage (µV)
100
50
0
−50
−100
−0.6
−0.4
−0.2
0.0
0.2
0.4
I (mA)
U_H
I

Are there other composites showing a sign-inverted effective Hall coefficient?



C. Kern *et al.*, arXiv:1806.04914 [cond-mat.mes-hall]



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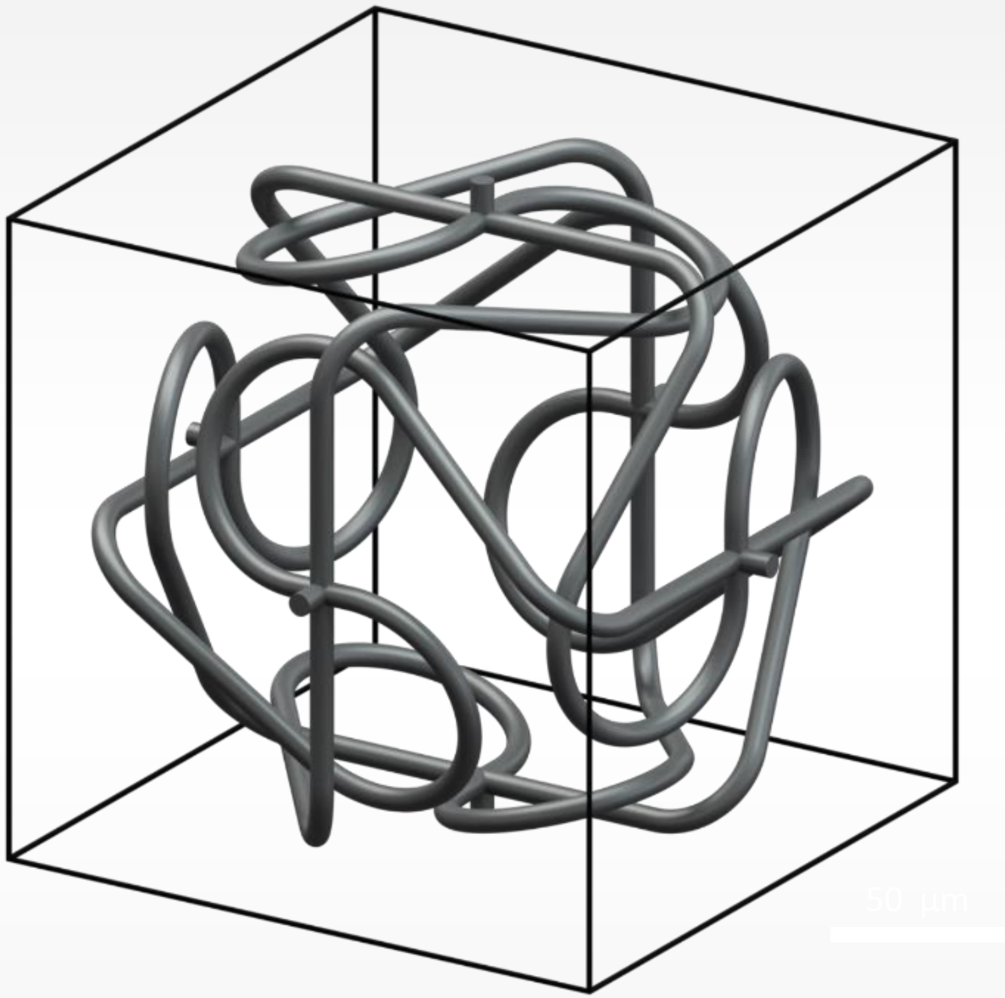


$$\mathrm{Cof}\,(\boldsymbol{\sigma}_0^*)\,\boldsymbol{A}_\mathrm{H}^* = \langle \mathrm{Cof}\,(\boldsymbol{\sigma}_0 \nabla \boldsymbol{\Phi})^\intercal\,\boldsymbol{A}_\mathrm{H} \rangle$$

C. Kern *et al.*, arXiv:1806.04914 [cond-mat.mes-hall]

Anisotropic structures

An antisymmetric Hall tensor

$$\boldsymbol{A}_{\mathrm{H}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & A_{23} \\ 0 & -A_{23} & 0 \end{pmatrix}$$

Current along $\hat{\boldsymbol{x}}$, magnetic field in the yz-plane

$$\boldsymbol{j} = j_x \hat{\boldsymbol{x}} \qquad \boldsymbol{b} = b_y \hat{\boldsymbol{y}} + b_z \hat{\boldsymbol{z}}$$

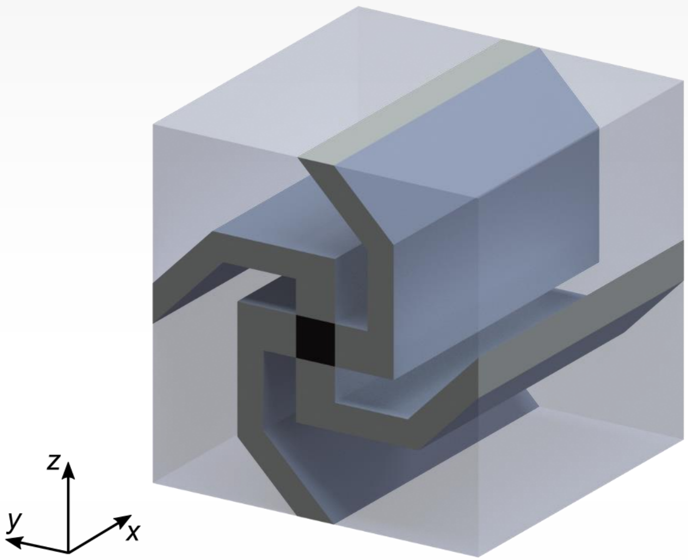
$$\longrightarrow \quad \boldsymbol{e}_{\mathrm{H}} = \mathscr{E}\left(\boldsymbol{A}_{\mathrm{H}} \boldsymbol{b}\right) \boldsymbol{j} = -A_{23} j_x \left(b_y \hat{\boldsymbol{y}} + b_z \hat{\boldsymbol{z}}\right)$$

The Hall electric field is parallel to the magnetic field: ***Parallel Hall effect***

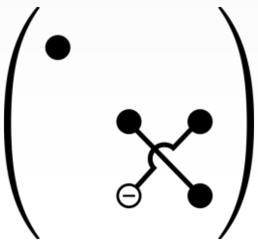
M. Briane and G. W. Milton, SIAM J. Appl. Math. **70**, 1810 (2010)

C. Kern *et al.*, Appl. Phys. Lett. **107**, 132103 (2015)

An antisymmetric Hall tensor



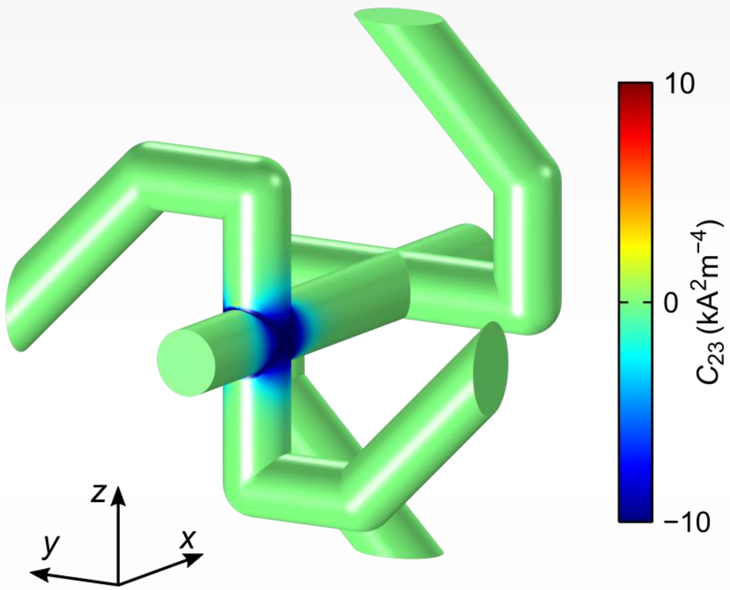
Tetragonal symmetry implies



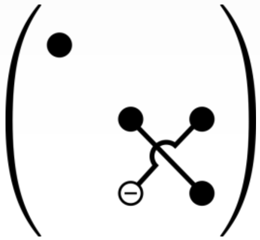
$$\boldsymbol{A}_{\mathrm{H}}^{*} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \boldsymbol{A}_{\mathrm{H}}^{0}$$

M. Briane and G. W. Milton, SIAM J. Appl. Math. **70**, 1810 (2010)

An antisymmetric Hall tensor

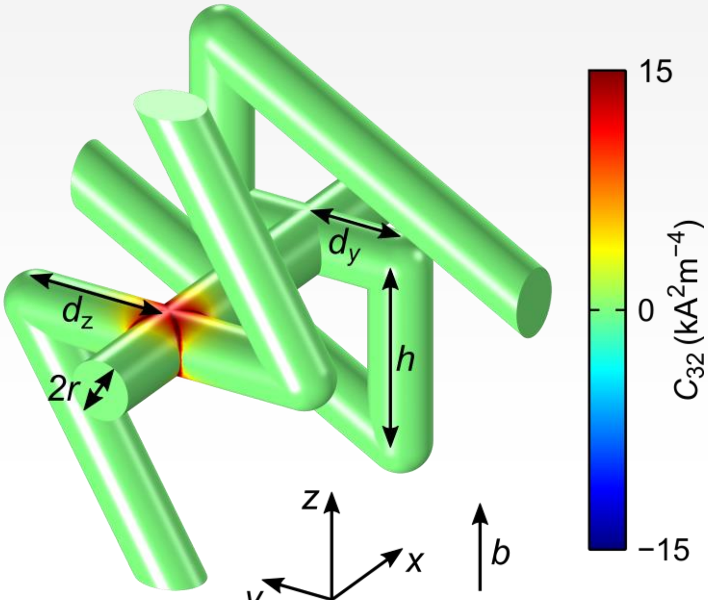


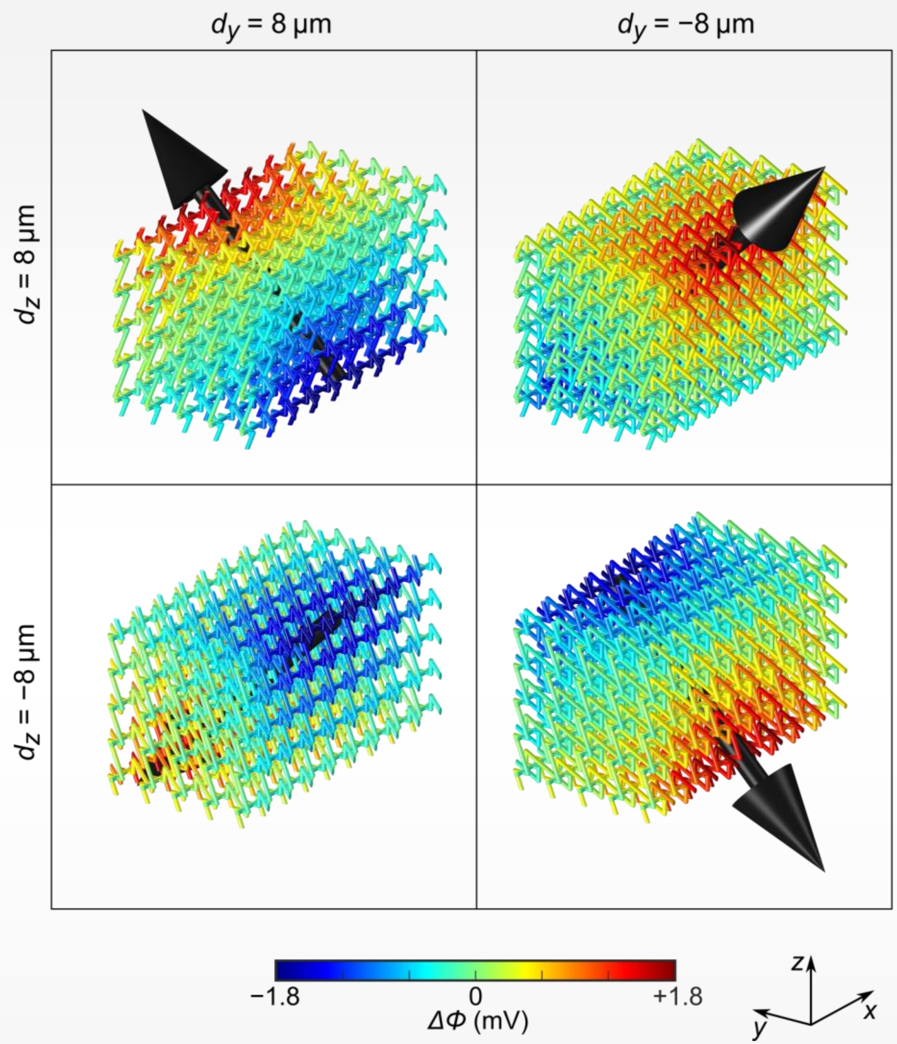
Tetragonal symmetry implies



$$\boldsymbol{A}_{\mathrm{H}}^{*} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.05 & 8.81 \\ 0 & -8.81 & 0.05 \end{pmatrix} A_{\mathrm{H}}^{0}$$

C. Kern *et al.*, arXiv:1806.04914 [cond-mat.mes-hall]



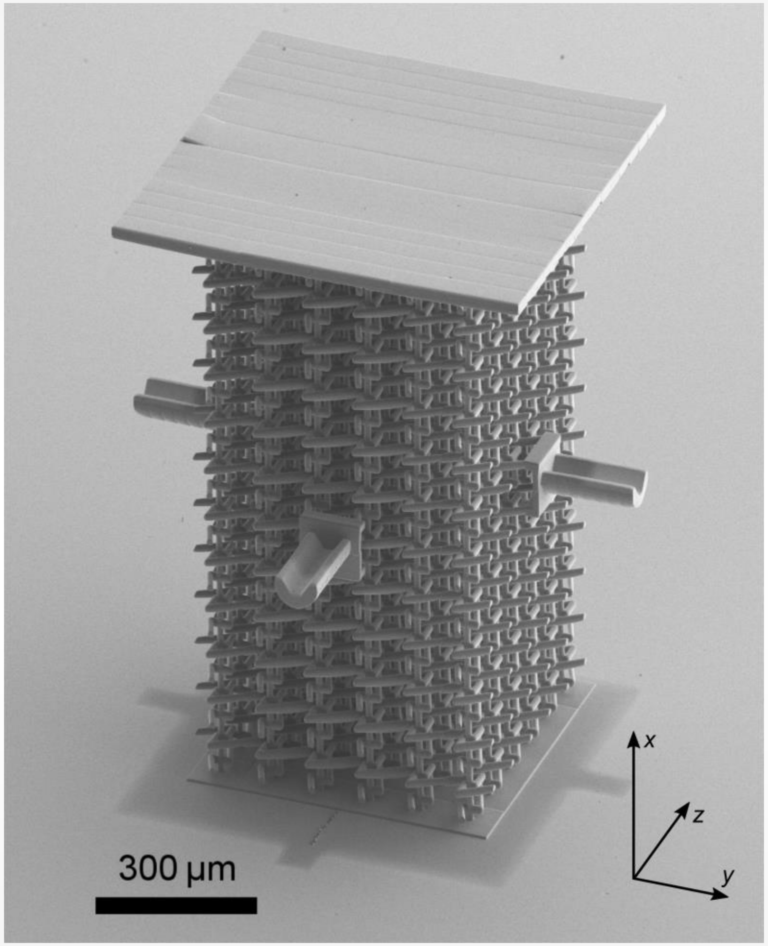


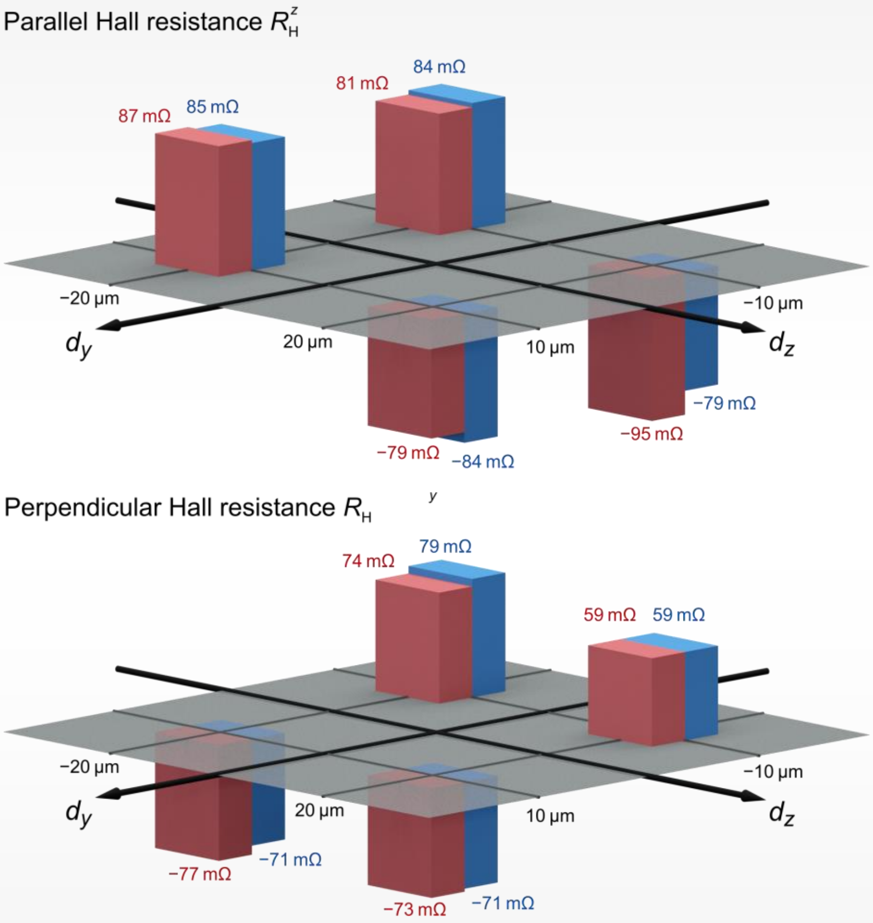
Monoclinic symmetry implies

$$\begin{pmatrix} \bullet & & \\ & \bullet & \bullet \\ & \bullet & \bullet \end{pmatrix}$$

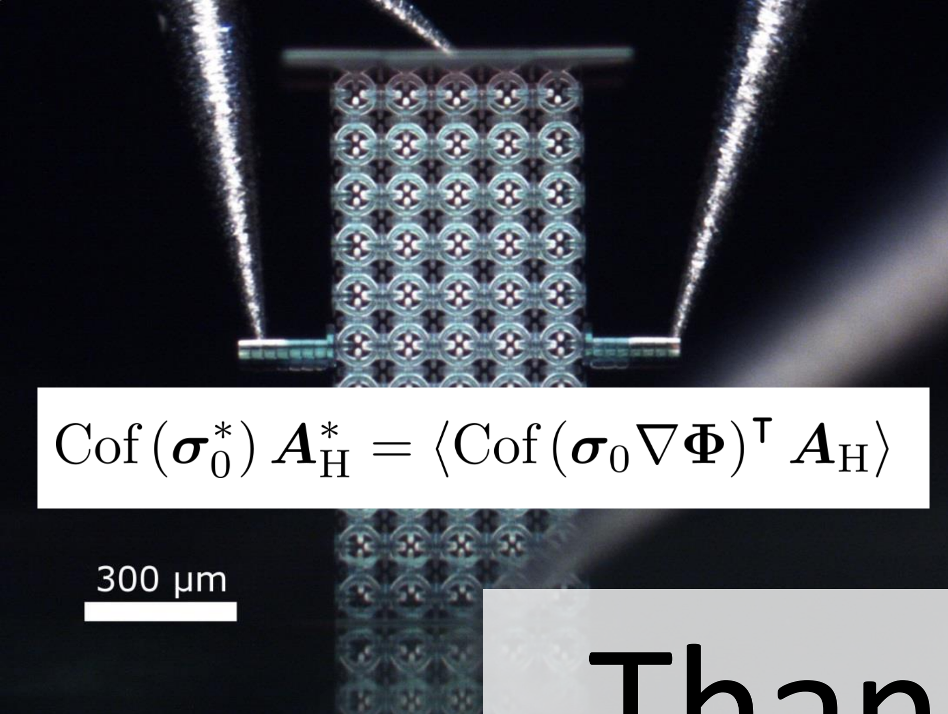
C. Kern *et al.*, Appl. Phys. Lett. **107**, 132103 (2015)

C. Kern *et al.*, arXiv:1806.04914 [cond-mat.mes-hall]

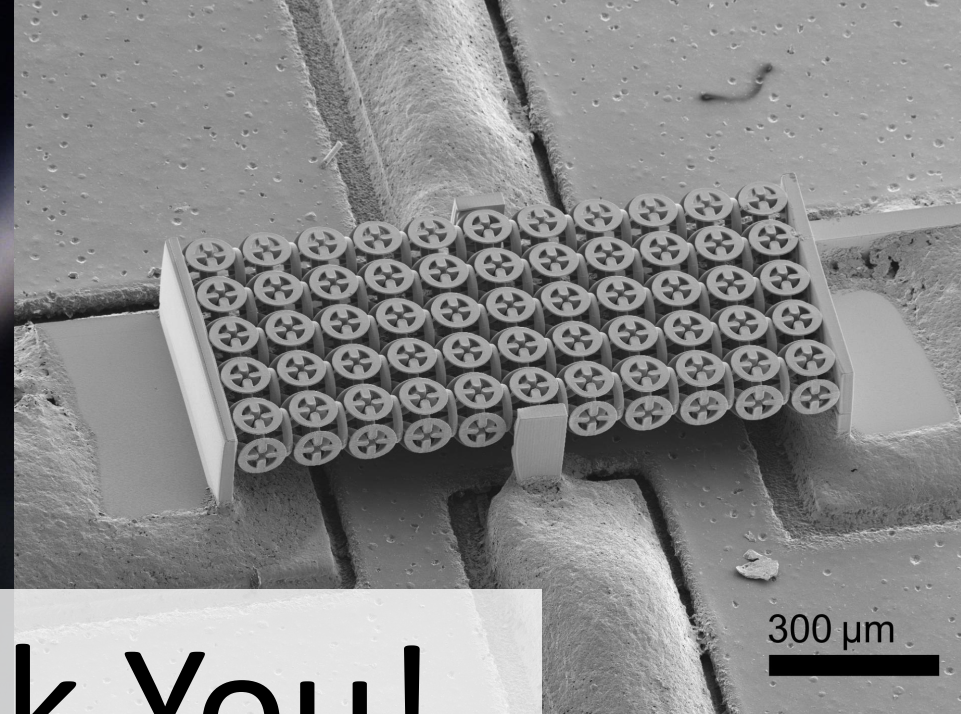




C. Kern *et al.*, Phys. Rev. Applied **7**, 044001 (2017)



$$\mathrm{Cof}\left(\boldsymbol{\sigma}_0^*\right) \boldsymbol{A}_{\mathrm{H}}^* = \left\langle \mathrm{Cof}\left(\boldsymbol{\sigma}_0 \nabla \boldsymbol{\Phi}\right)^{\mathsf{T}} \boldsymbol{A}_{\mathrm{H}} \right\rangle$$



Thank You!