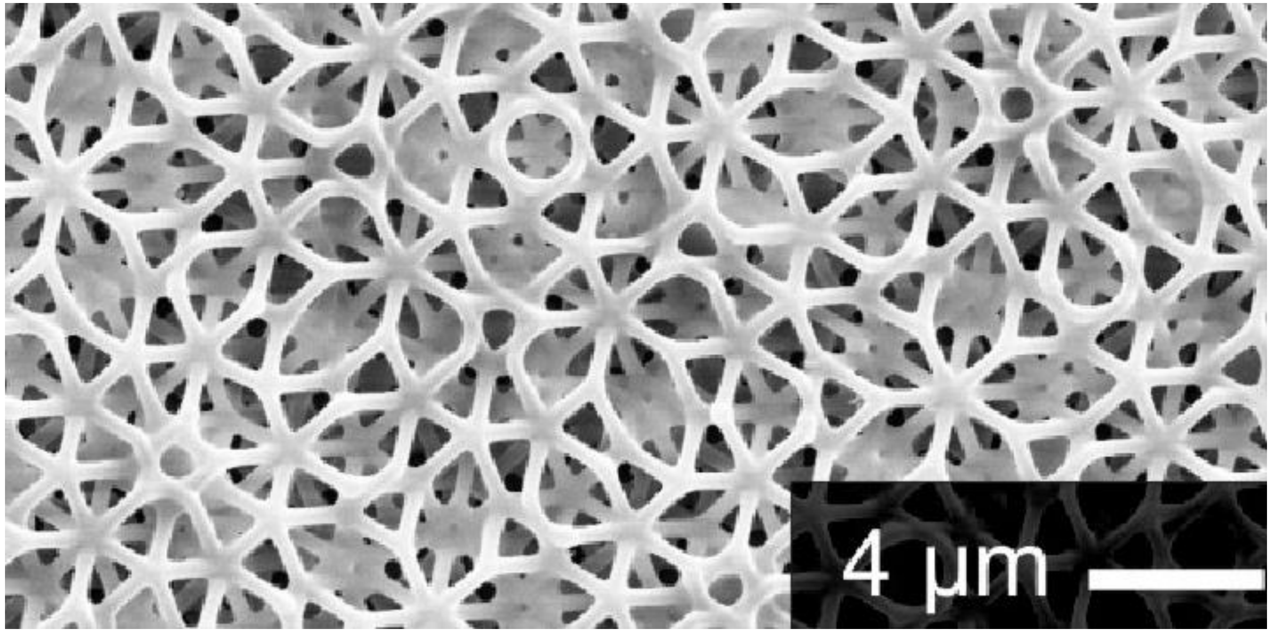
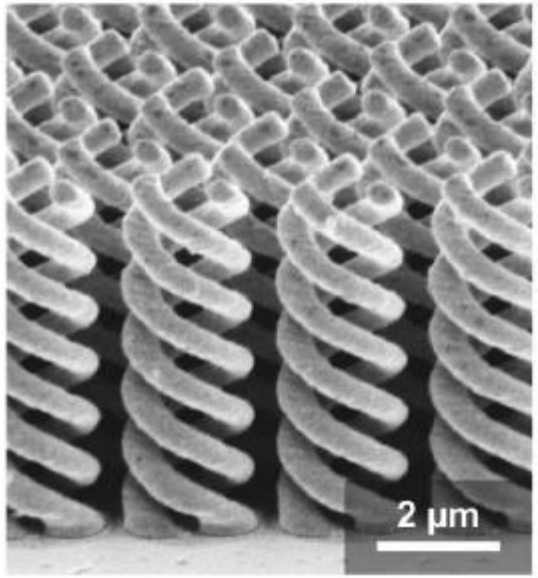
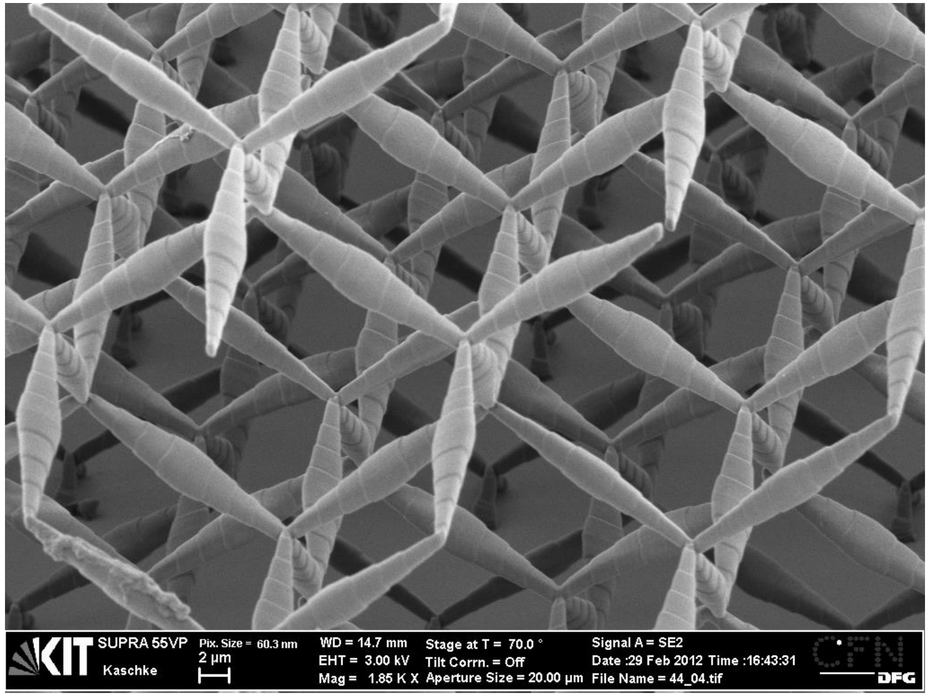
Metamaterials

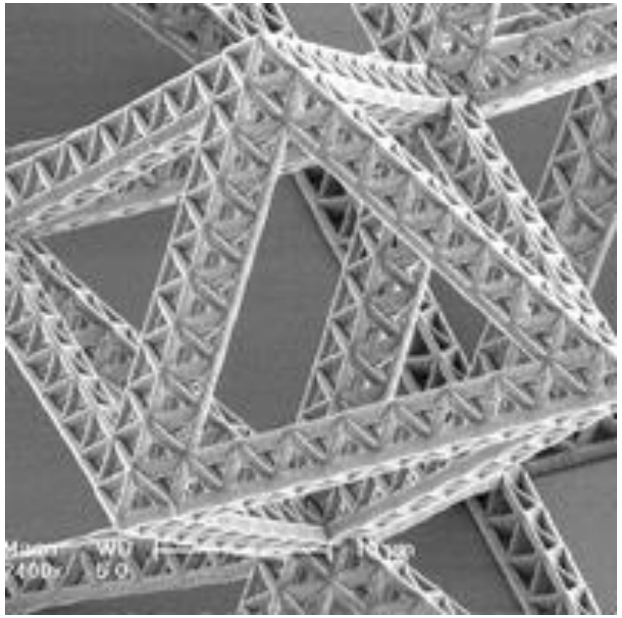
Graeme Milton, University of Utah

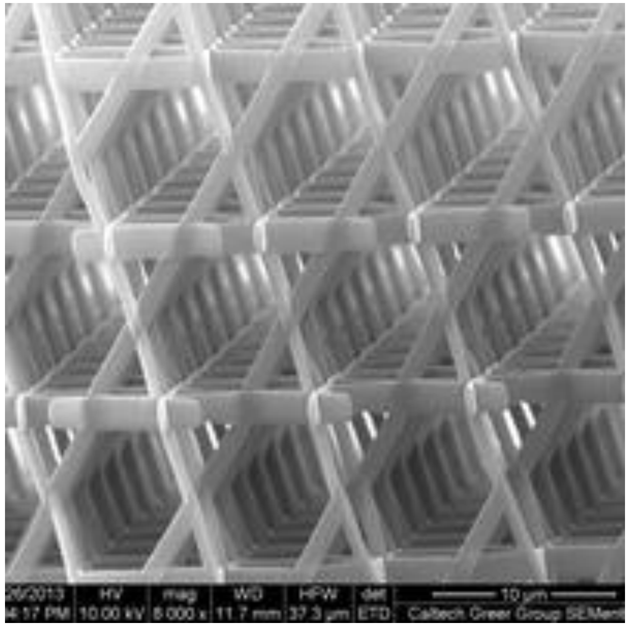


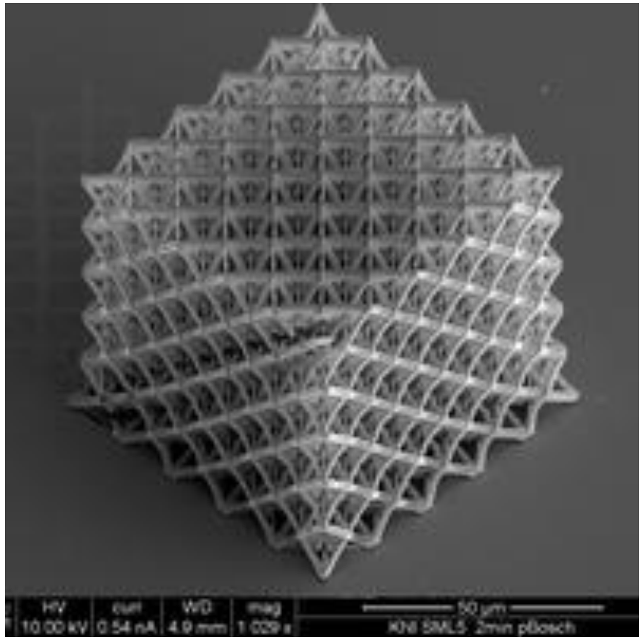




Group of
Martin Wegener







Group of Julia Greer

Walser 1999:

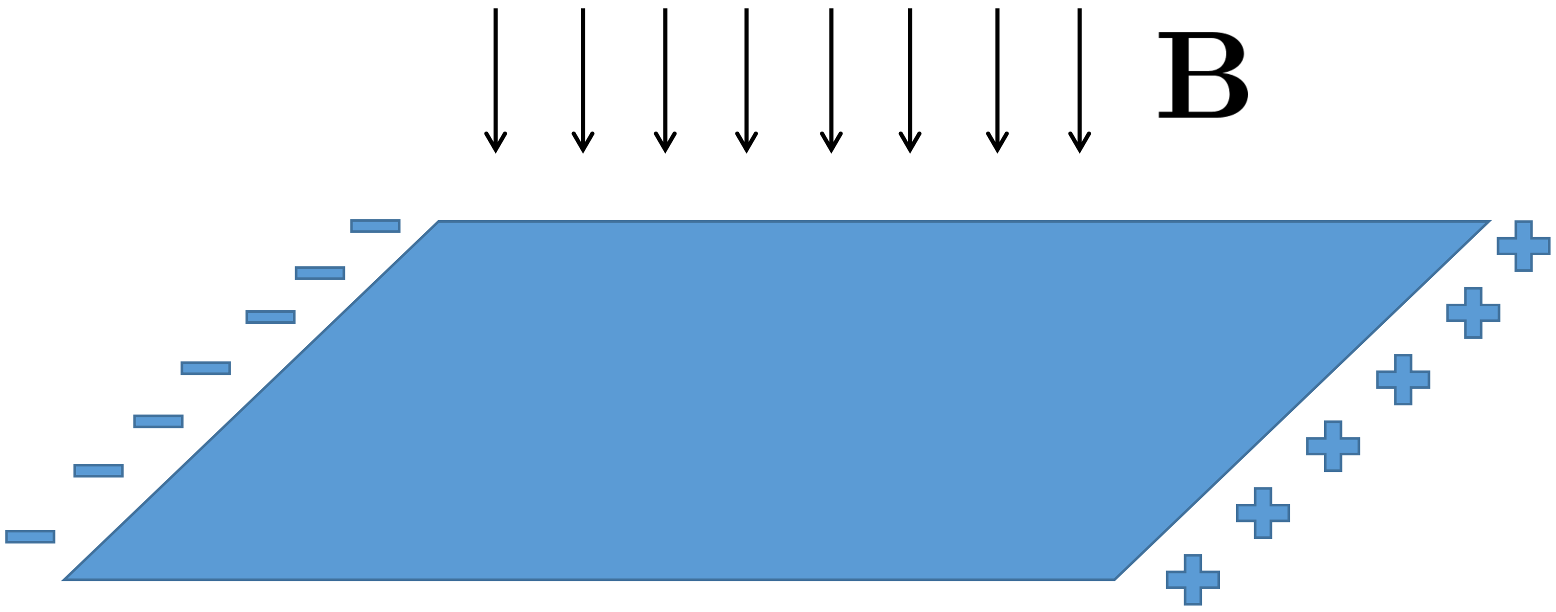
Macroscopic composites having a manmade, three-dimensional, periodic cellular architecture designed to produce an optimized combination, not available in nature, of *two or more responses* to specific excitation.

Browning and Wolf 2001:

Metamaterials are a new class of ordered composites that exhibit exceptional properties not readily observed in nature.

It's constantly a surprise to find what properties a composite can exhibit.

One interesting example:



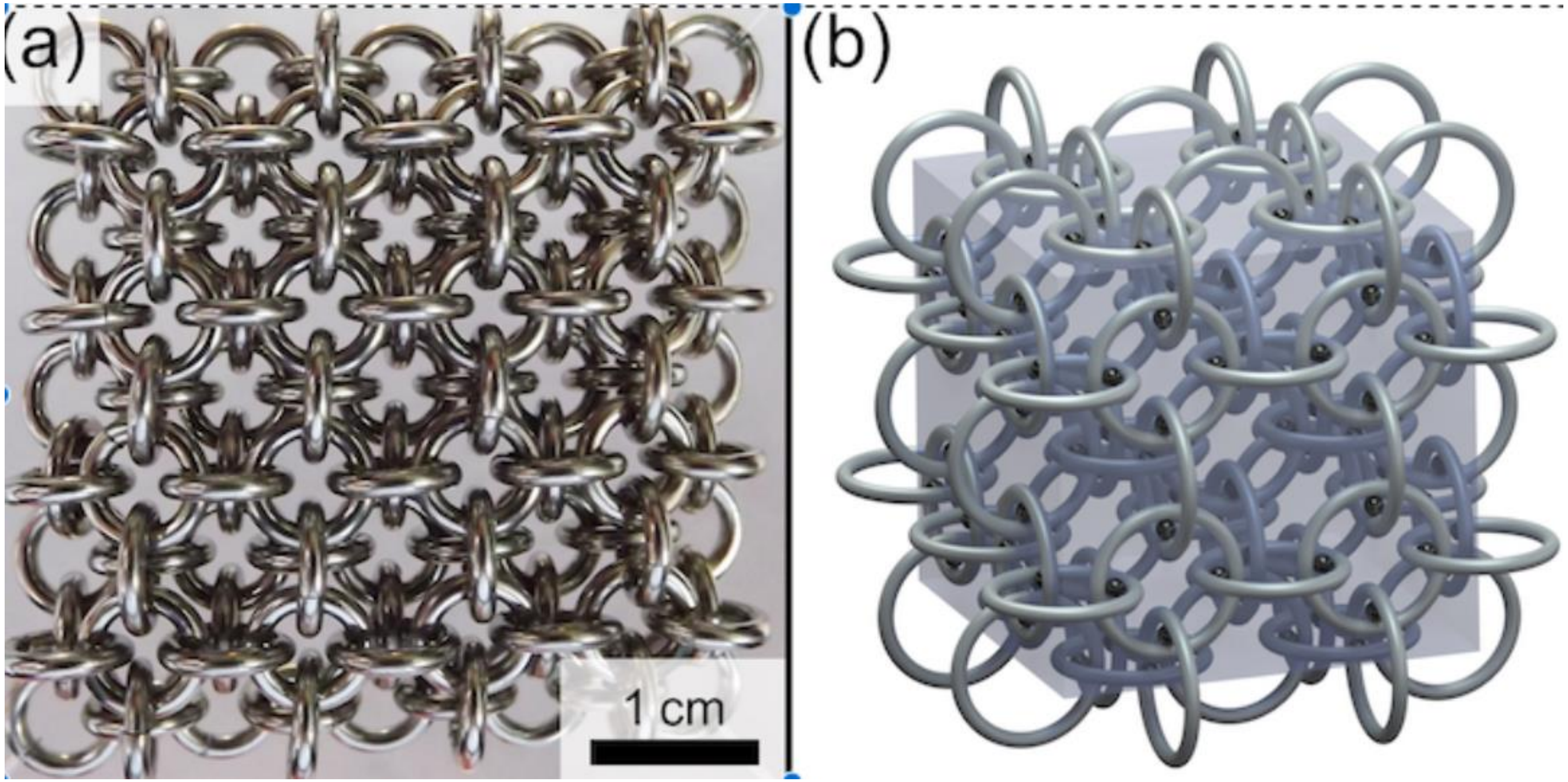
Hall Voltage

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

In elementary physics textbooks one is told that in classical physics the sign of the Hall coefficient tells one the sign of the charge carrier.

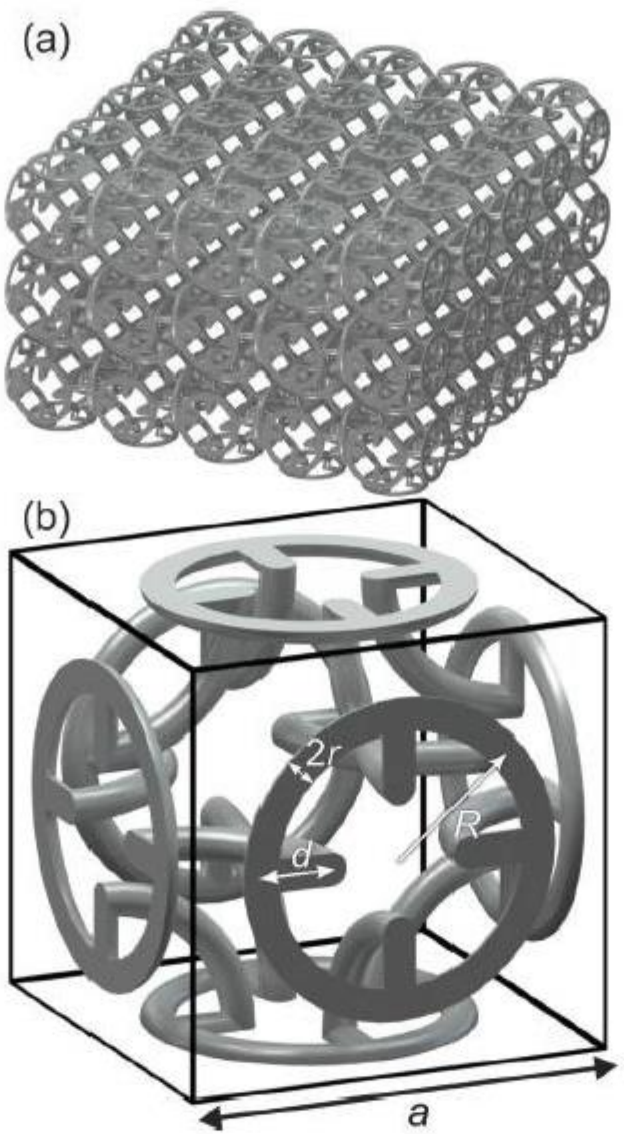
However there is a counterexample!

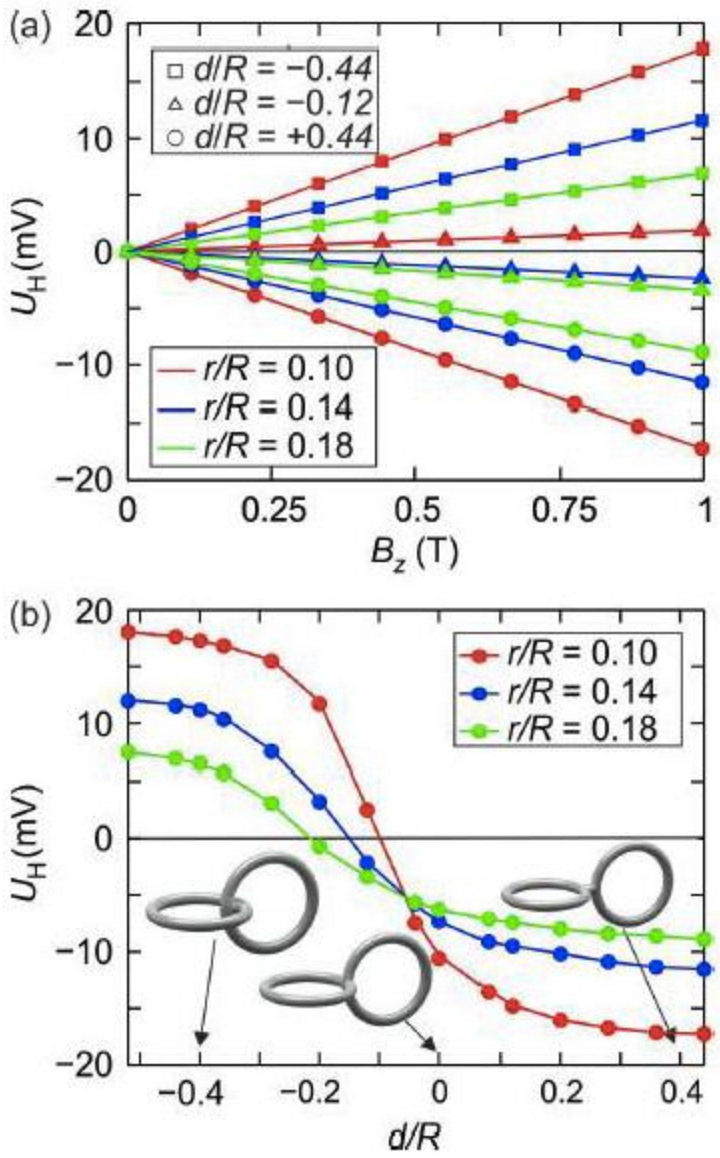
Geometry suggested by artist Dylon Whyte

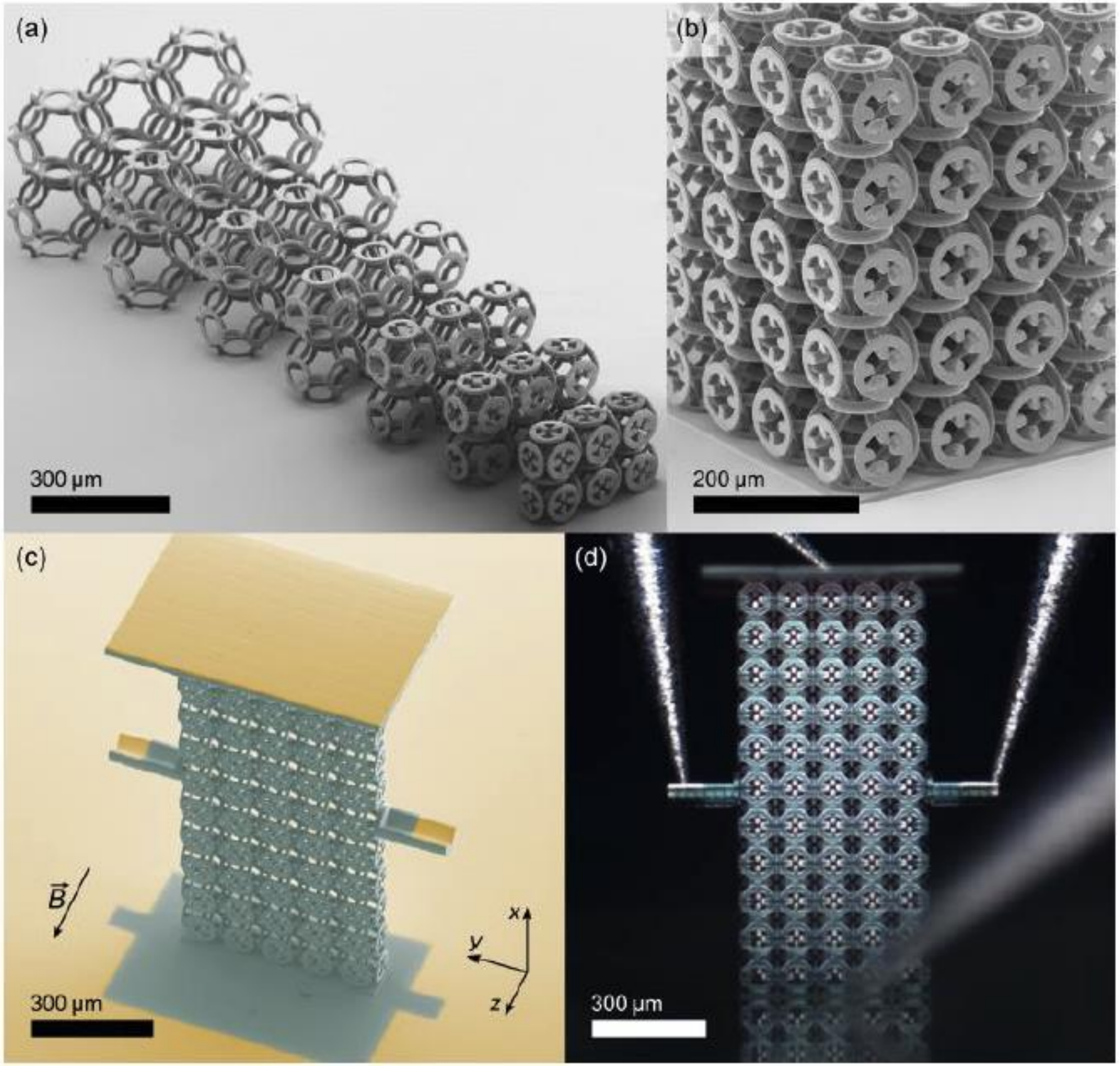


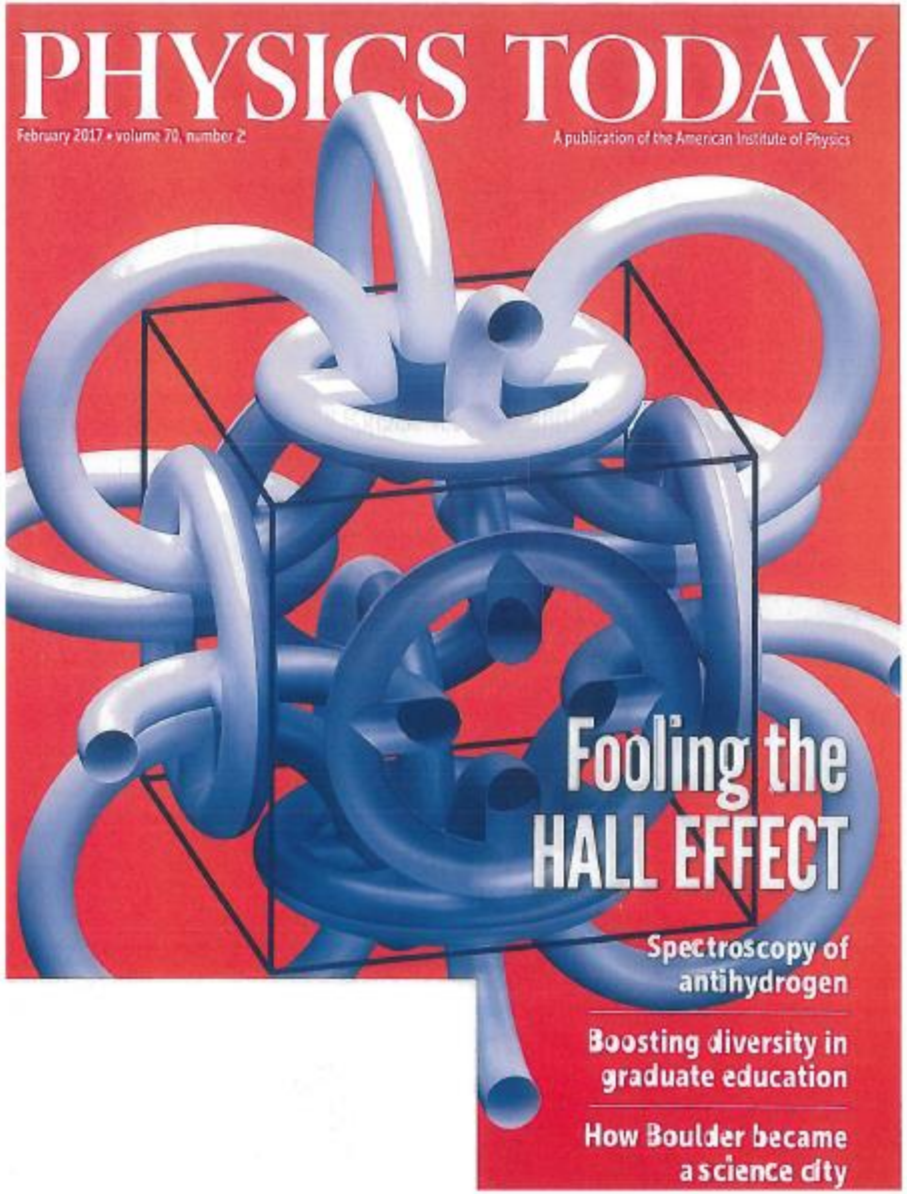
A material with cubic symmetry having a Hall Coefficient opposite to that of the constituents (with Marc Briane)

Simplification of Kadic et.al. (2015)



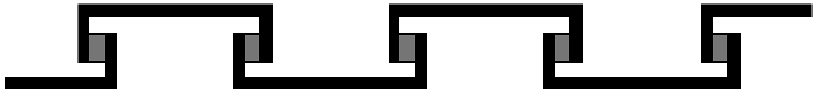


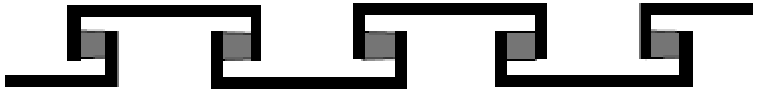


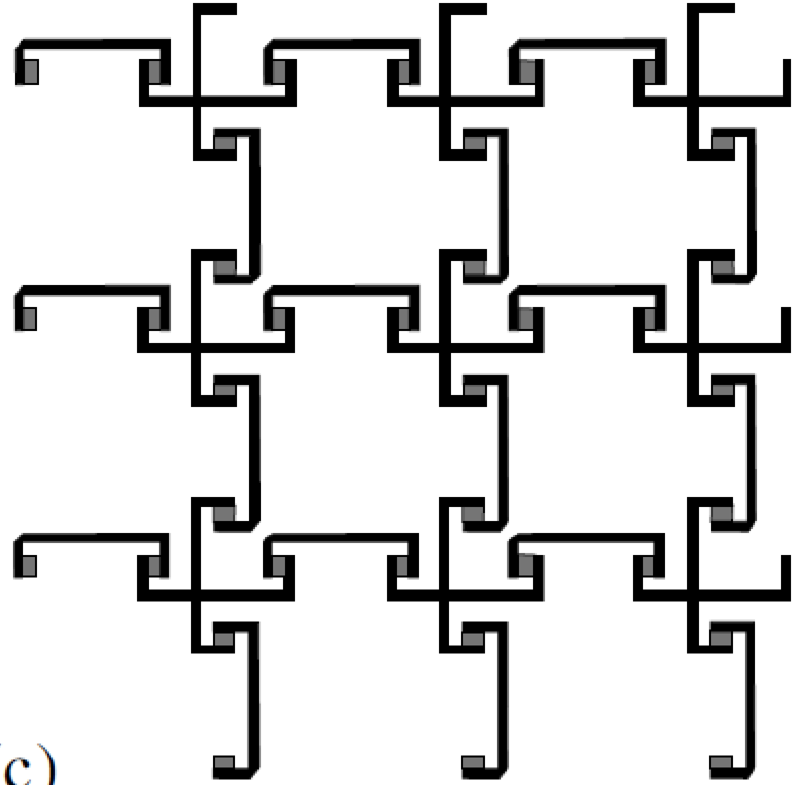


Experimental Realization of Kern, Kadic, Wegener

Another example: negative expansion from positive expansion

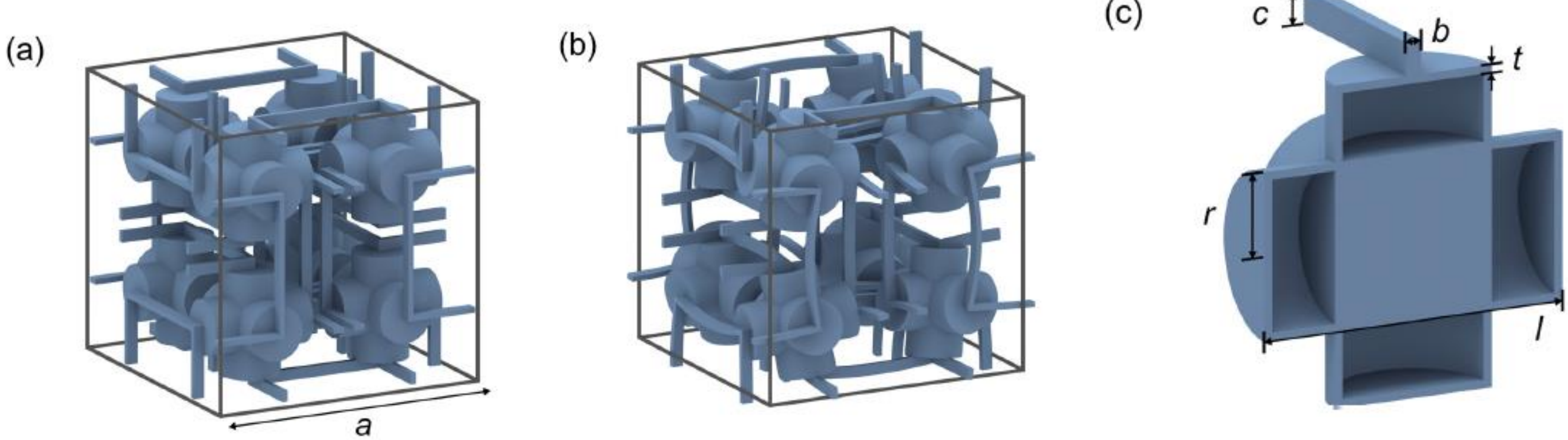

(a)


(b)


(c)

Original designs: Lakes (1996); Sigmund & Torquato (1996, 1997)

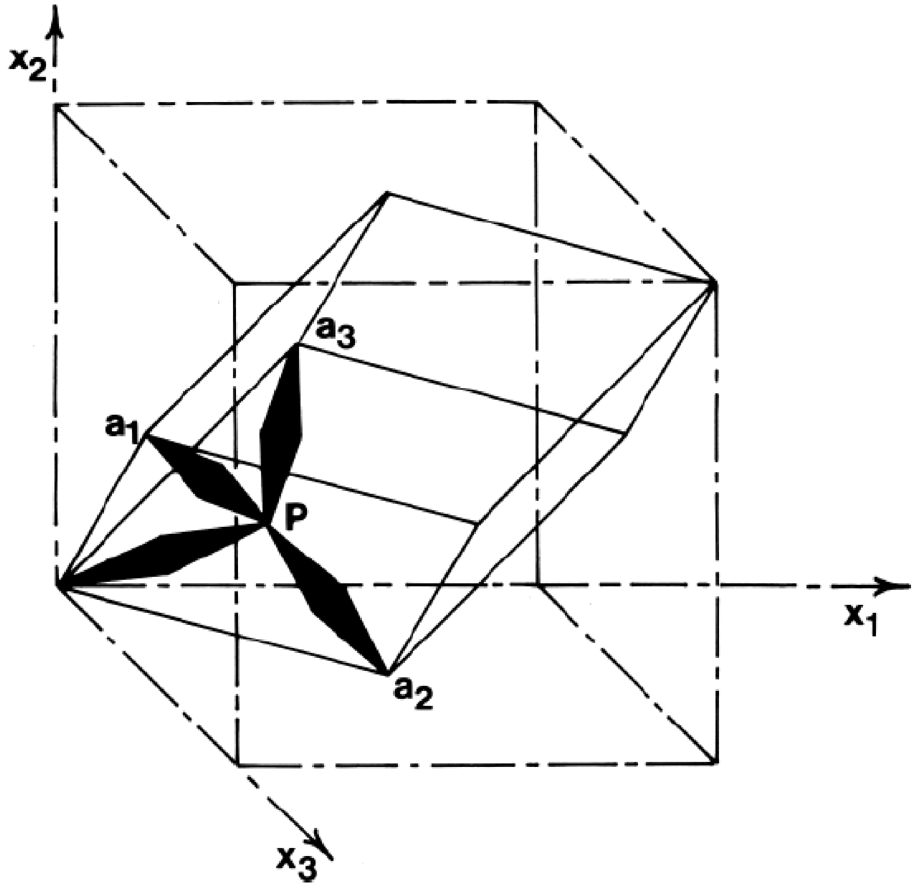
One can get a similar effect for poroelasticity



Qu, et.al 2017

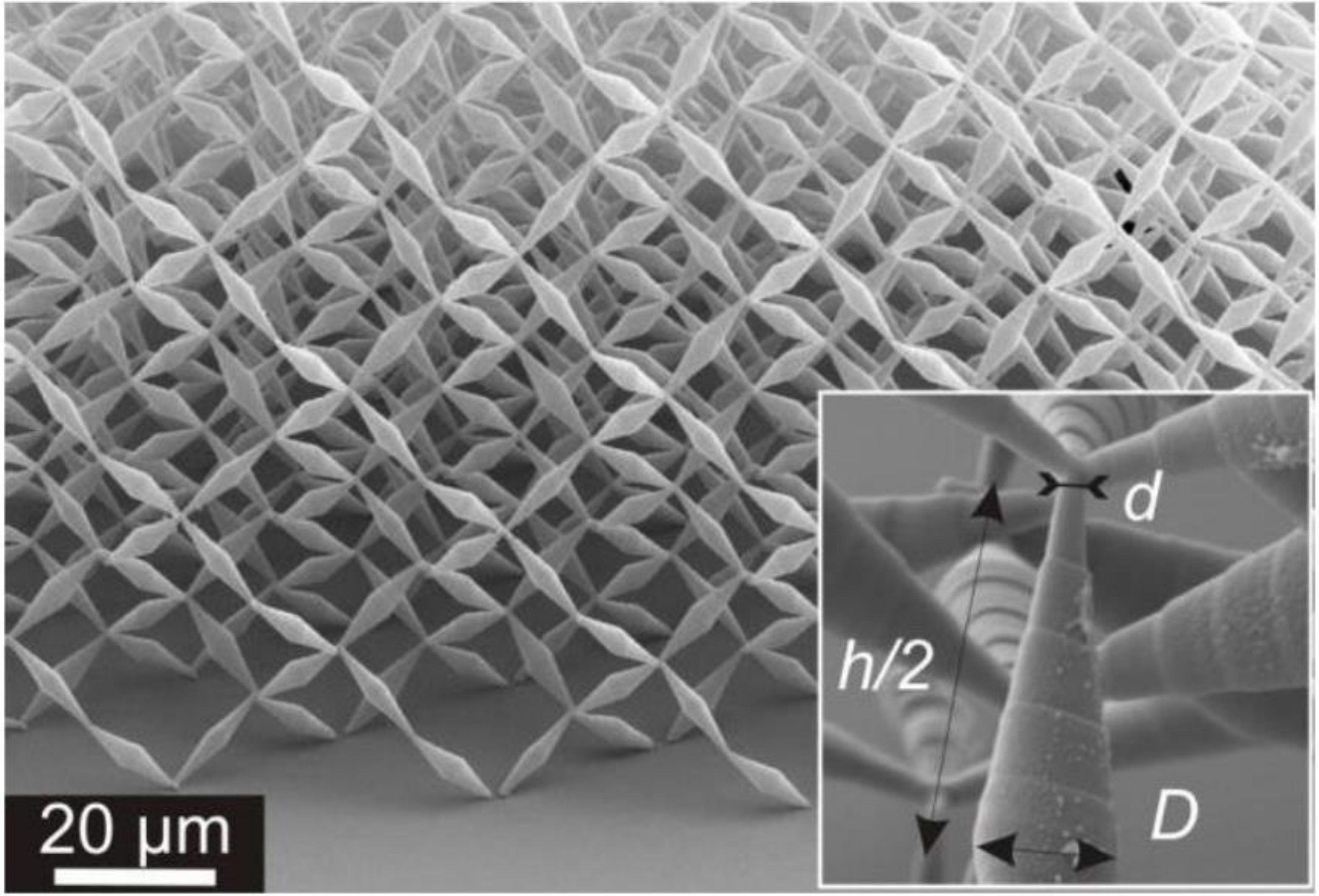
New classes of elastic materials (with Cherkaev, 1995)

A three dimensional pentamode material which can support any prescribed loading



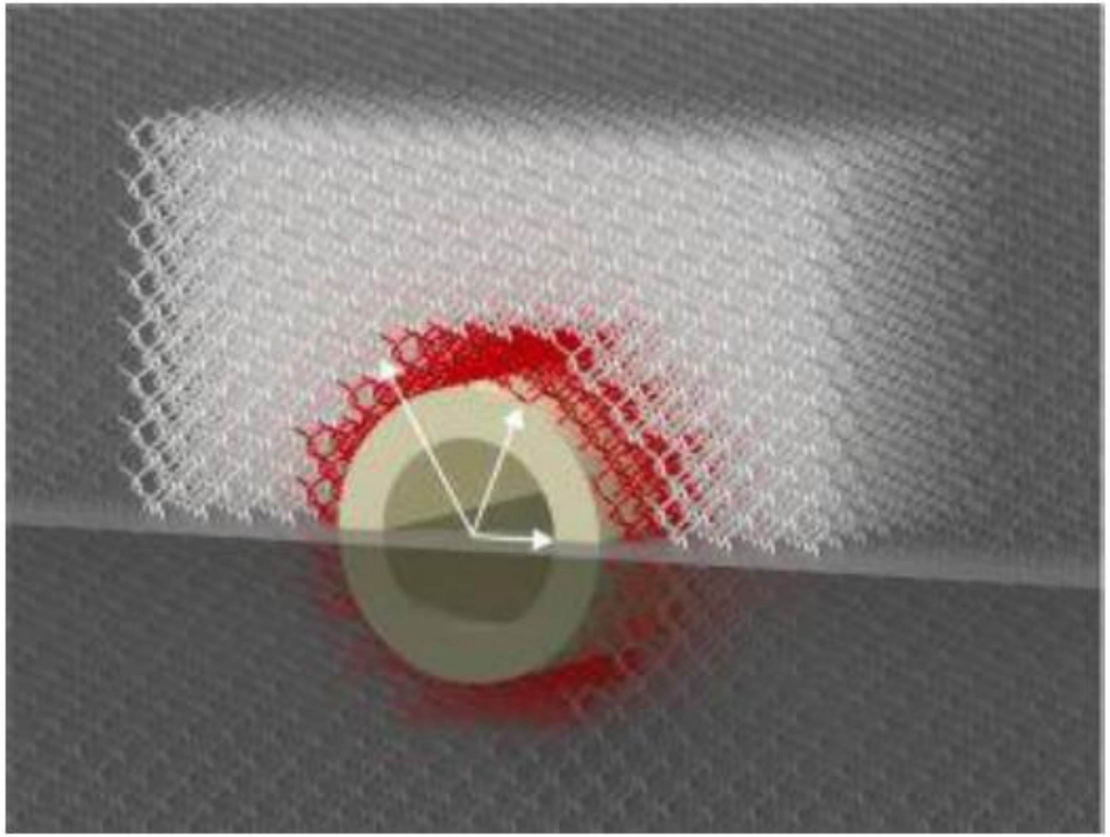
Like a fluid it only supports one loading, unlike a fluid that loading may be anisotropic

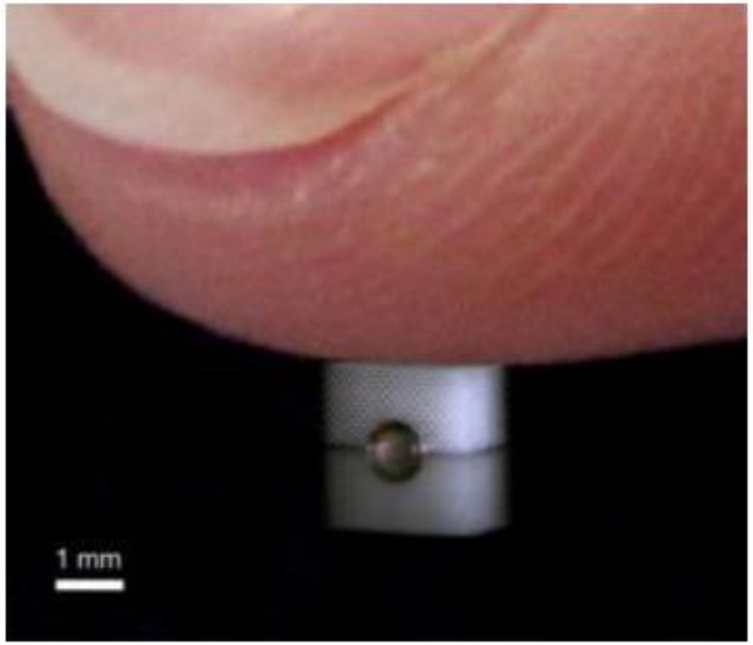
Realization of Kadic et.al. 2012



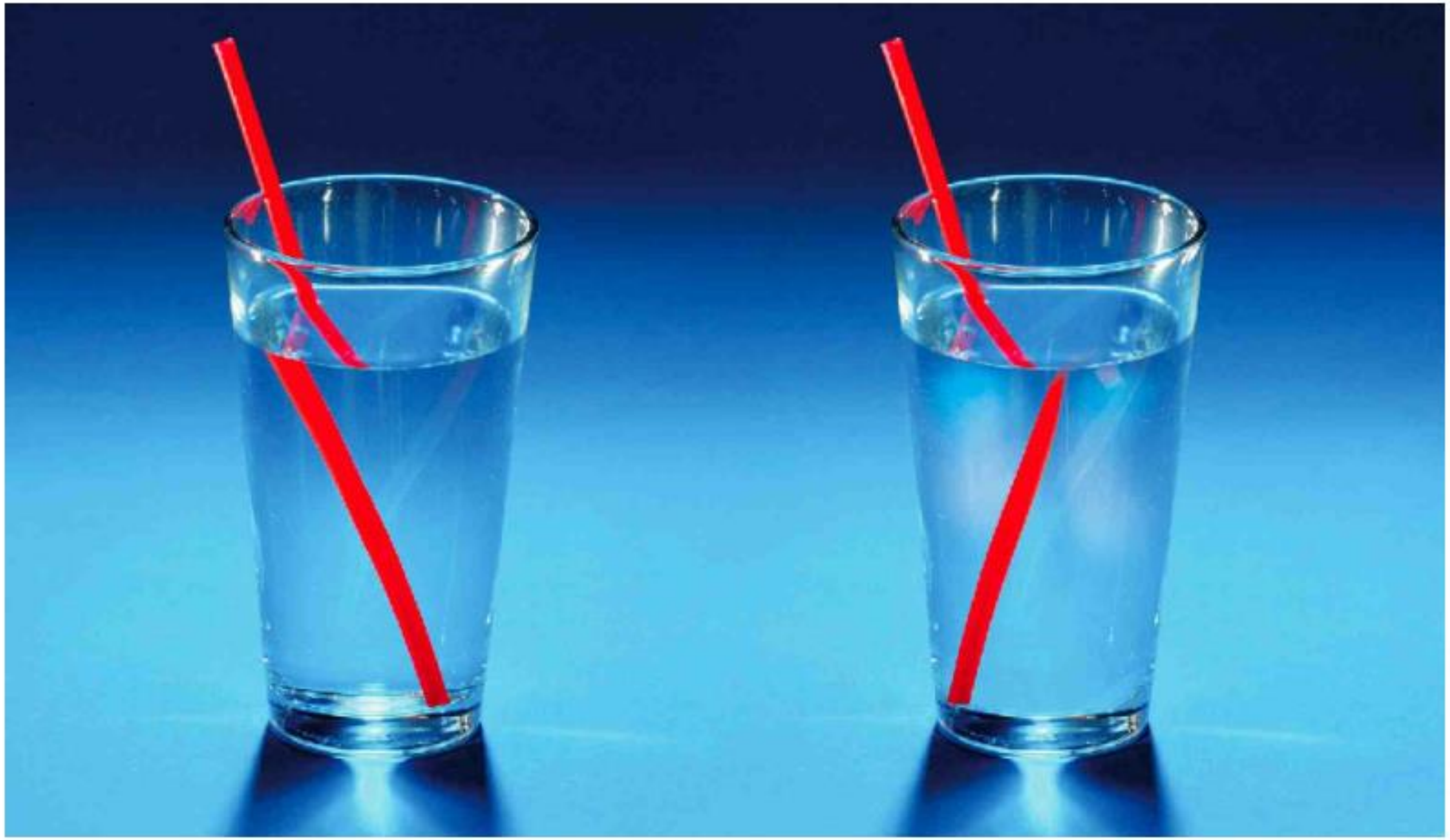
Application of Pentamodes:

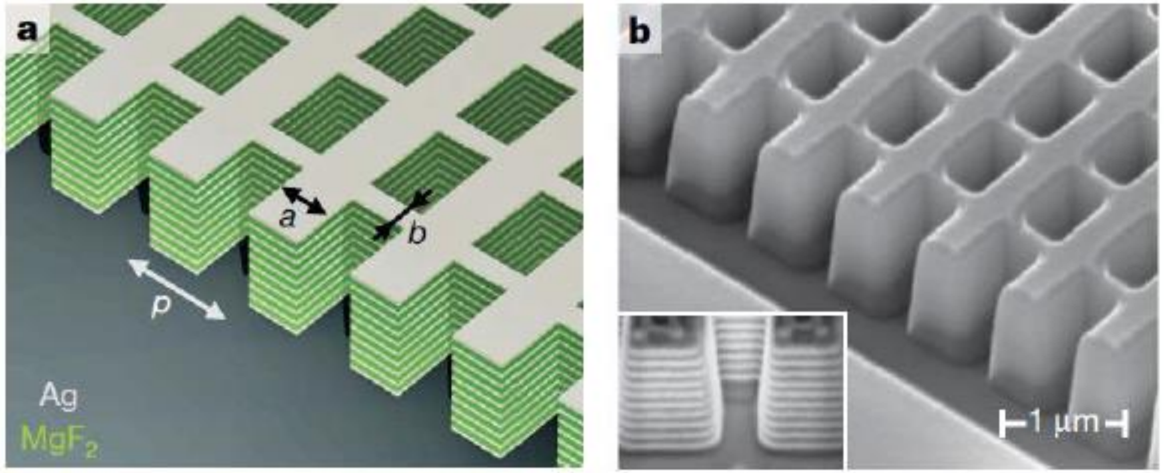
Cloak making an object “unfeelable”:
Buckmann et. al. (2014)

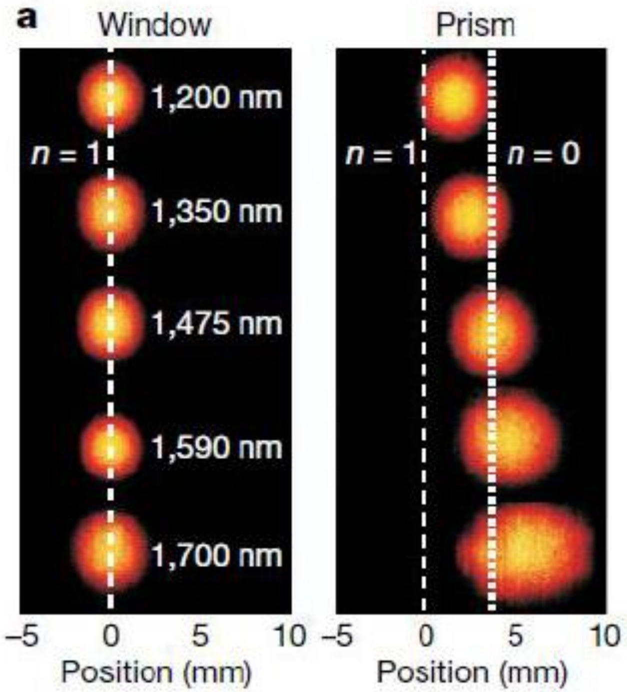


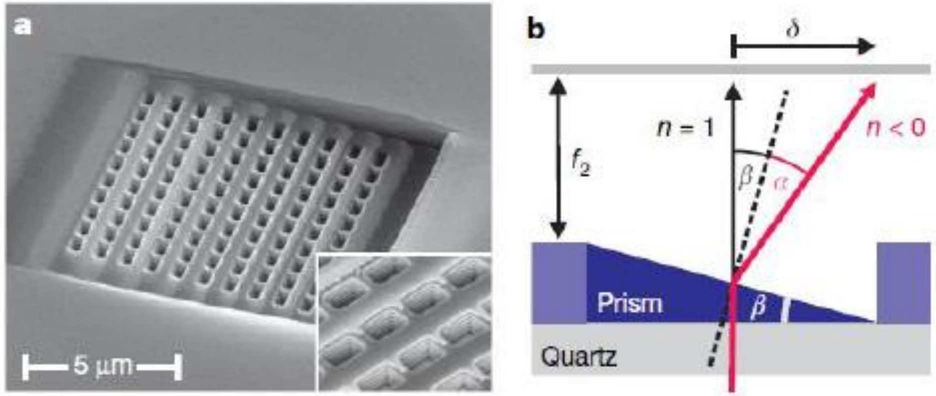


Negative Refraction Simulation: Hess 2008

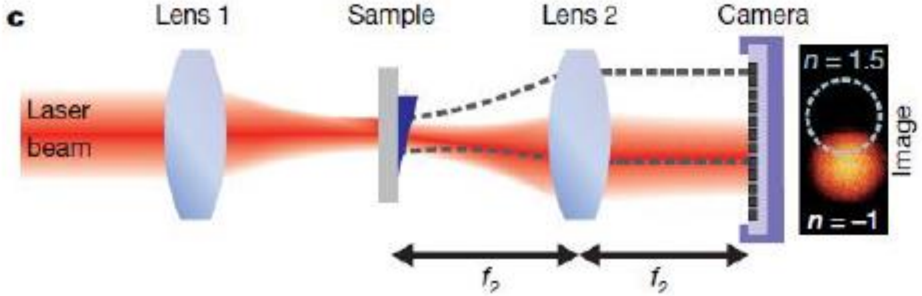


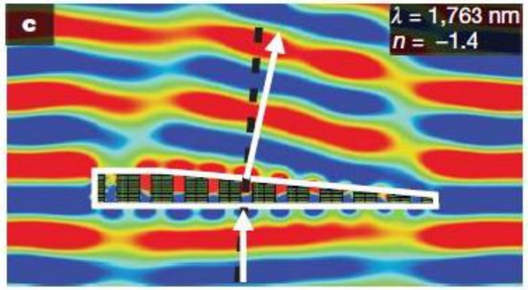




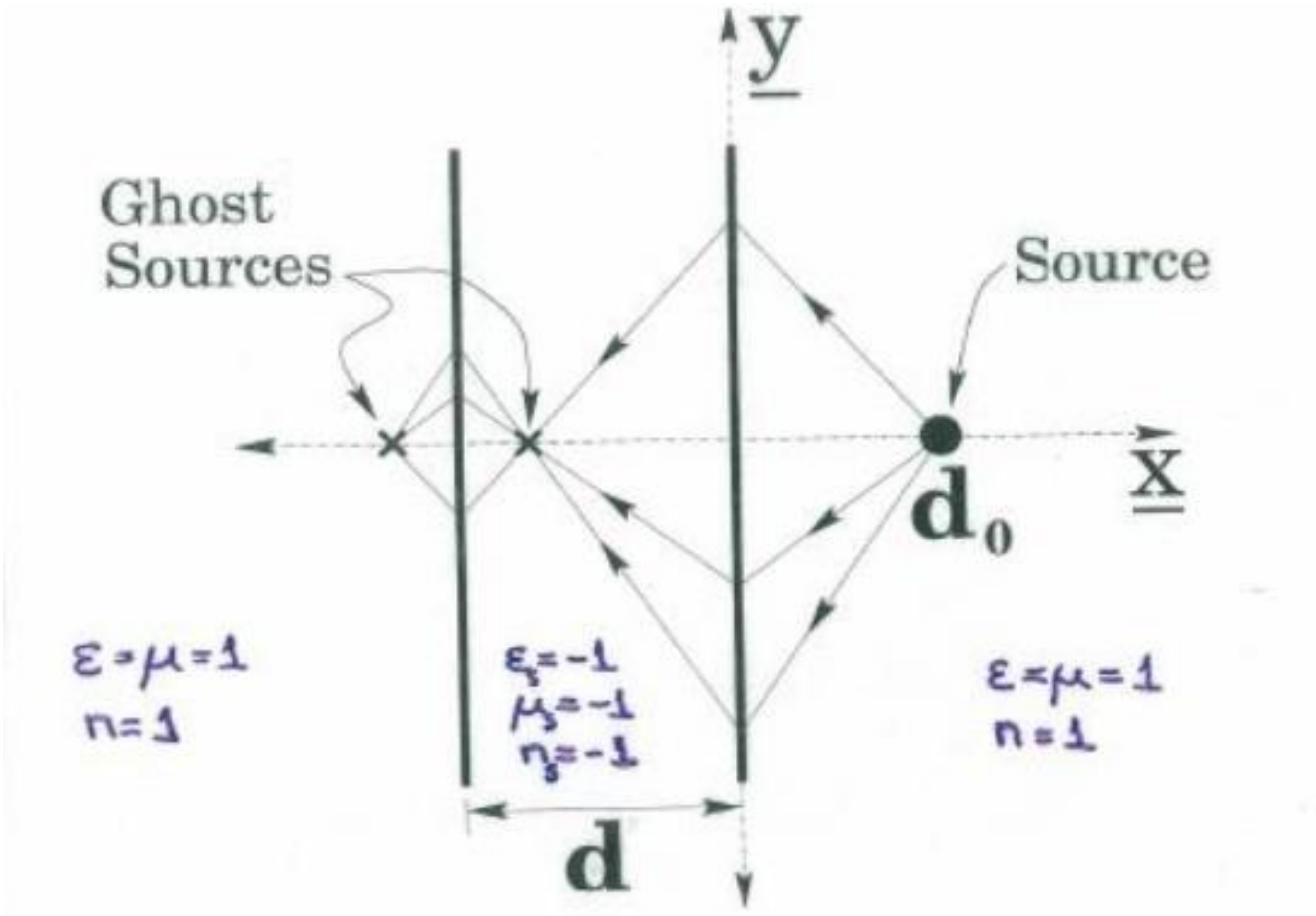


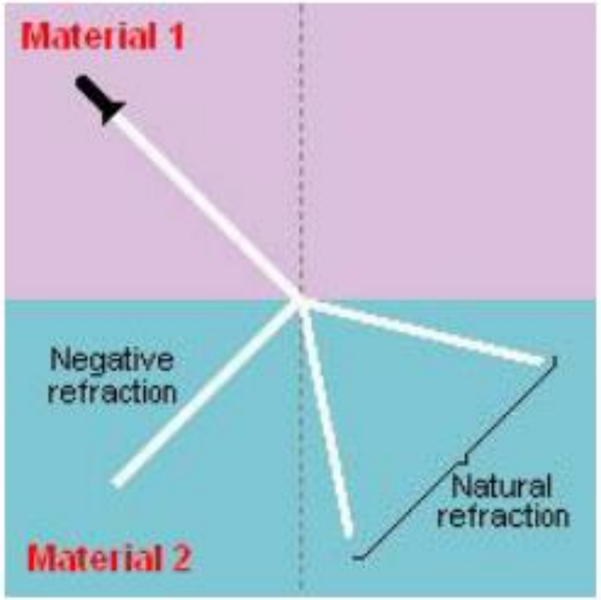
Negative refraction at optical frequencies: Valentine et. al.(2008)





Focusing beyond the diffraction limit: the superlens (Pendry, 2000)





Wrong Picture

PHYSICAL REVIEW B VOLUME 49, NUMBER 12 15 MARCH 1994-II

Optical and dielectric properties of partially resonant composites

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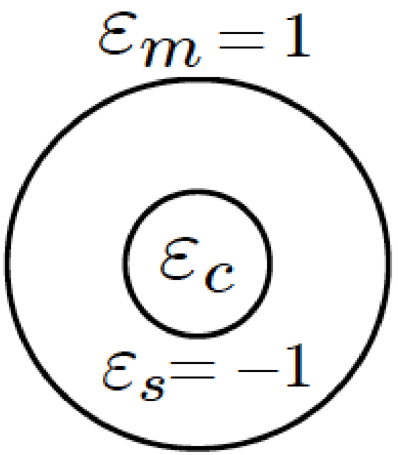
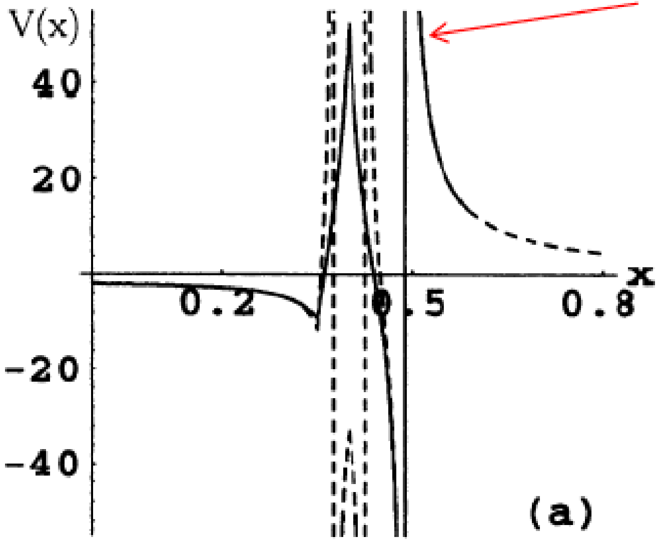
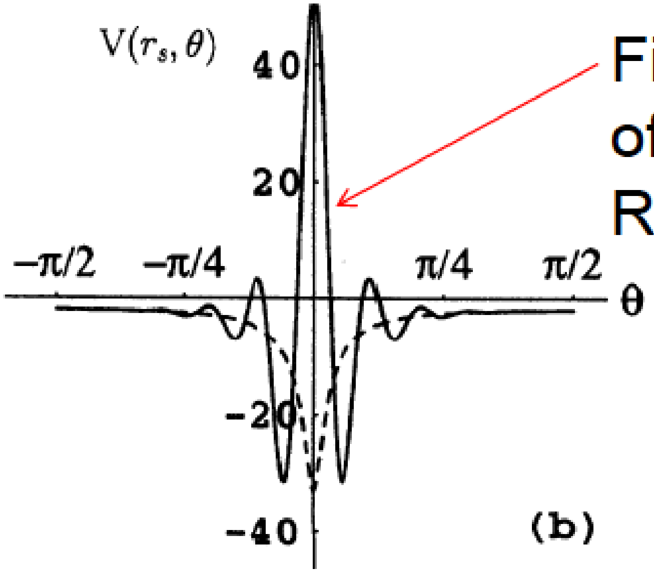
(Received 2 November 1993)



First Discovery of a Ghost Source

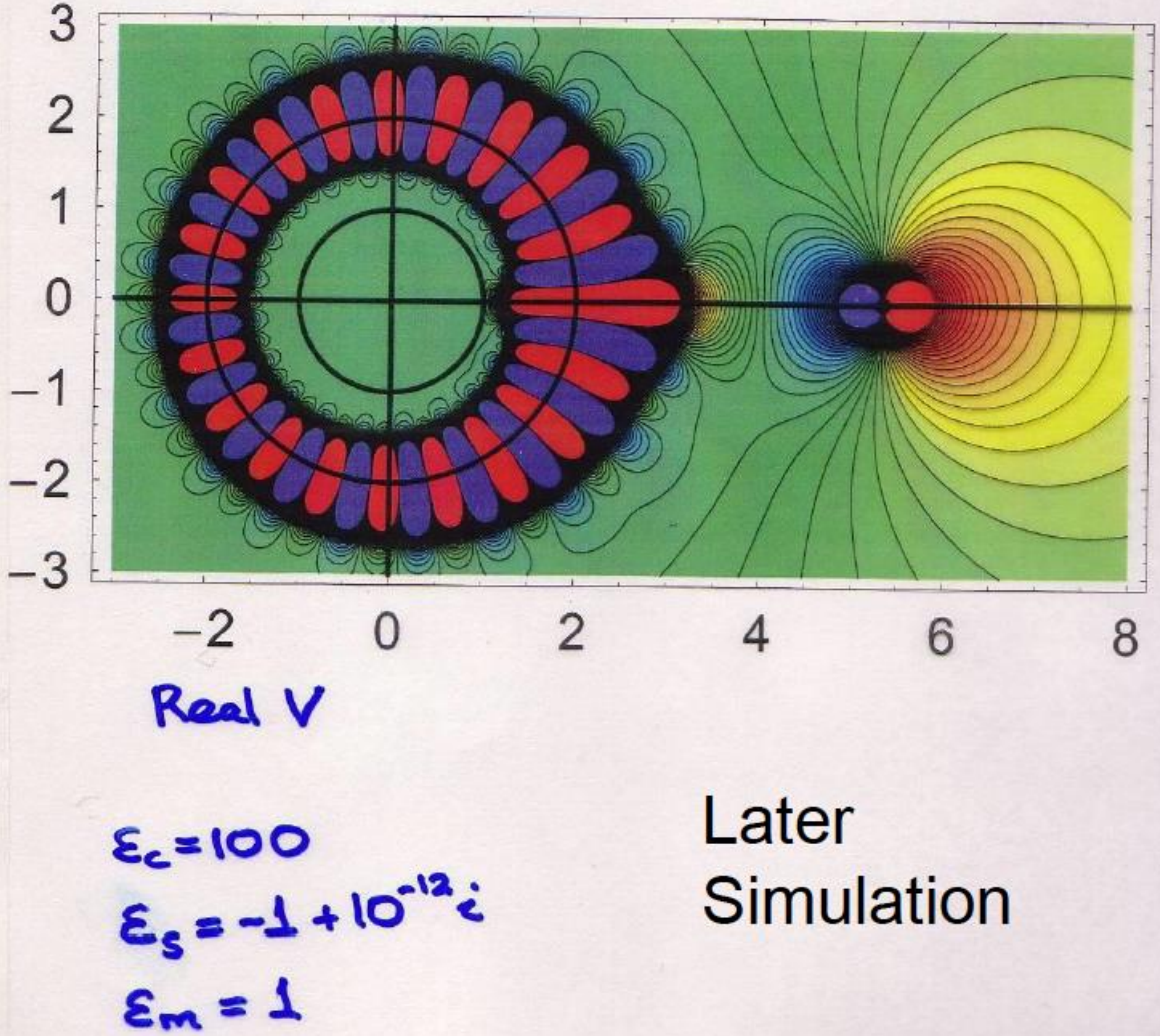


(Shell radius=0.4, Core radius=0.35)

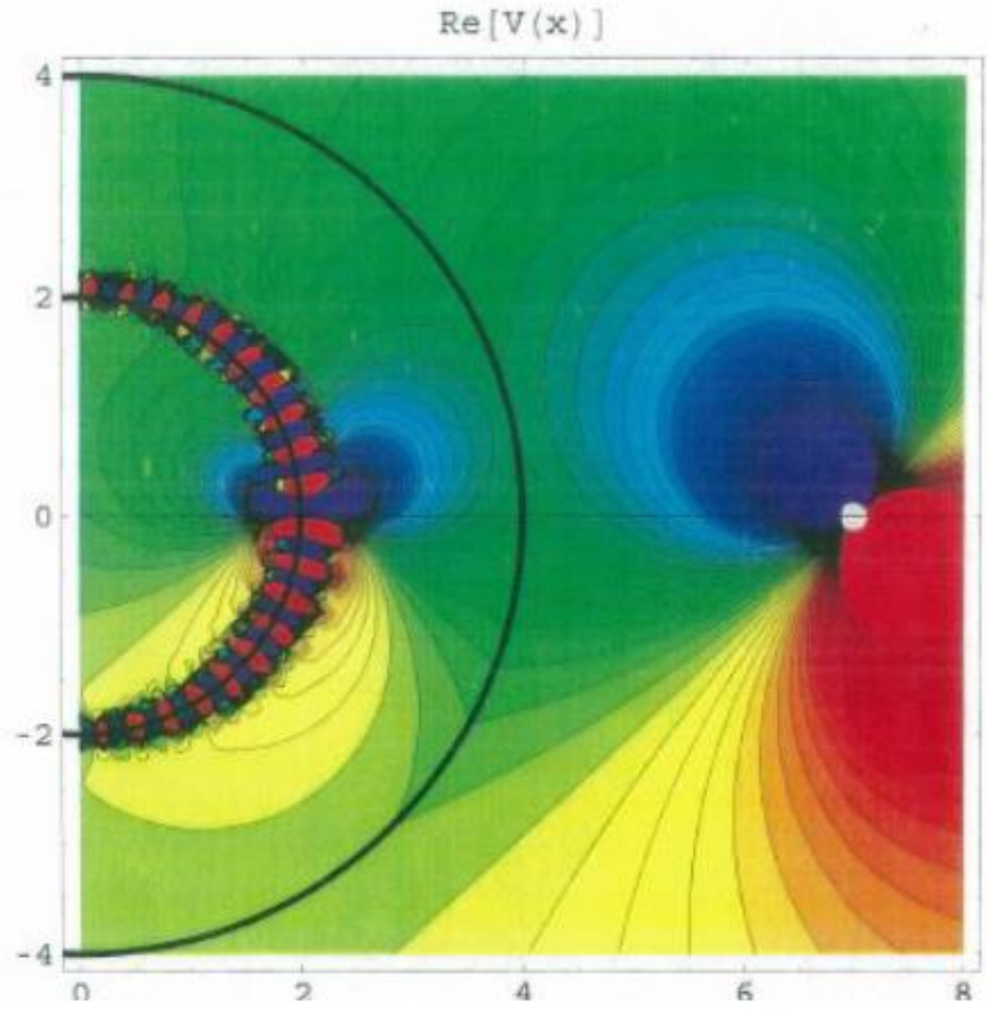


First Discovery of Anomalous Resonance

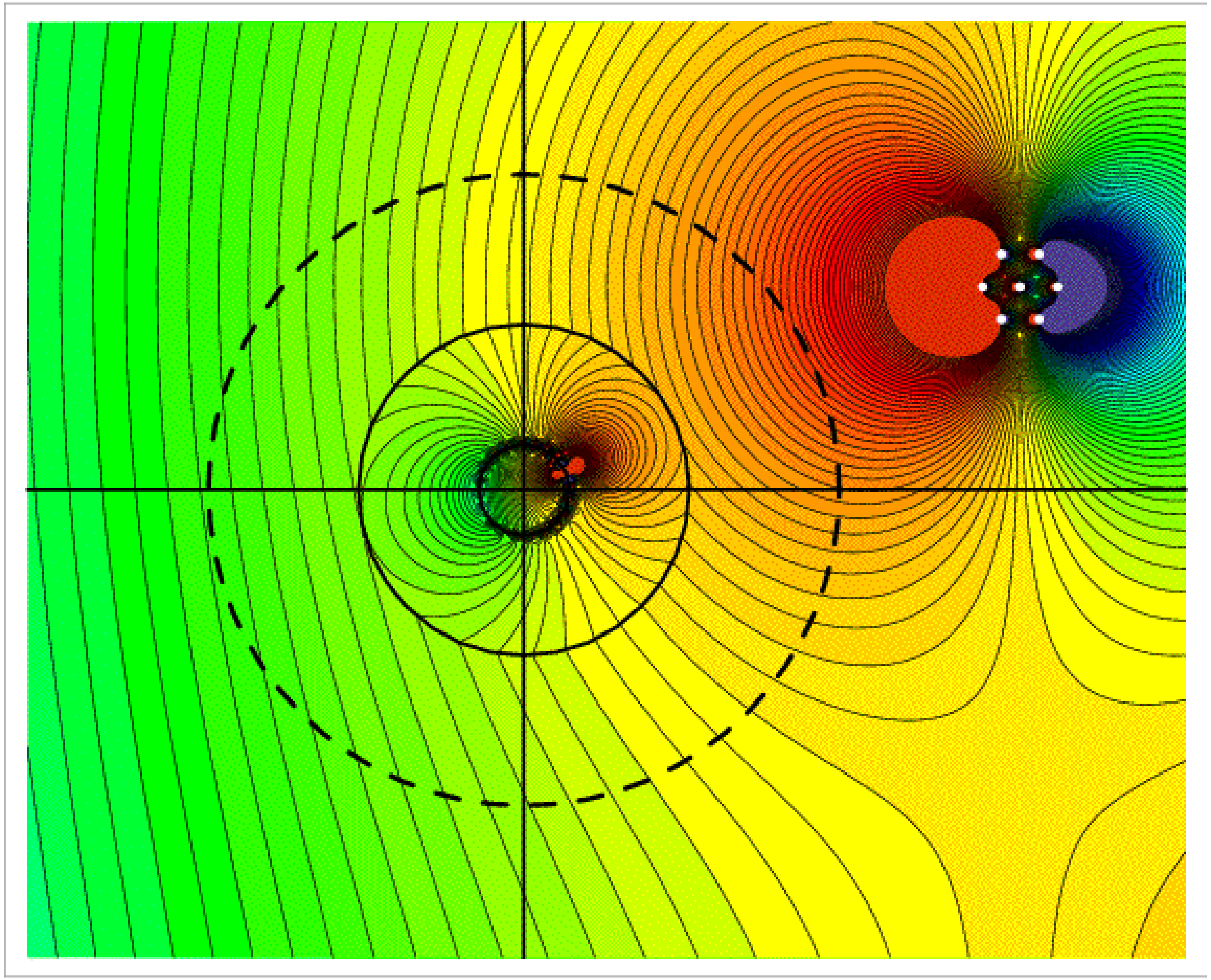
Ghost sources and anomalous resonance are the essential mechanisms that explain superlensing.



Later Simulation



When the shell was hollow we found it was completely invisible to any applied field

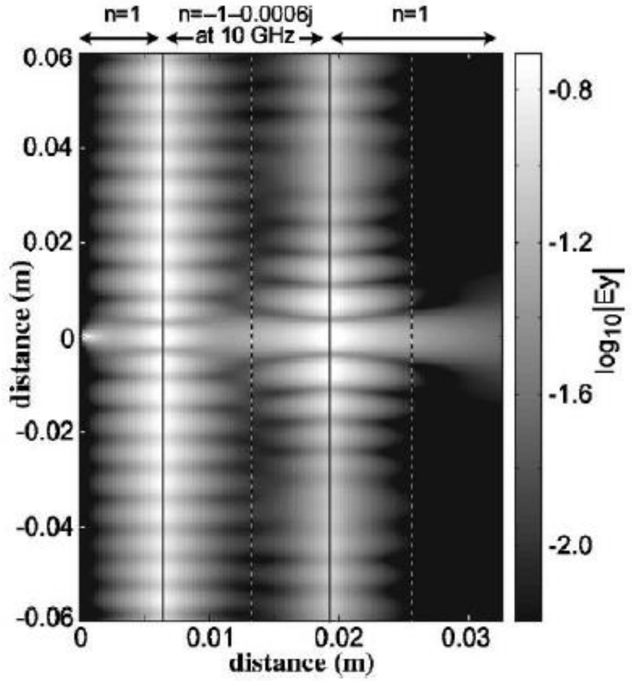


Cloaking due to anomalous resonance
With Botten, Mcphedran
Nicorovici 2006,2007

Many other works
in particular
by Hoai Minh Nguyen

Similarly for the perfect lens there are anomalously resonant regions:

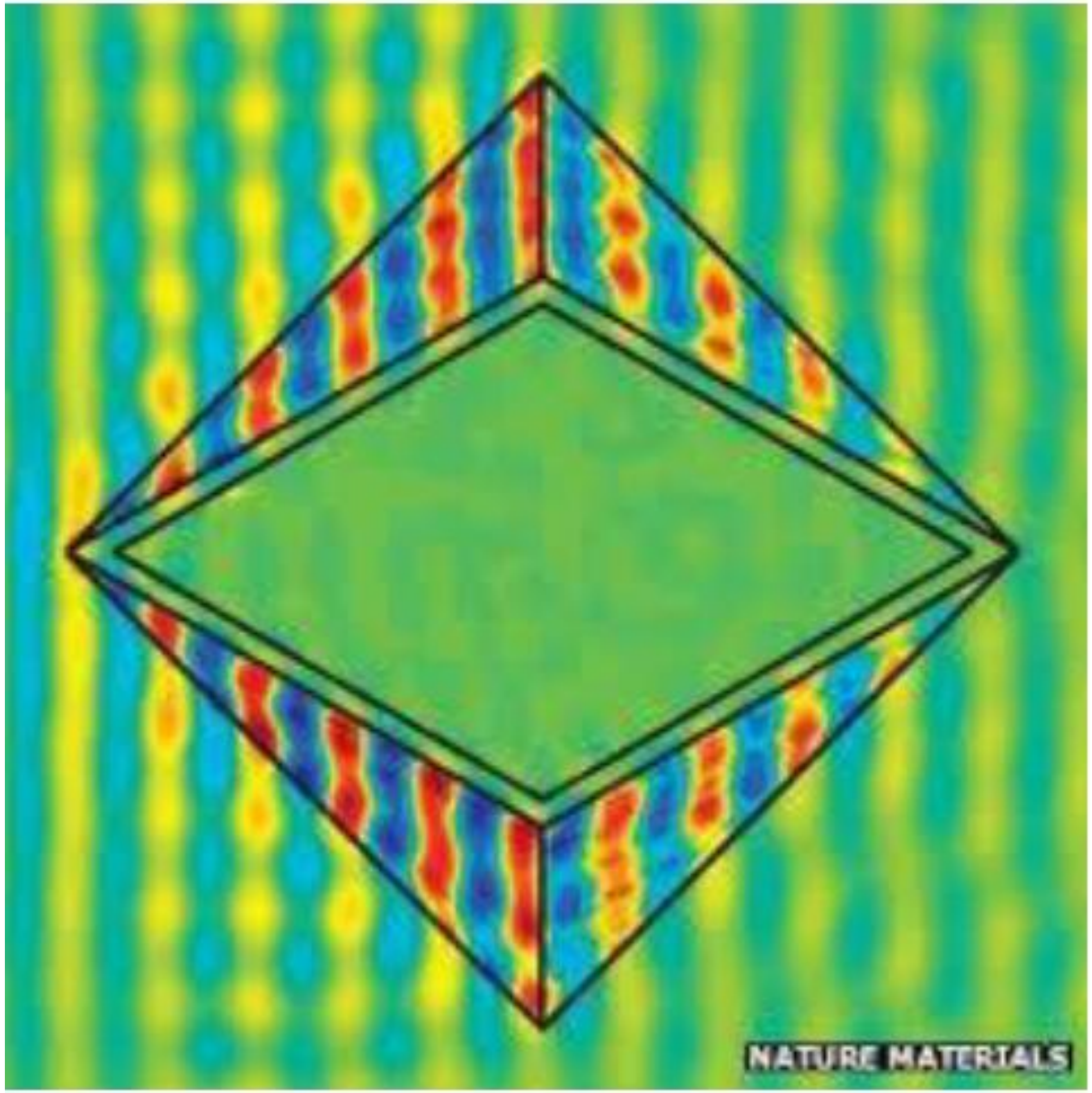
Work by Garcia and Nieto-Vesperinas (2002) and Pokrovsky and Efros (2002) indicated large fields between the ghost sources.

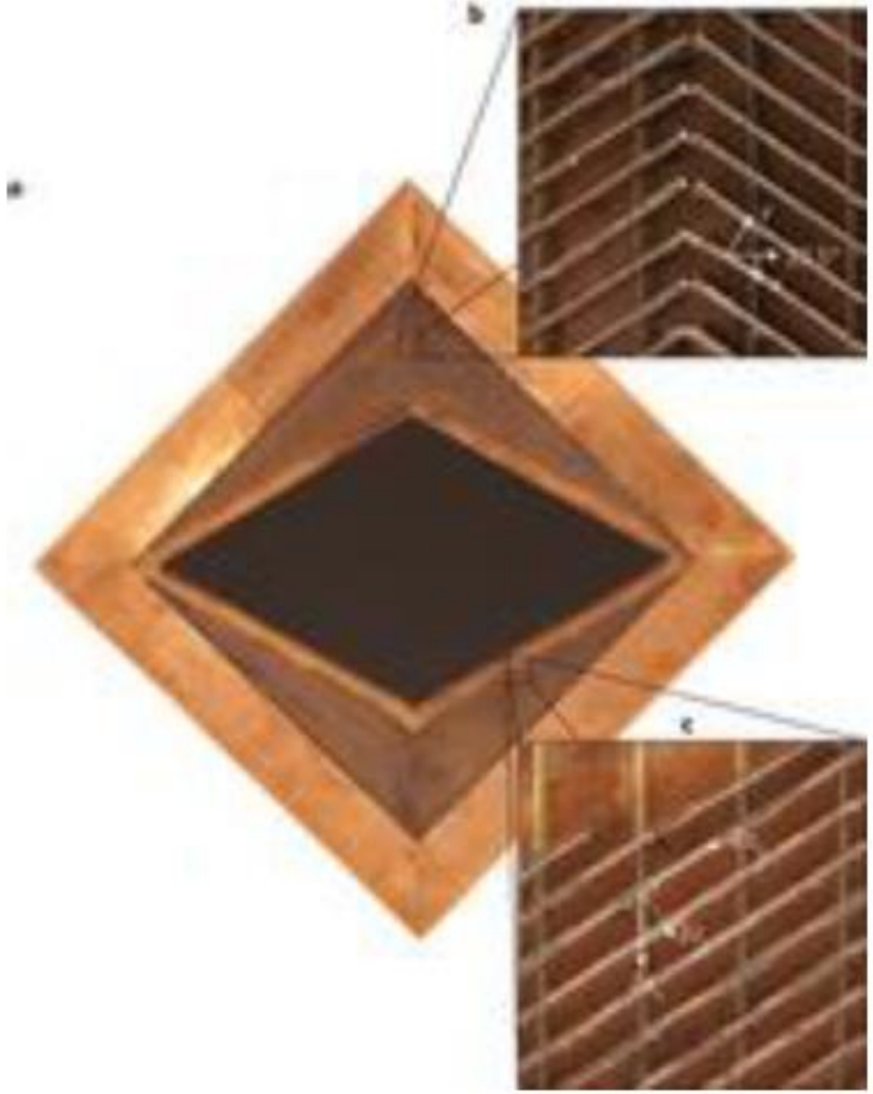


Correct Picture

Numerical Results of Cummer (2003) showing the anomalously resonant regions on both sides of the lens

In fact instead of getting perfect transmission sometimes the transmission is zero!





Unidirectional Cloak: Landy and Smith (2013)

An important parallel:

Maxwell's Equations:

$$\frac{\partial}{\partial x_i}\left(C_{ijk\ell}\frac{\partial E_\ell}{\partial x_k}\right) = \{\omega^2 \boldsymbol{\varepsilon}\mathbf{E}\}_j$$

$$C_{ijk\ell} = e_{ijm}e_{k\ell n}\{\boldsymbol{\mu}^{-1}\}_{mn}$$

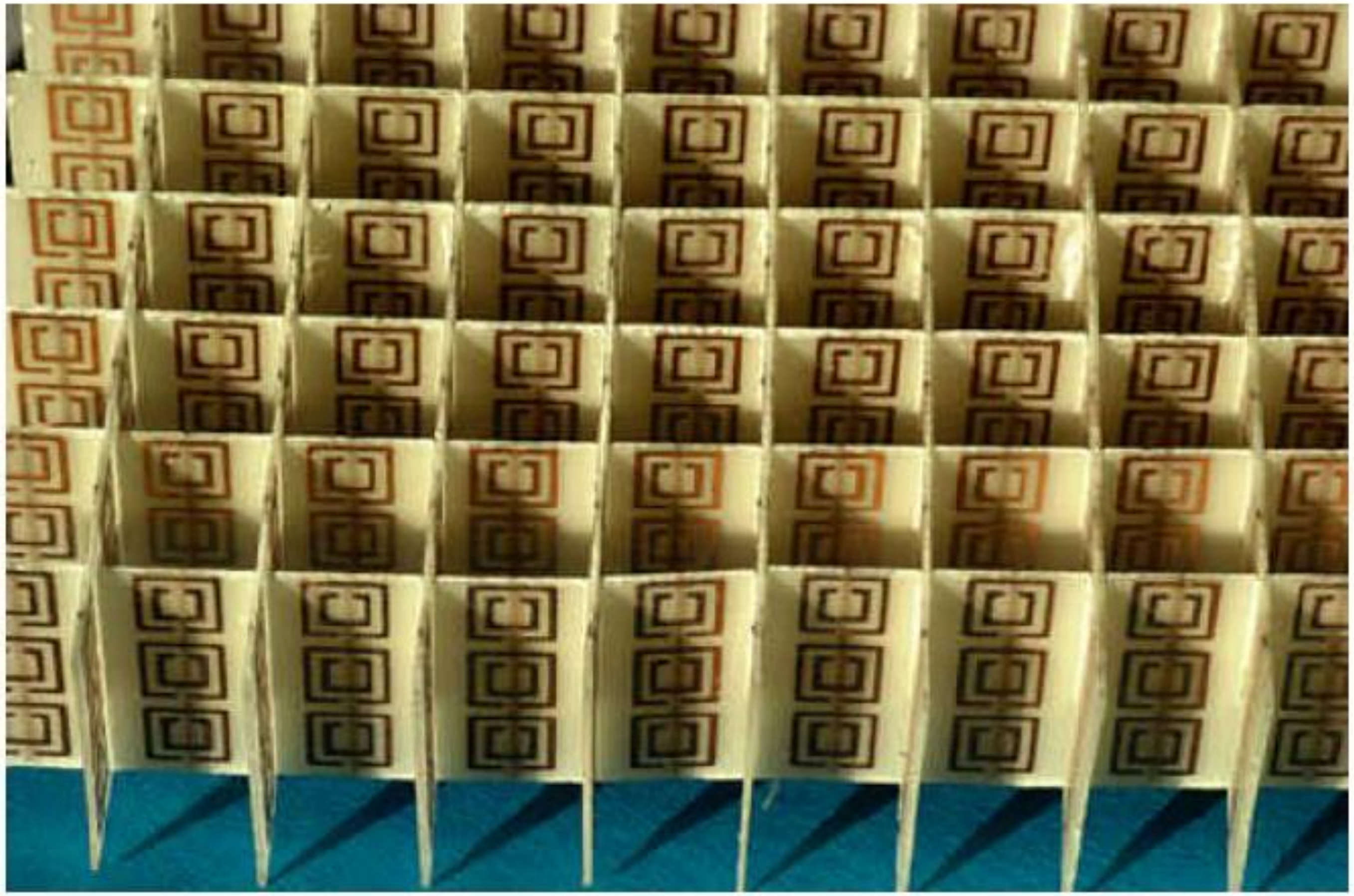
Continuum Elastodynamics:

$$\frac{\partial}{\partial x_i}\left(C_{ijk\ell}\frac{\partial u_\ell}{\partial x_k}\right) = -\{\omega^2 \boldsymbol{\rho}\mathbf{u}\}_j$$

Suggests that $\varepsilon(\omega)$ and $\boldsymbol{\rho}(\omega)$ might have similar properties

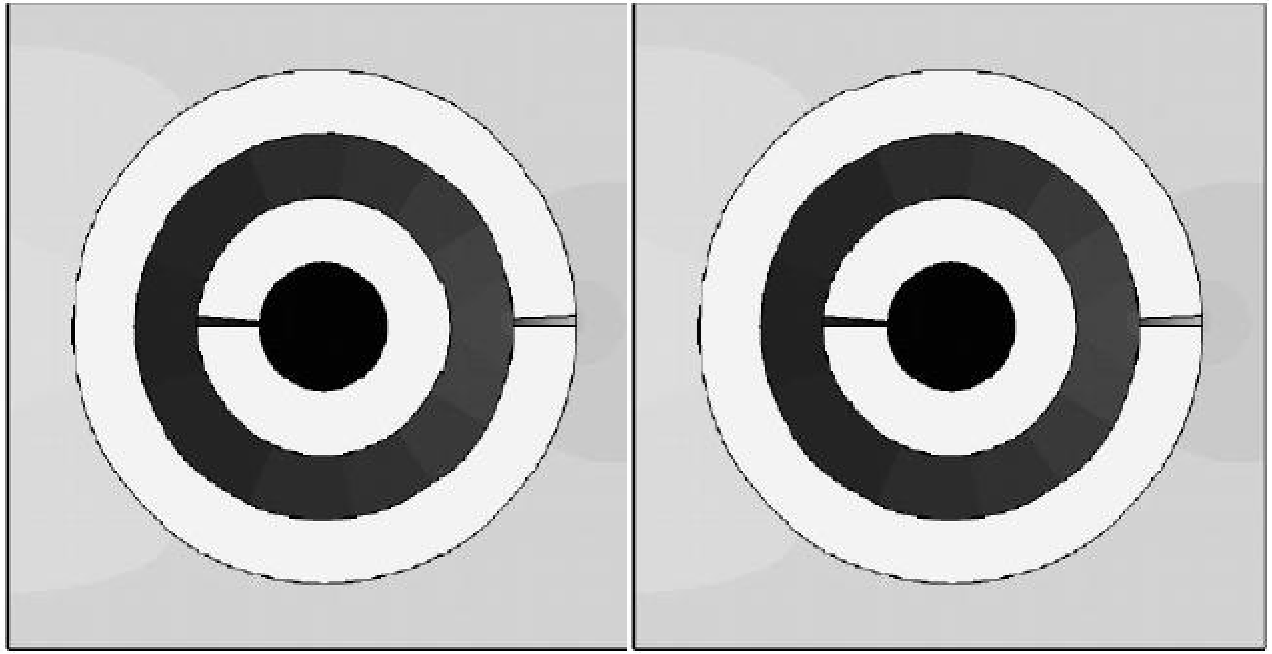
Specifically a similar dependence on frequency

There is a close connection between negative density and negative magnetic permeability



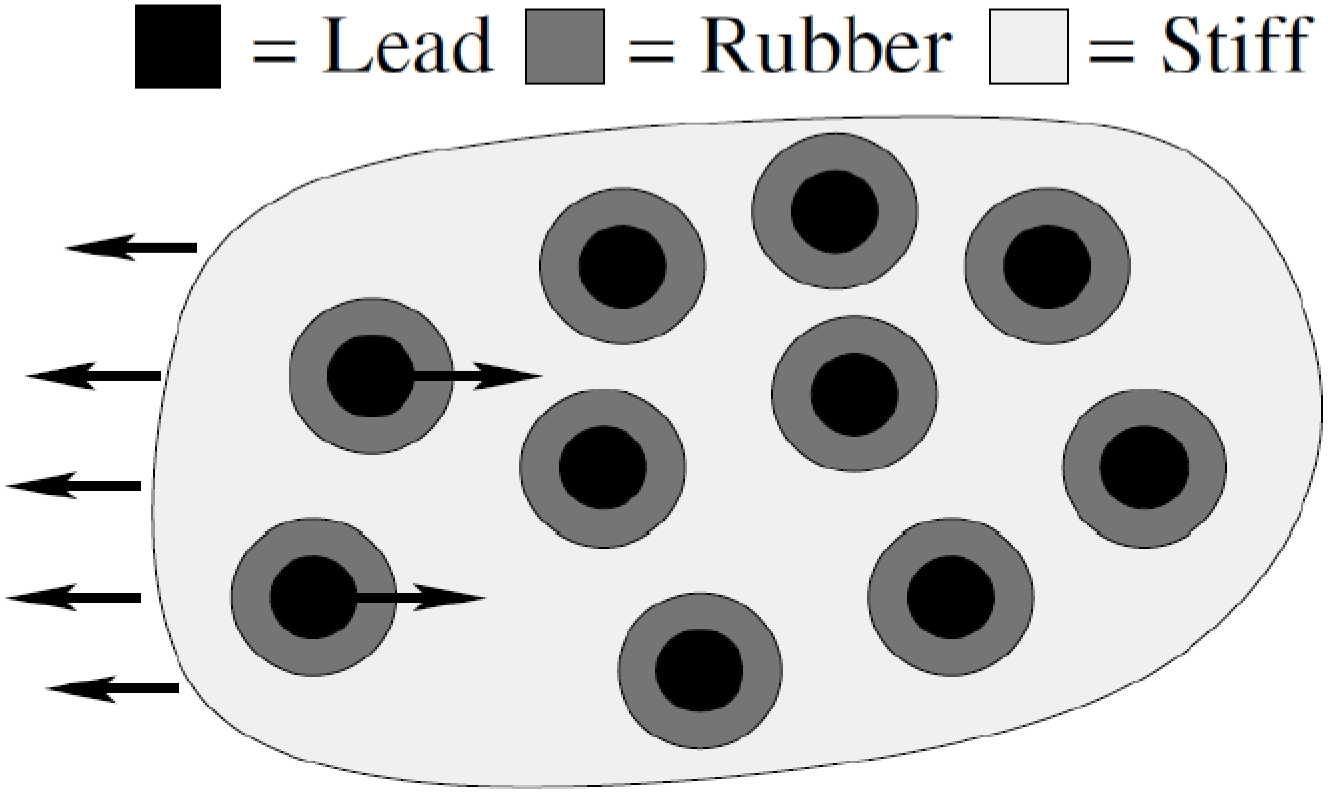
Split ring structure of David Smith

In two dimensions the Helmholtz equation describes both antiplane elastodynamics and TE (or TM) electrodynamics

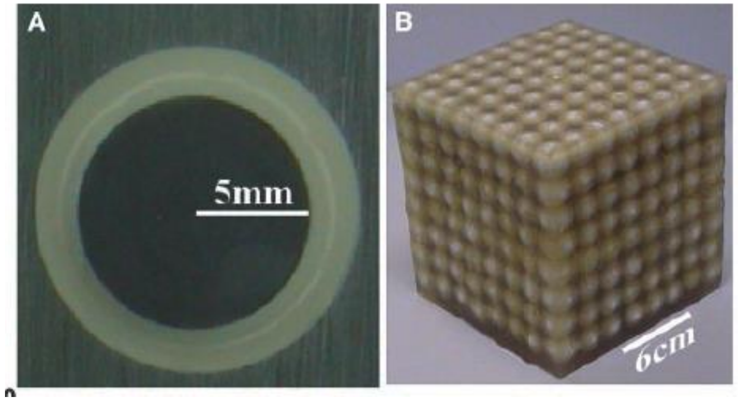


Split ring resonantor structure behaves as an acoustic band gap material (Movchan and Guenneau, 2004)

Sheng, Zhang, Liu, and Chan (2003) found that materials could exhibit a negative effective density over a range of frequencies

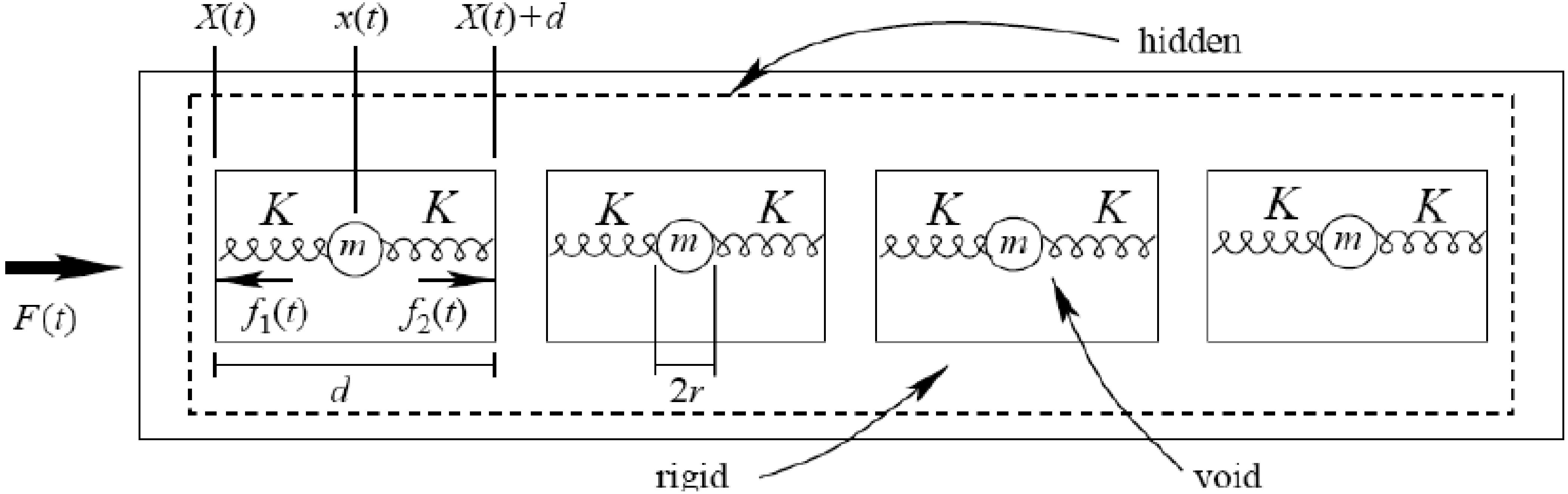


Experiment: Liu et. al (2000)



Mathematically the observation goes back to Zhikov (2000) also Bouchitte & Felbacq (2004)

A simplified one-dimensional model:



$$\hat{P} = M\hat{V}, \quad \text{with} \quad M = M_0 + \frac{2Knm}{2K - m\omega^2},$$

(With John Willis)

Early work recognizing anisotropic and negative densities

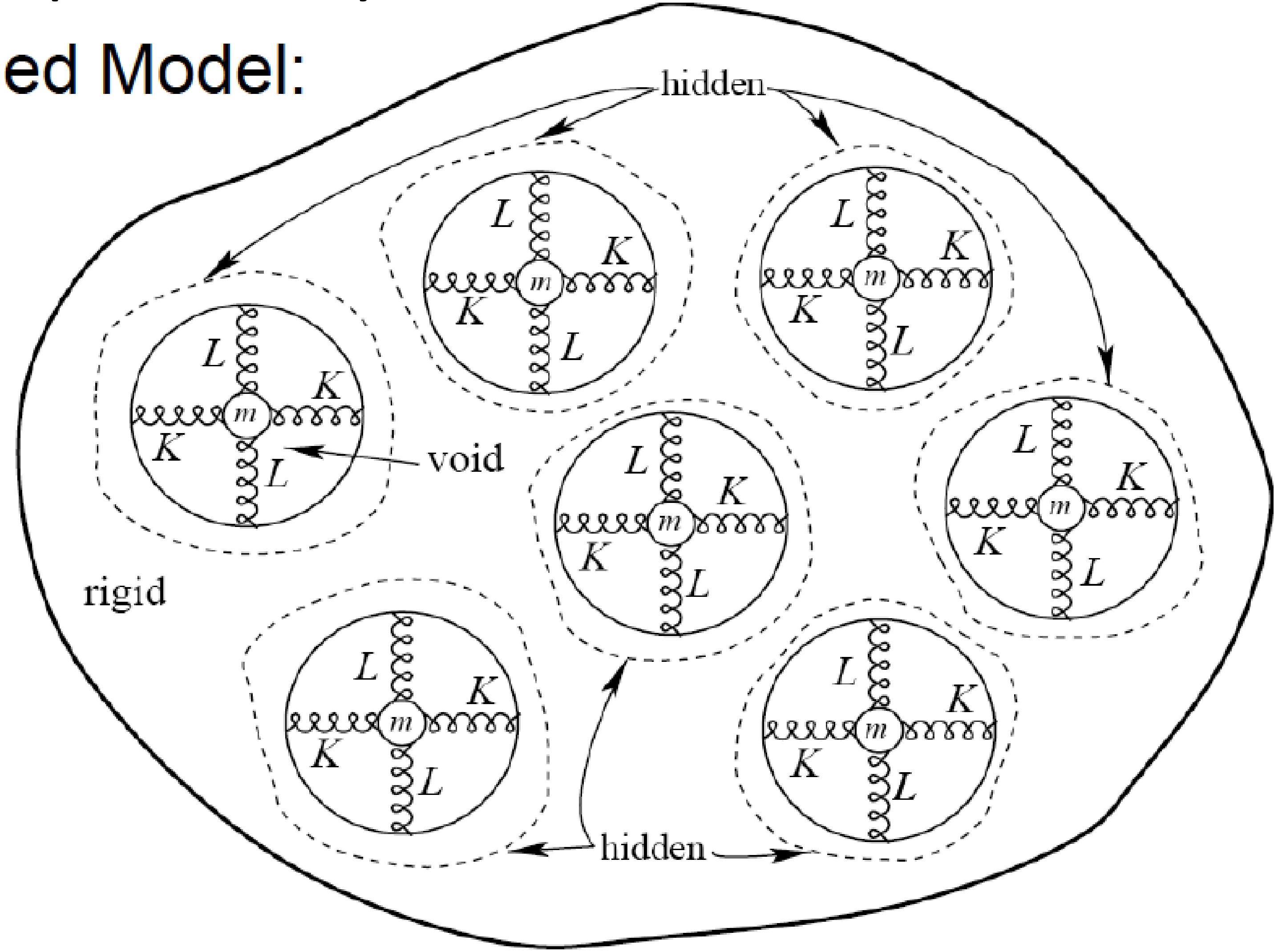
Aurialt and Bonnet (1994, 1995)

“The monochromatic macroscopic behavior is elastic, but with an effective density ϱ^{eff} of tensorial character and depending on the pulsation”

"hatched areas correspond to negative densities ϱ^{eff},
i.e., to stopping bands."

Anisotropic Density

Simplified Model:

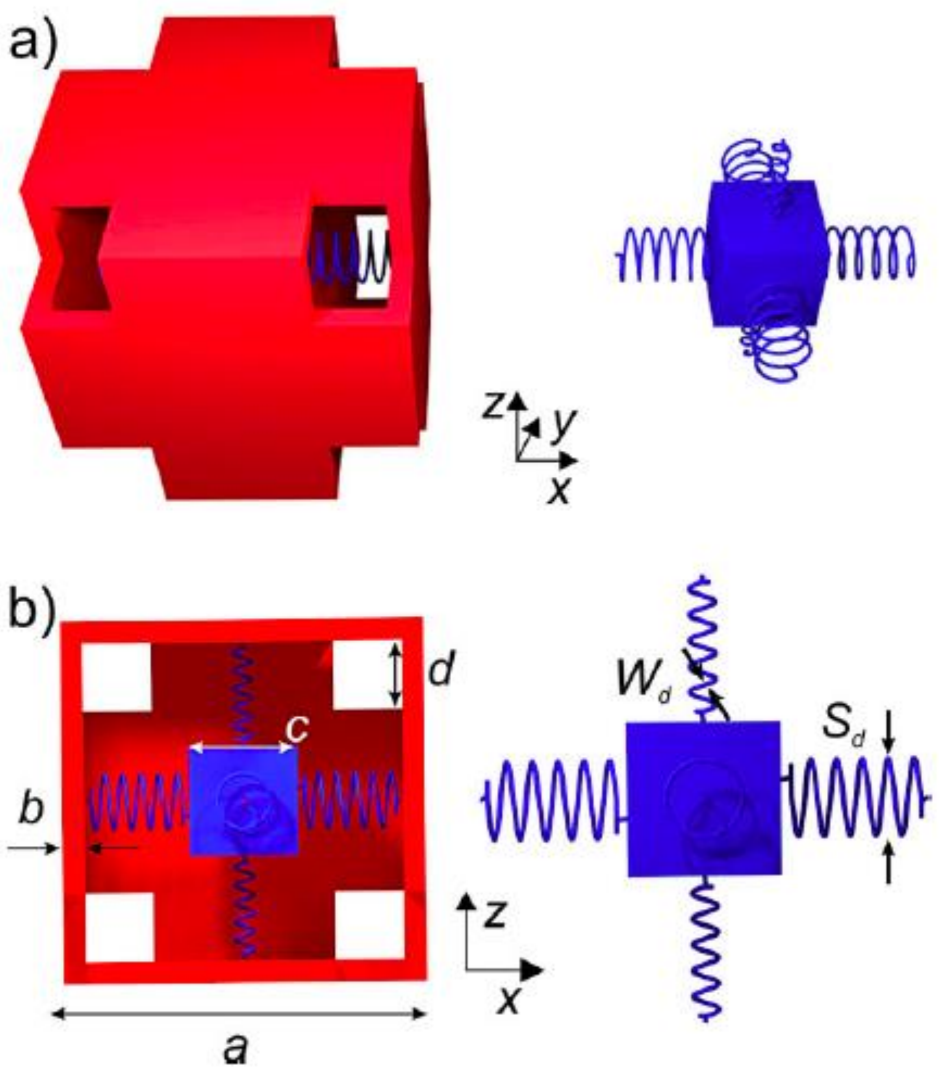


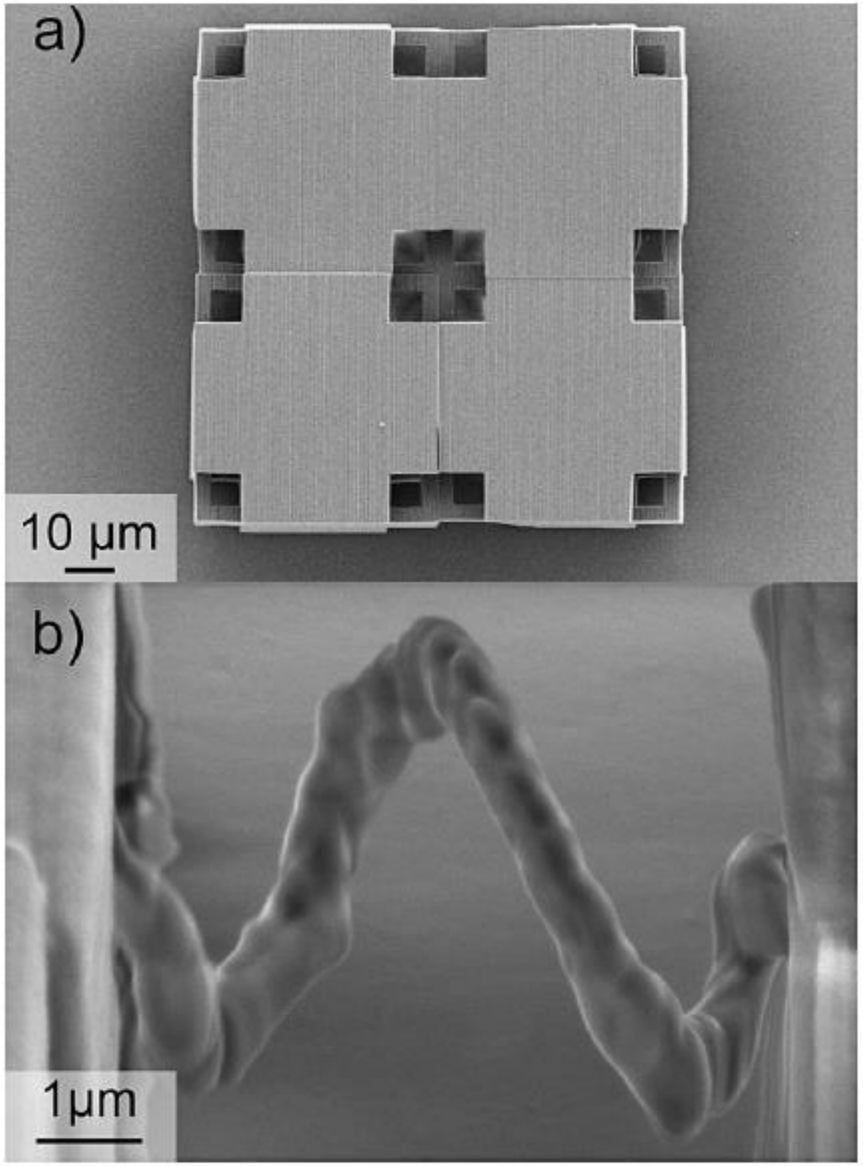
Anisotropic density in layered materials: Schoenberg and Sen (1983)

The springs could have some damping in which case the mass will be complex

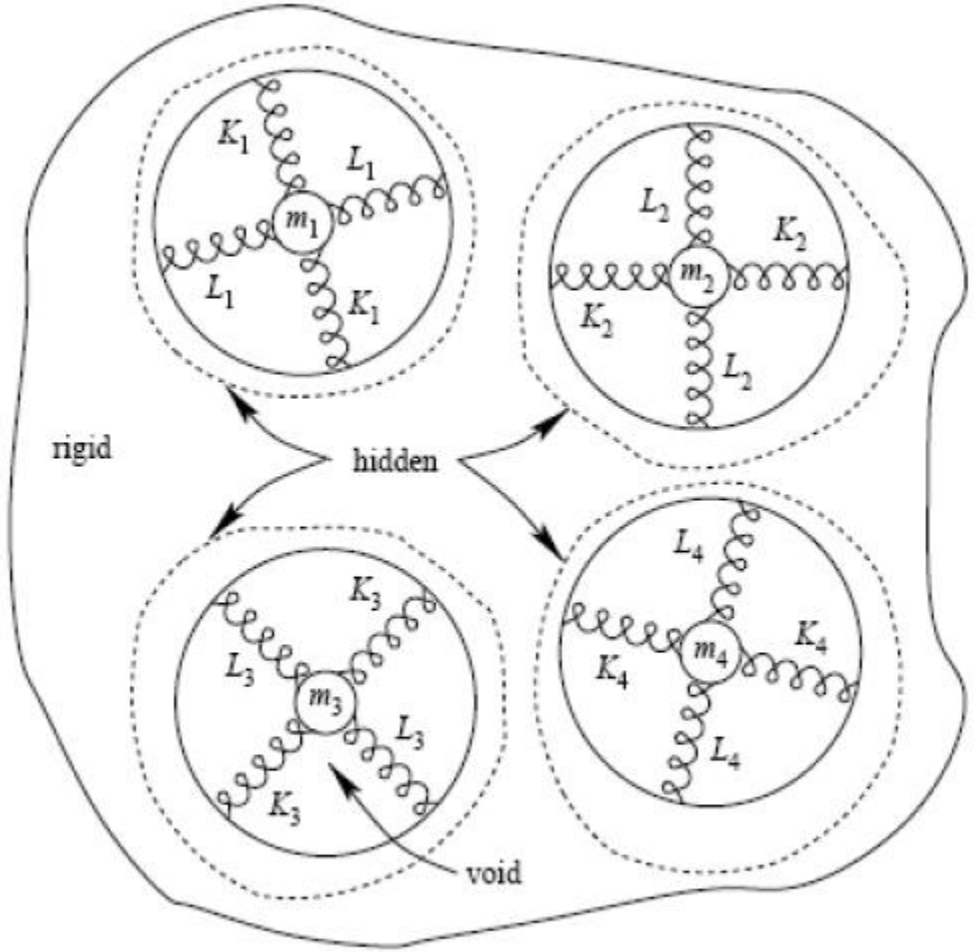
(With John Willis)

Realized by Buckmann, et.al., 2015





Seemingly rigid body



Eigenvectors of the effective mass density can rotate with frequency

(With John Willis)

What do we learn?
For materials with microstructure, Newton's law

$$F = ma$$

needs to be replaced by

$$F(t) = \int_{-\infty}^{t} K(t' - t)a(t')\, dt'$$

It takes some time for the internal masses to respond to the macroscopically applied force.

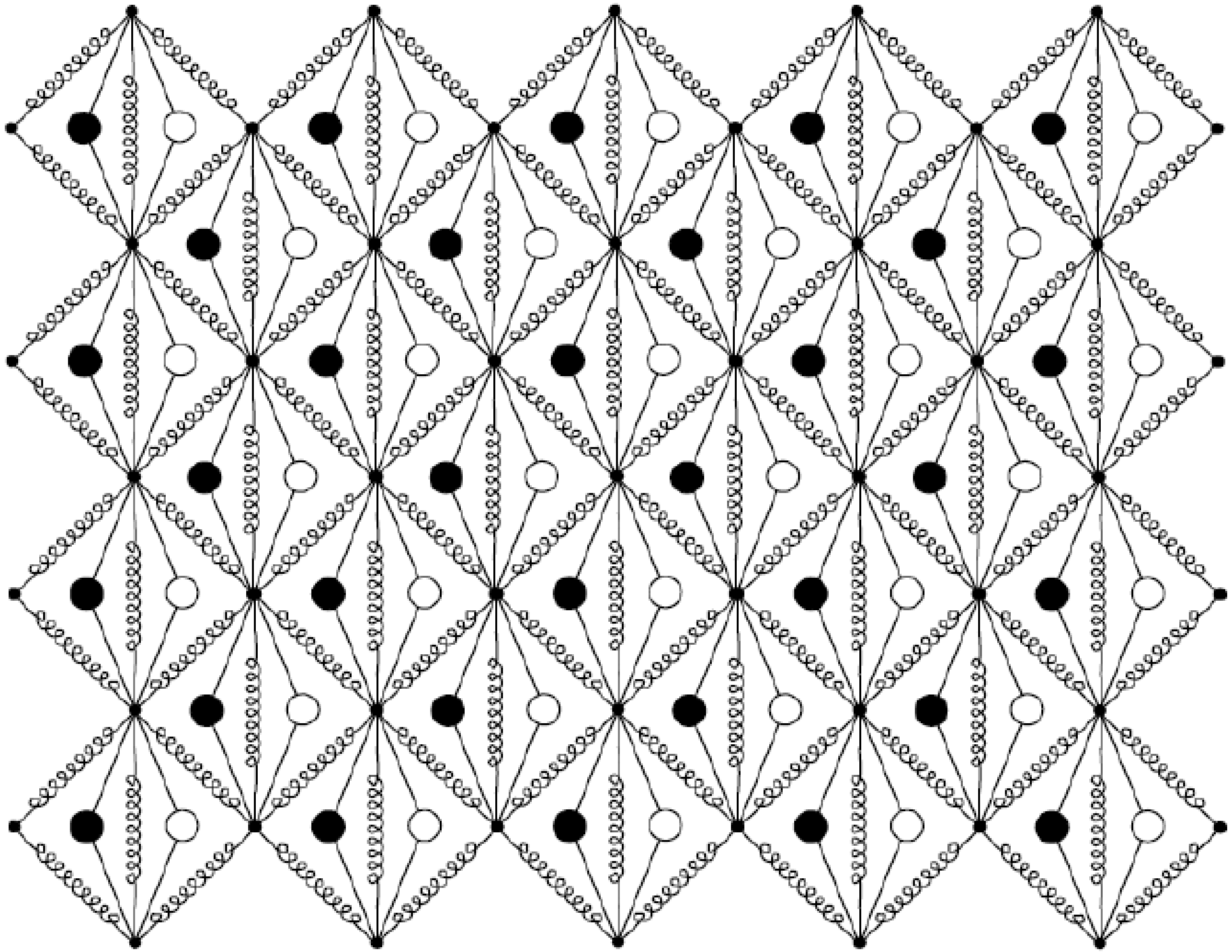
(With John Willis)

Models for the Willis equations

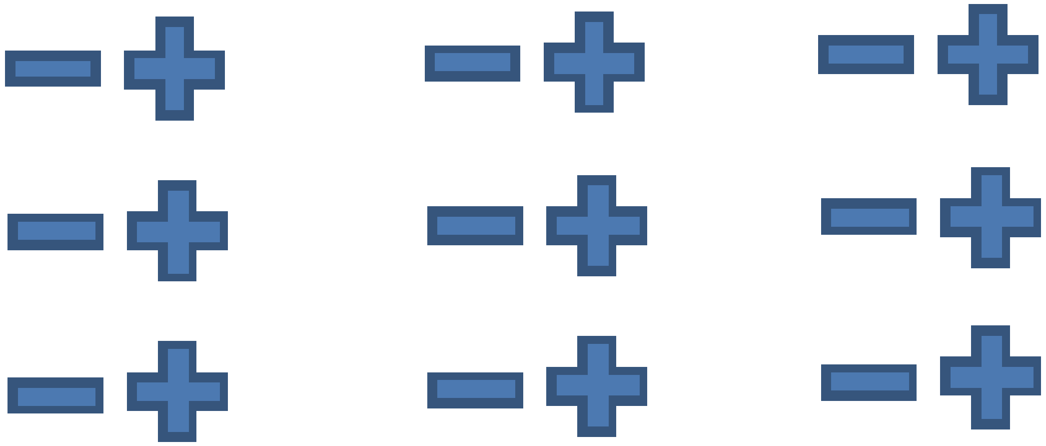
$$\begin{pmatrix} \boldsymbol{\sigma} \\ \mathbf{p} \end{pmatrix} = \begin{pmatrix} \mathcal{C} & \mathbf{S} \\ \mathbf{D} & \boldsymbol{\rho} \end{pmatrix} \begin{pmatrix} \nabla \mathbf{u} \\ \mathbf{v} \end{pmatrix}$$

σ –Stress
$\mathbf{p}$–Momentum
$\mathbf{u}$–Displacement
$\mathbf{v}$–Velocity

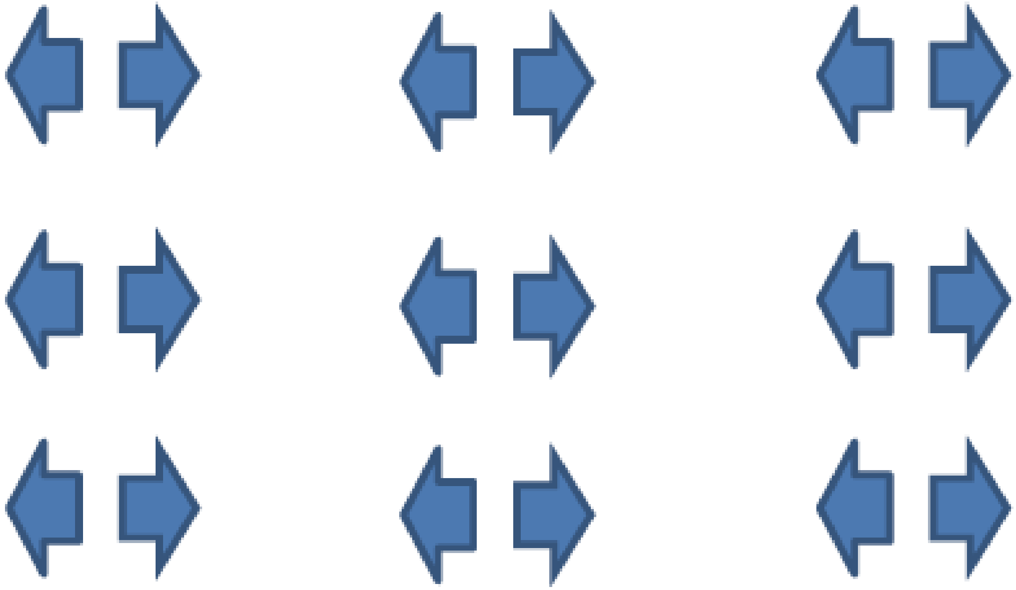
Analog of the bianisotropic equations of electromagnetism



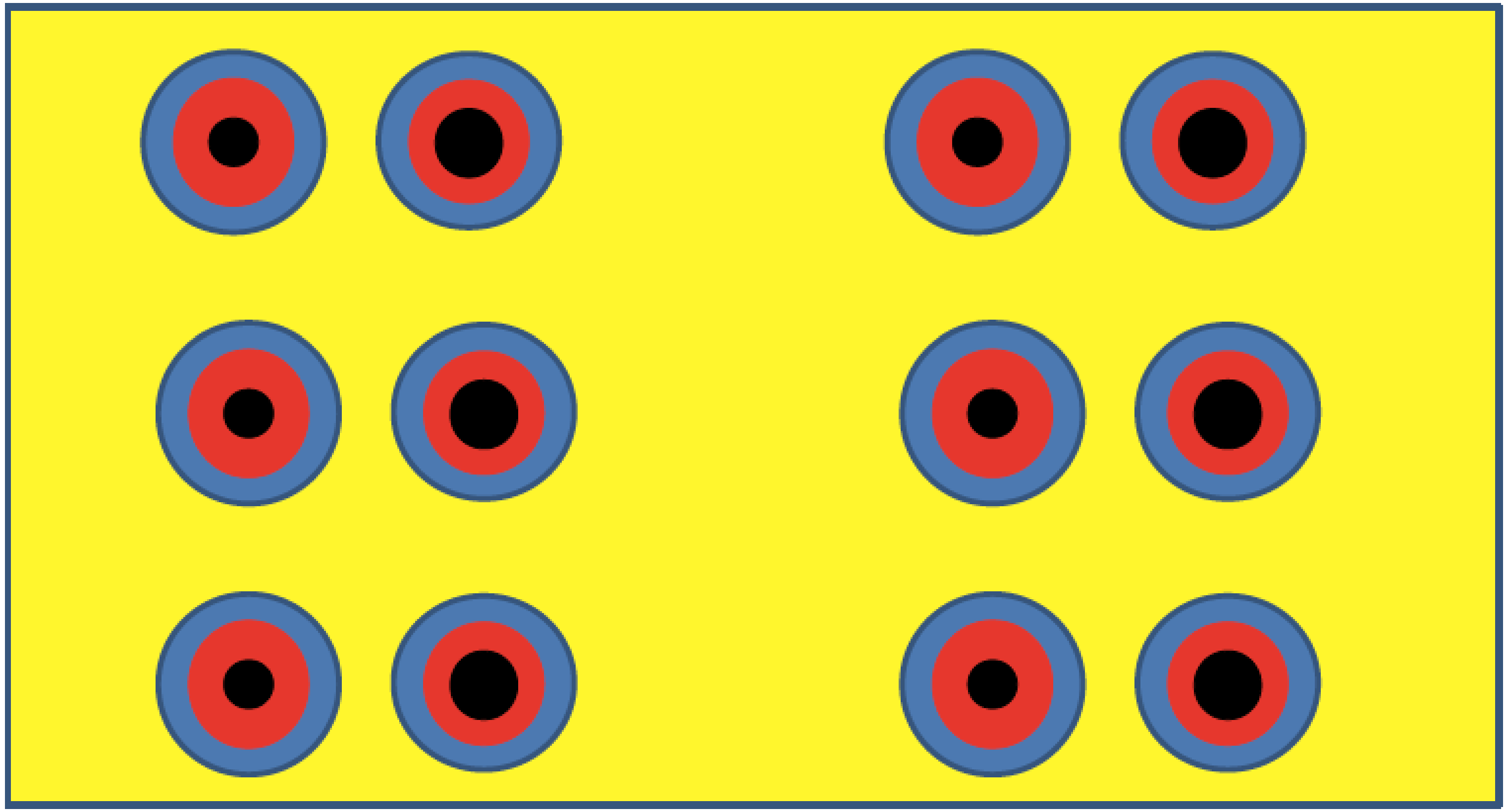
The Black circles have positive effective mass
The White circles have negative effective mass



Electric dipole array generates polarization field



Force dipole array generates stress field



Yellow=Compliant, Blue=Stiff
Red=Rubber, Black=Lead

Time harmonic acceleration with no strain gives stress: Example of a Willis material

Linear elastic equations under a Galilean transformation

$$\begin{pmatrix} \frac{\partial \boldsymbol{\sigma}}{\partial t} \\ \nabla \cdot \boldsymbol{\sigma} \end{pmatrix} = \underbrace{\begin{pmatrix} -\mathcal{C}(\mathbf{x}) & 0 \\ 0 & \rho(\mathbf{x}) \end{pmatrix}}_{\mathbf{Z}(\mathbf{x})} \begin{pmatrix} -\frac{1}{2}\left[\nabla \mathbf{v} + \nabla \mathbf{v}^T\right] \\ \frac{\partial \mathbf{v}}{\partial t} \end{pmatrix} \qquad x_4 = -t,$$

$$\overline{\nabla} = \begin{pmatrix} \nabla \\ \frac{\partial}{\partial x_4} \end{pmatrix} = \begin{pmatrix} \nabla \\ -\frac{\partial}{\partial t} \end{pmatrix}, \quad J_{ik} = -\frac{\partial \sigma_{ik}}{\partial t}, \quad \text{for } i,k = 1,2,3, \quad J_{4k} = -\{\nabla \cdot \boldsymbol{\sigma}\}_k,$$

$\overline{\nabla} \cdot \mathbf{J} = 0, \quad \mathbf{J} = \mathbf{Z}\overline{\nabla}\mathbf{v}.$ (looks a bit like conductivity)

Galilean transformation: $\overline{\mathbf{x}}' = \mathbf{A}\overline{\mathbf{x}}, \quad \text{with } \mathbf{A} = \begin{pmatrix} \mathbf{I} & \mathbf{w} \\ 0 & 1 \end{pmatrix},$

$$\begin{pmatrix} \frac{\partial \boldsymbol{\sigma}'}{\partial t'} \\ \nabla' \cdot \boldsymbol{\sigma}' \end{pmatrix} = \begin{pmatrix} \mathcal{I} & \mathbf{w}\mathbf{I} \\ 0 & \mathbf{I} \end{pmatrix} \begin{pmatrix} \frac{\partial \boldsymbol{\sigma}}{\partial t} \\ \nabla \cdot \boldsymbol{\sigma} \end{pmatrix} = \begin{pmatrix} \frac{\partial \boldsymbol{\sigma}}{\partial t} + \mathbf{w}(\nabla \cdot \boldsymbol{\sigma})^T \\ \nabla \cdot \boldsymbol{\sigma} \end{pmatrix},$$

$$\begin{pmatrix} -\nabla' \mathbf{v}' \\ \frac{\partial \mathbf{v}'}{\partial t'} \end{pmatrix} = \begin{pmatrix} \mathcal{I} & 0 \\ \mathbf{I}\mathbf{w}^T & \mathbf{I} \end{pmatrix}^{-1} \begin{pmatrix} -\nabla \mathbf{v} \\ \frac{\partial \mathbf{v}}{\partial t} \end{pmatrix} = \begin{pmatrix} -\nabla \mathbf{v} \\ \frac{\partial \mathbf{v}}{\partial t} + \mathbf{w}^T \nabla \mathbf{v} \end{pmatrix},$$

$$\begin{aligned} \mathbf{Z}'(\overline{\mathbf{x}}') &= \begin{pmatrix} \mathcal{I} & \mathbf{w}\mathbf{I} \\ 0 & \mathbf{I} \end{pmatrix} \mathbf{Z}(\mathbf{x}) \begin{pmatrix} \mathbf{I} & 0 \\ \mathbf{I}\mathbf{w}^T & \mathbf{I} \end{pmatrix} \\ &= \begin{pmatrix} -\mathcal{C}(\mathbf{x}) + \mathbf{w}\rho(\mathbf{x})\mathbf{w}^T & \mathbf{w}\rho(\mathbf{x}) \\ \rho(\mathbf{x})\mathbf{w}^T & \rho(\mathbf{x}) \end{pmatrix}. \end{aligned}$$

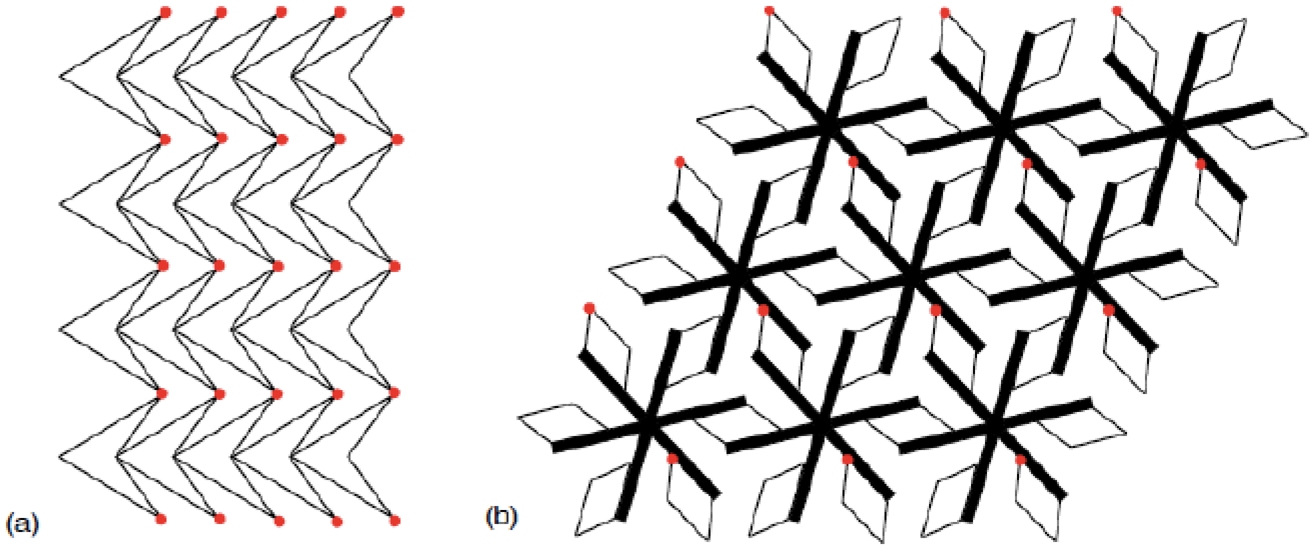
Has Willis type couplings!
Also a non-symmetric stress

"Transformation Optics" that dates to Dolin (1961)

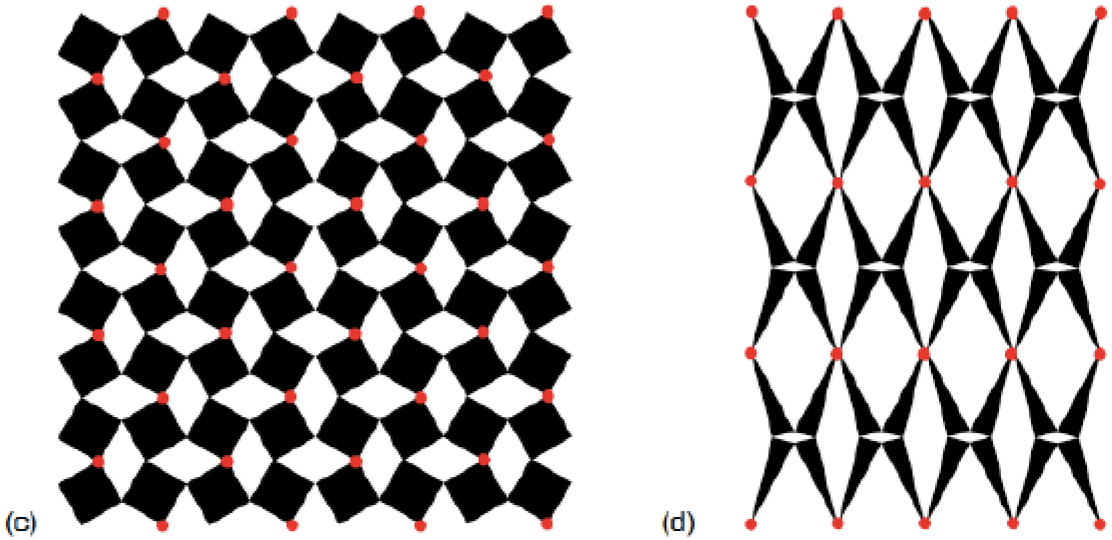
Also Guoliang Huang, et.al.

Unimode and Bimode Affine Materials

Examples of nonlinear 2d unimode materials

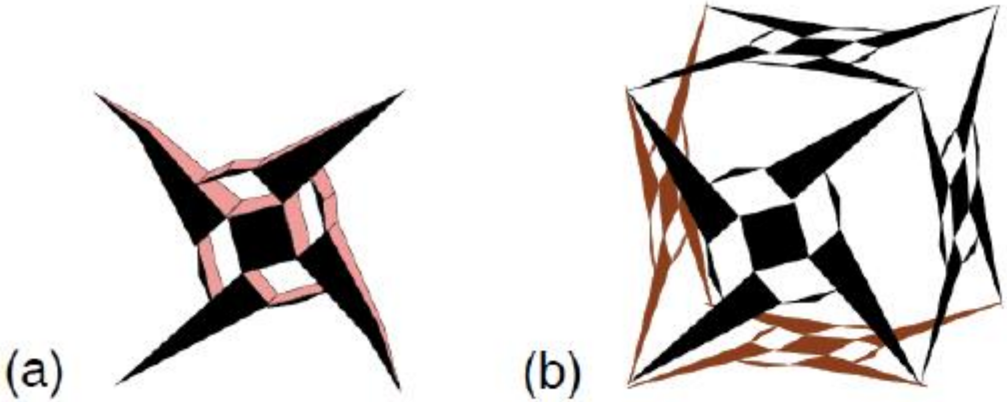


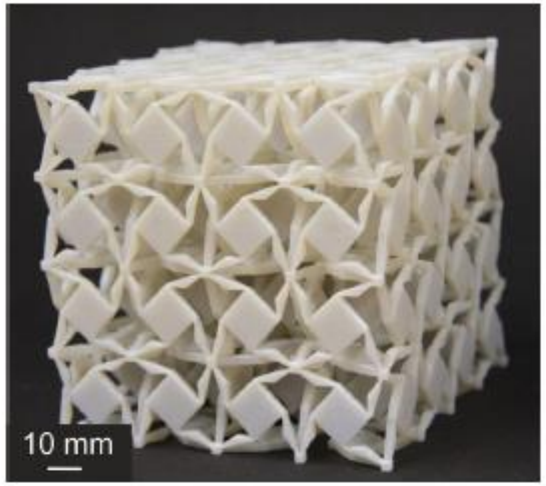
Larsen et. al.　　Milton

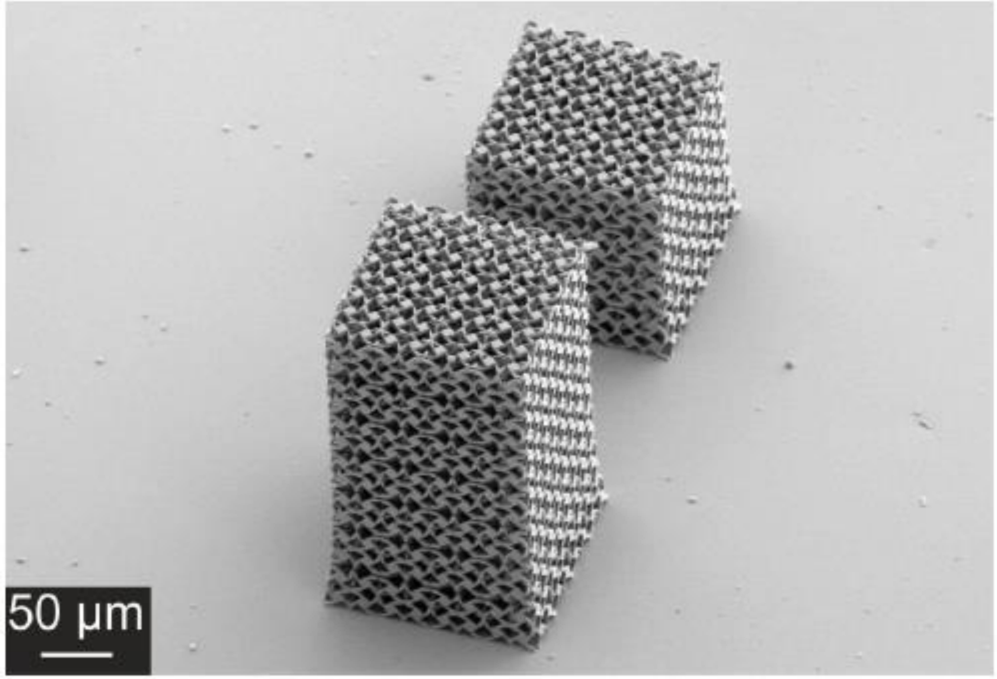


Grima and Evans

Three Dimensional Dilational materials

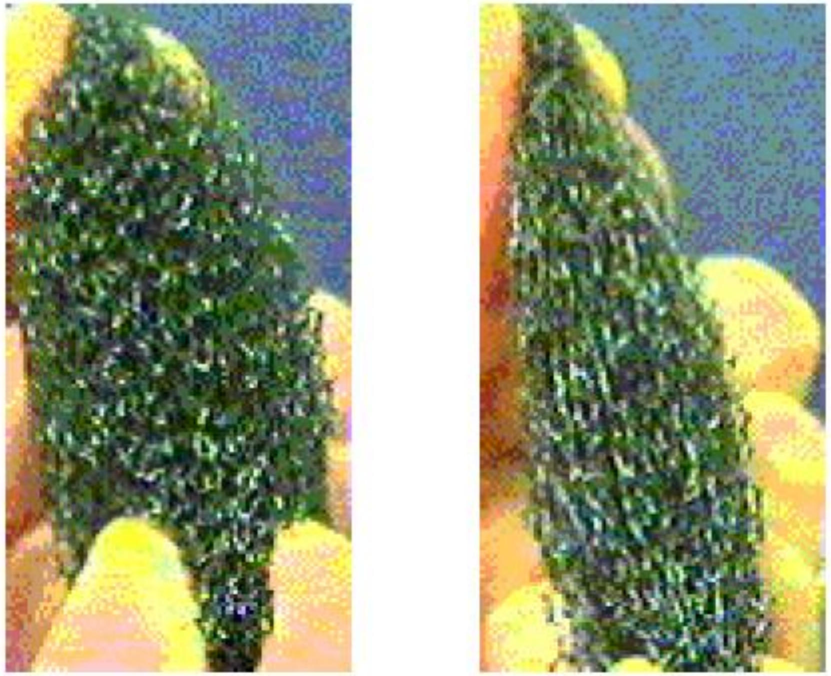






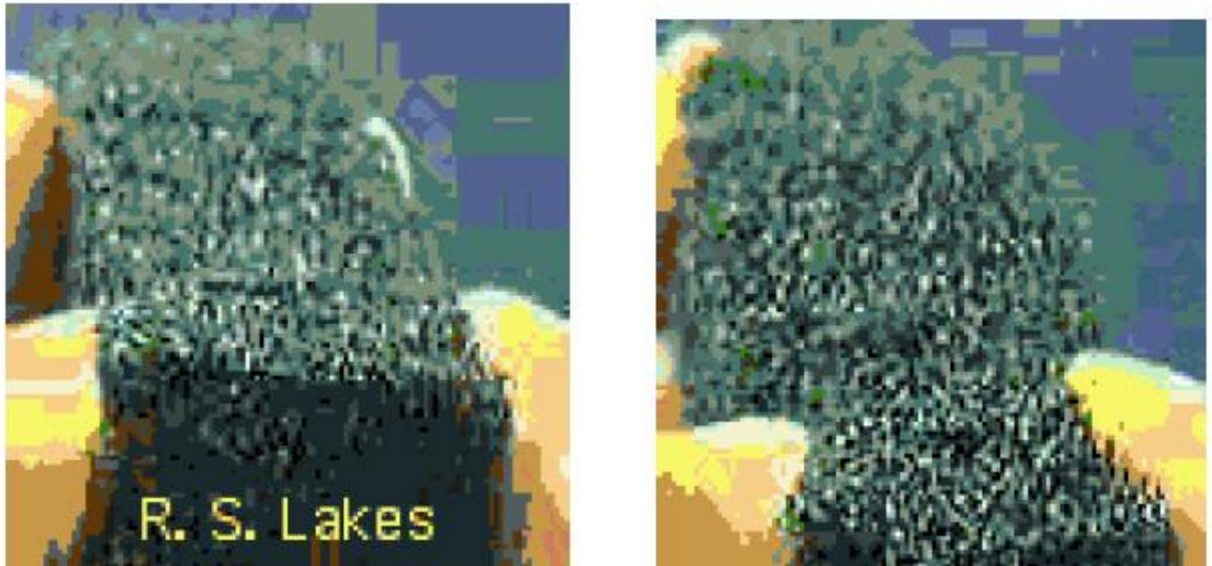
Buckmann,, Schittny, Thiel, Kadic, Milton Wegener (2014)

Experiment of R. Lakes (1987)

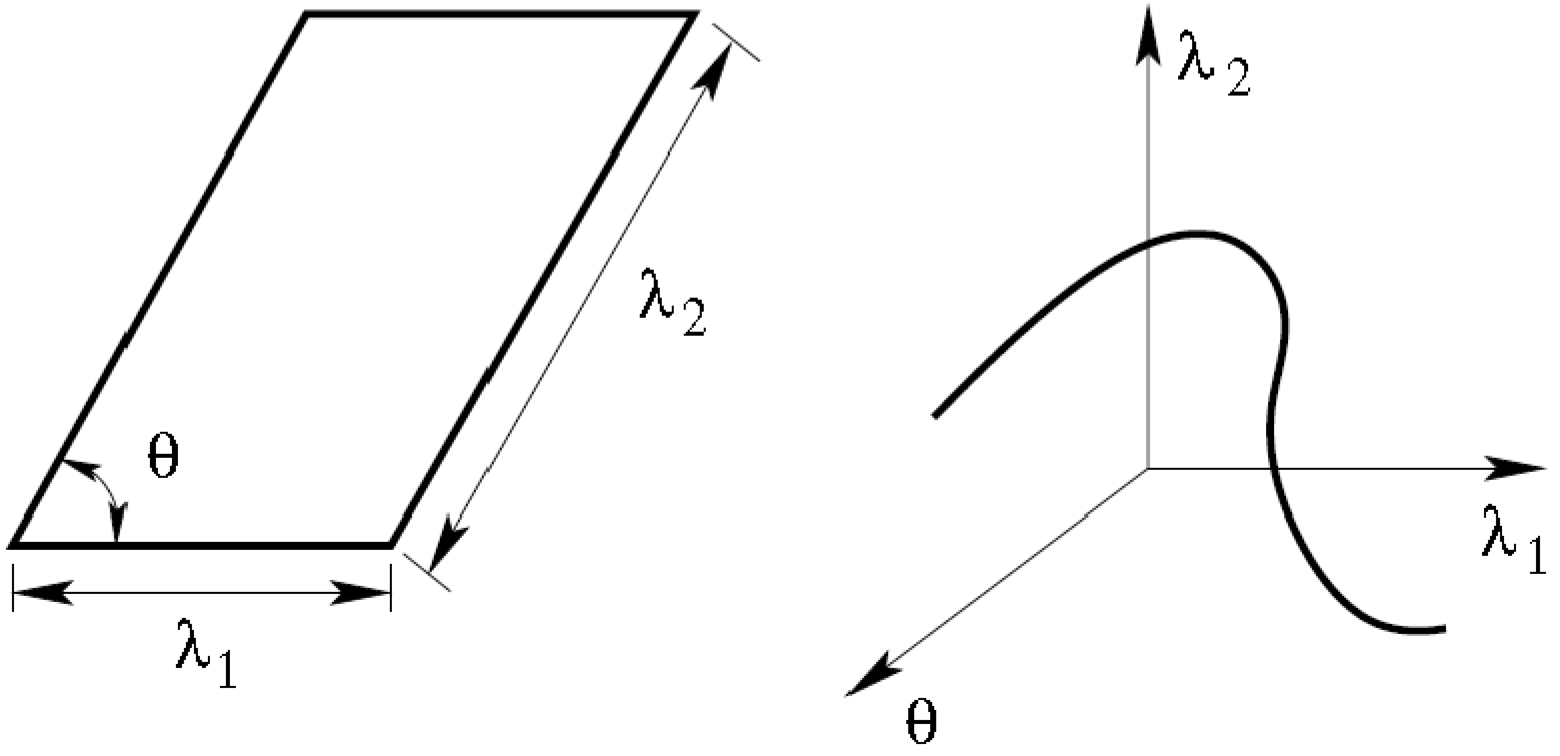


Normal Foam

These are ideal "Auxetic" materials



Unimode:

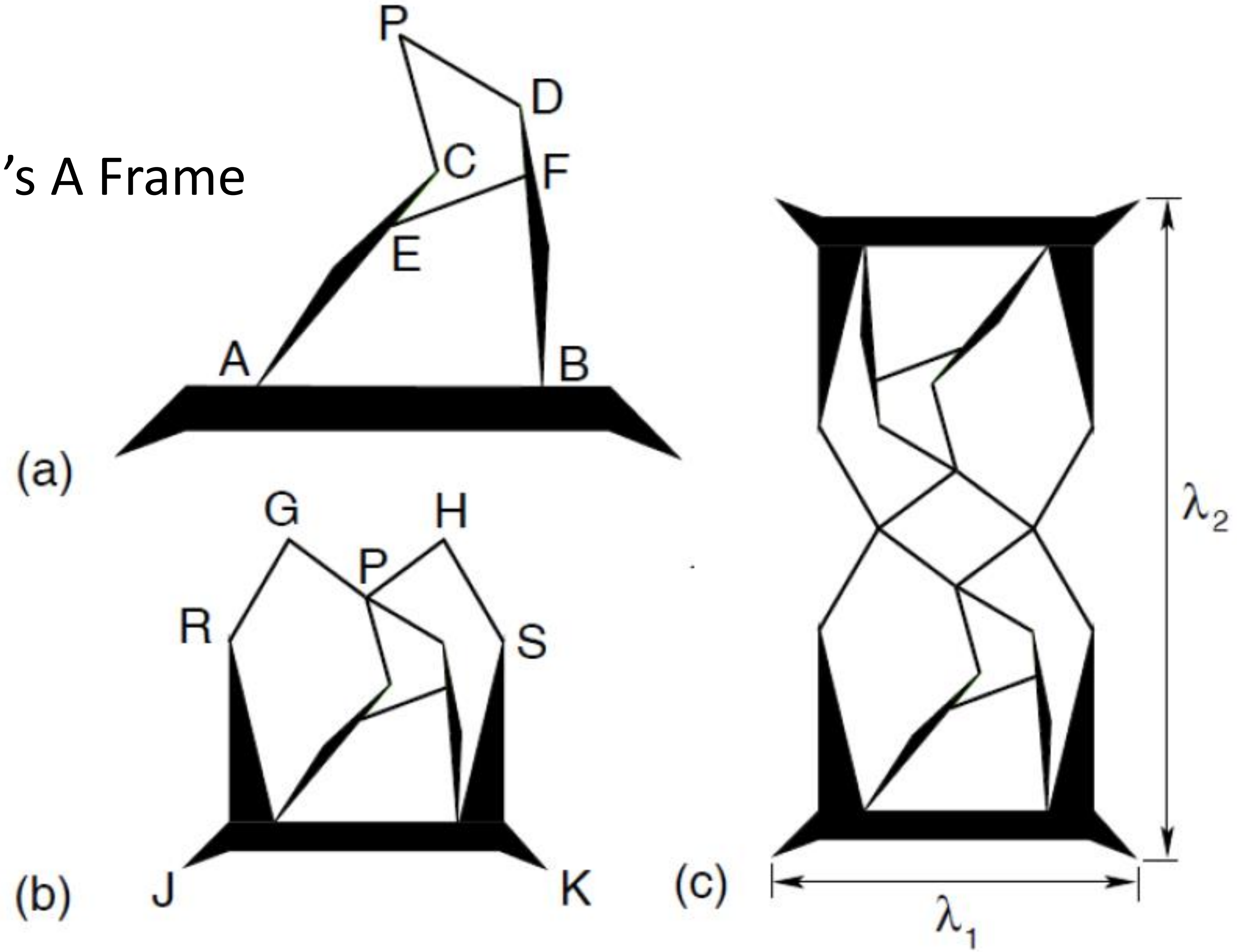


What trajectories $\lambda_1(t) = \lambda_2(t) = \theta(t)$ are realizable? (Answer: any trajectory!)

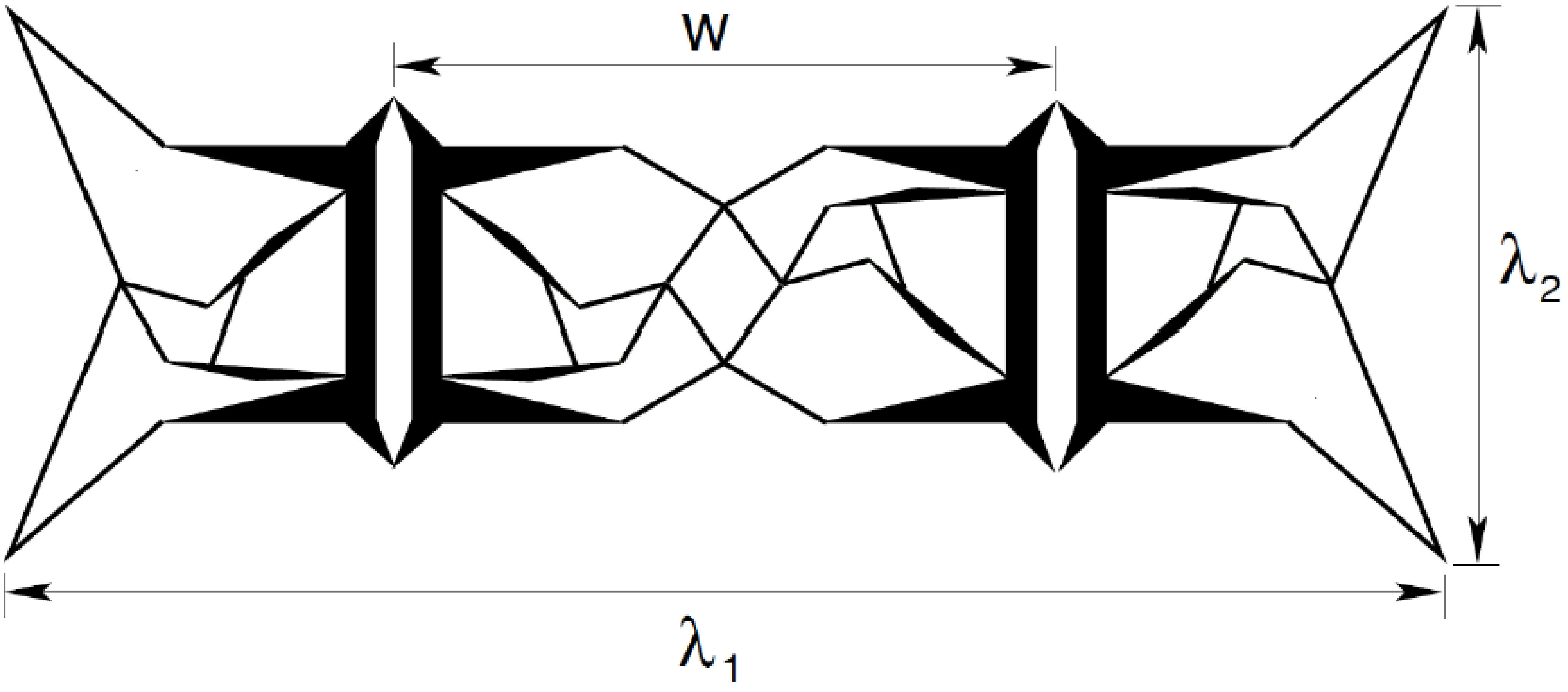
In a bimode material there is a surface of realizable motions.

Cell of the perfect expander: a unimode material

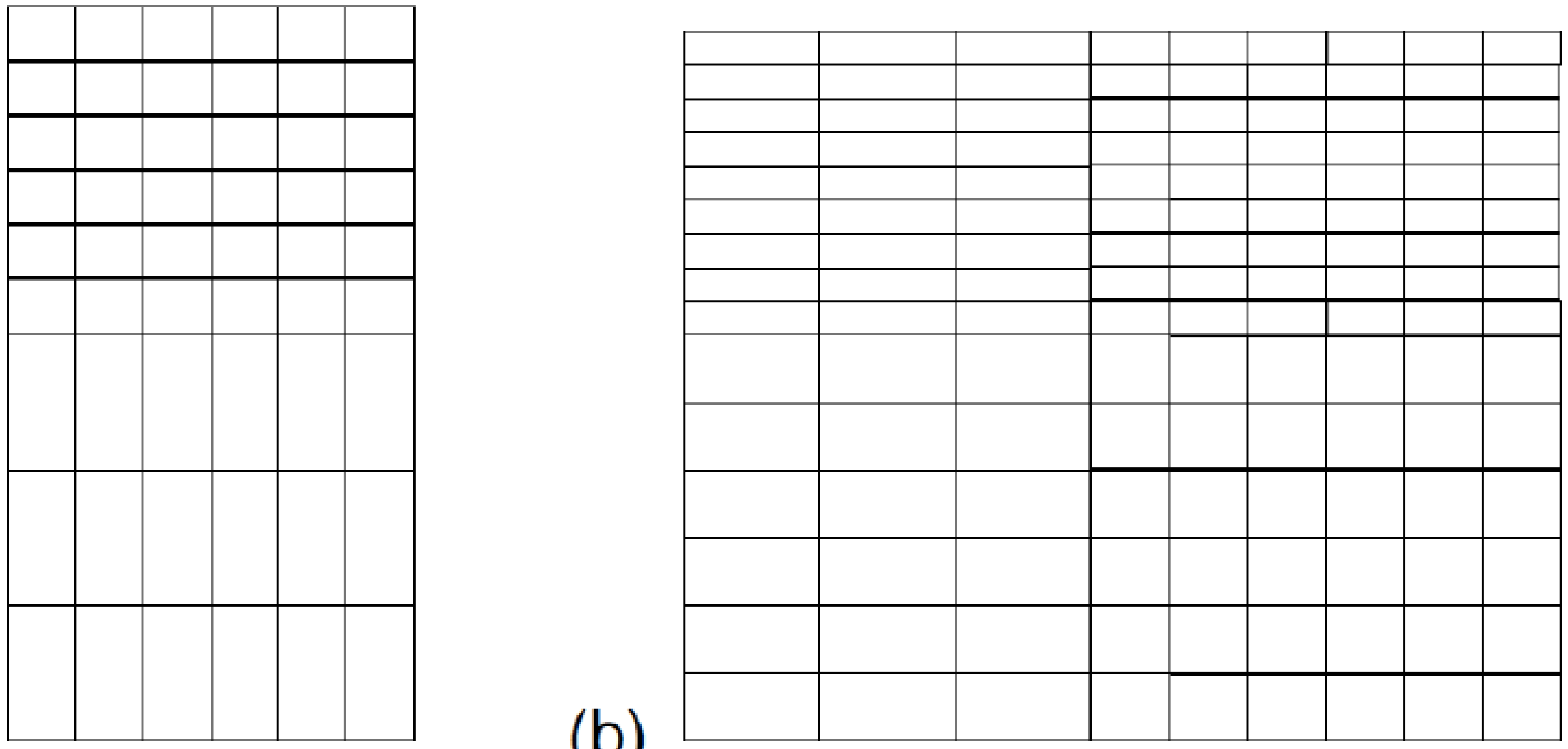
Hart's A Frame



Cell of a bimode material



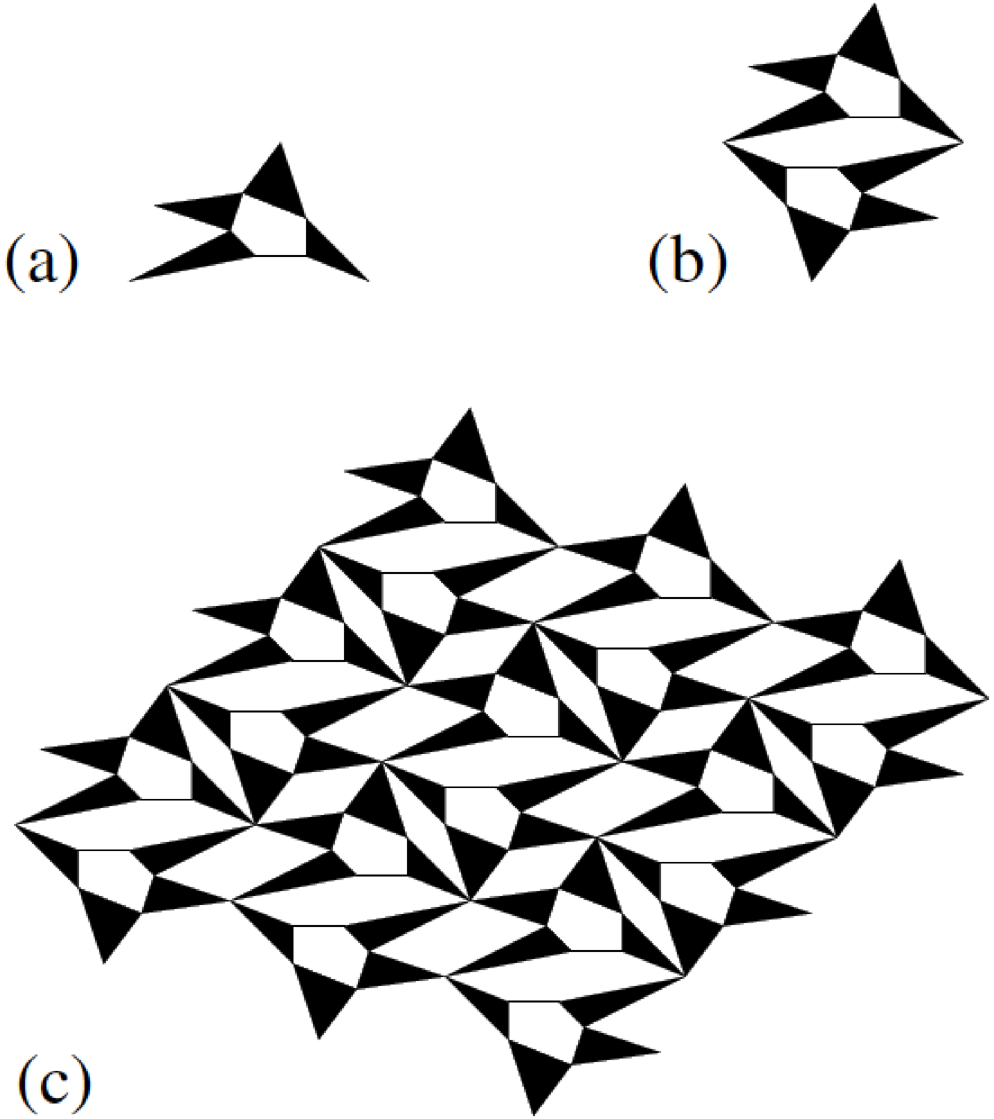
However neither are affine materials:



(a) (b)

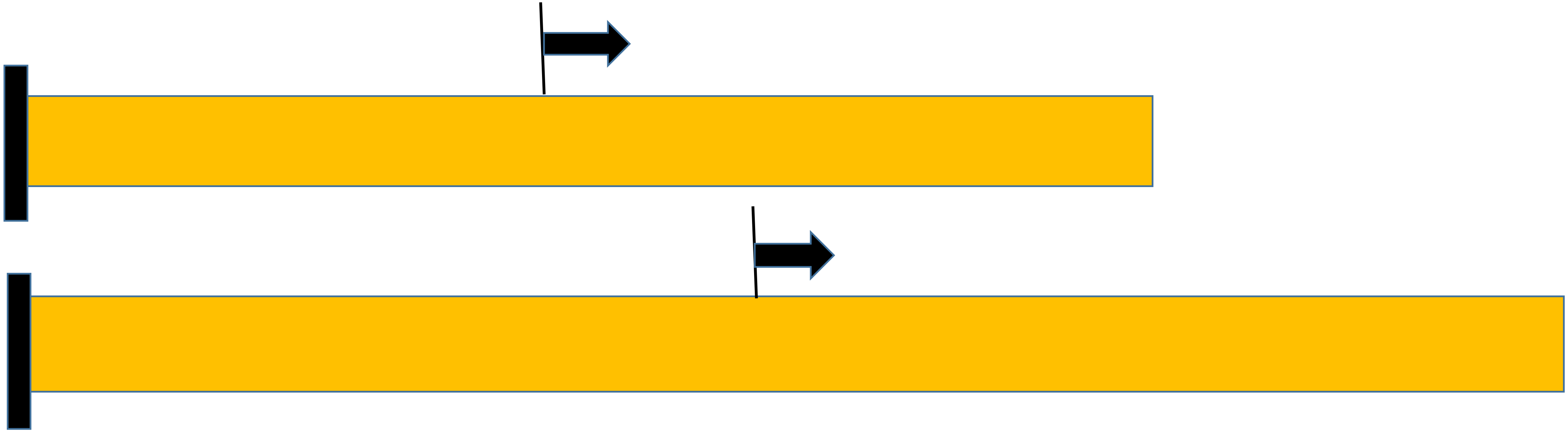
So can one get affine bimode materials?

Bimode material for which the only easy modes of deformations are affine ones

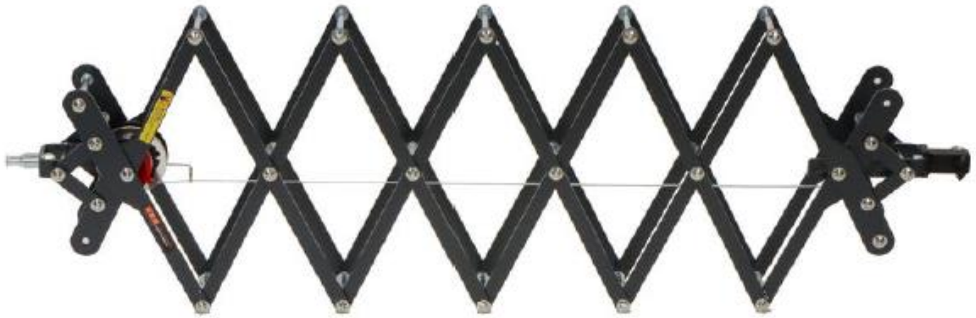


Characteristic Feature of Affine Materials:

They dislike strain gradients



Example of Pierre Seppecher
Like a Pantograph:



Field Patterns: A new type of Wave

Ornella Mattei and Graeme Milton,
Department of Mathematics, The University of Utah


설계 시
상담
32
8-0732
Space&Time
Architecture

Space-time microstructures

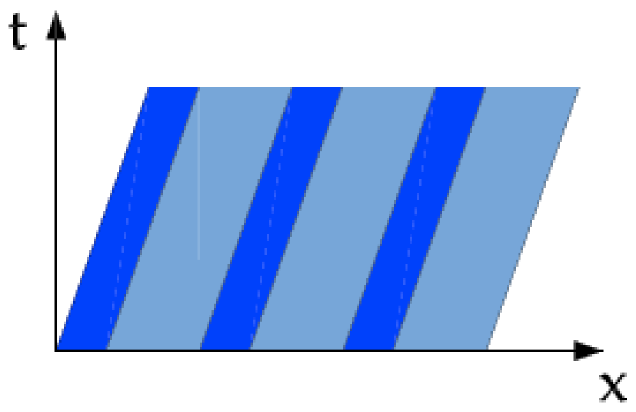
$$(a\,u_t)_t - (b\,u_x)_x = 0$$

Static materials: $a = a(x)$ and $b = b(x)$

Space-time microstructures: $a = a(x, t)$ and $b = b(x, t)$

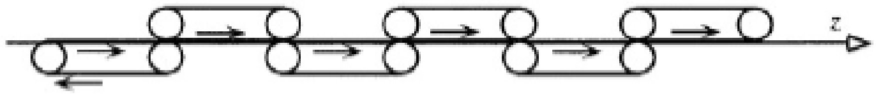
Activated materials:

The property pattern moves



Kinetic materials:

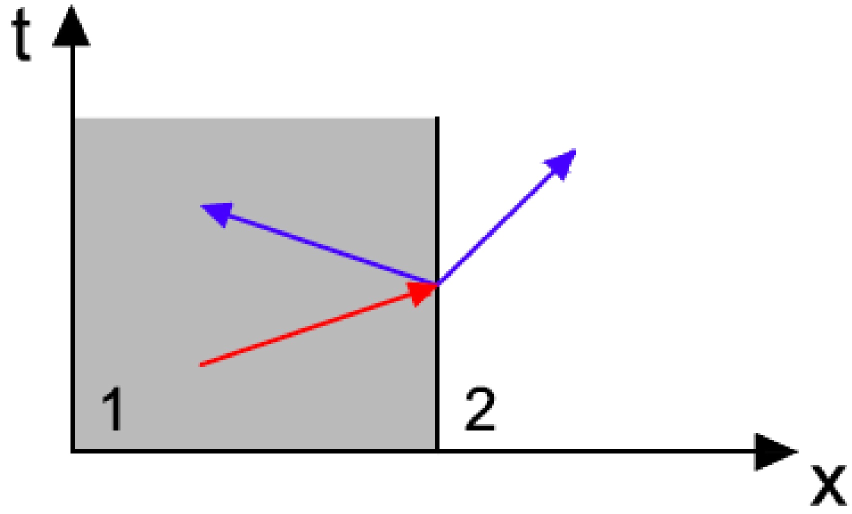
The material itself moves



[K.A. Lurie, An Introduction to the Mathematical Theory of Dynamic Materials (2007)]

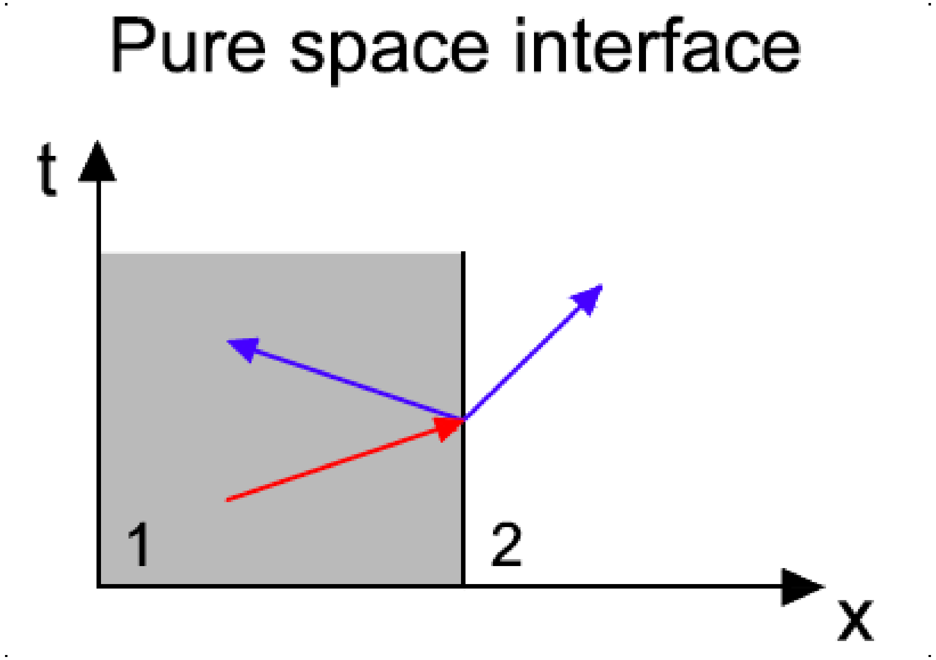
Dynamic composites

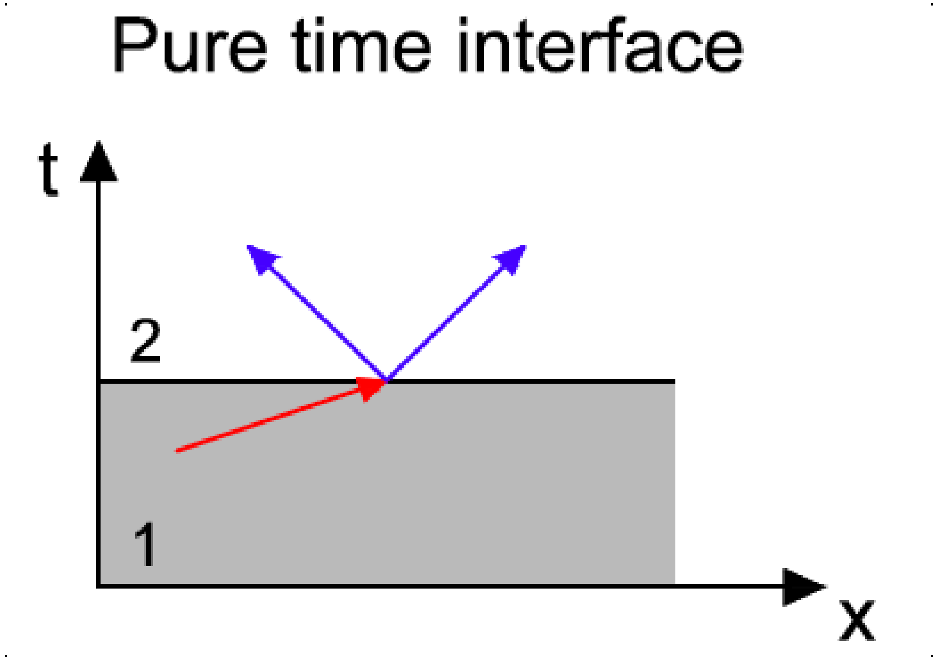
Pure space interface



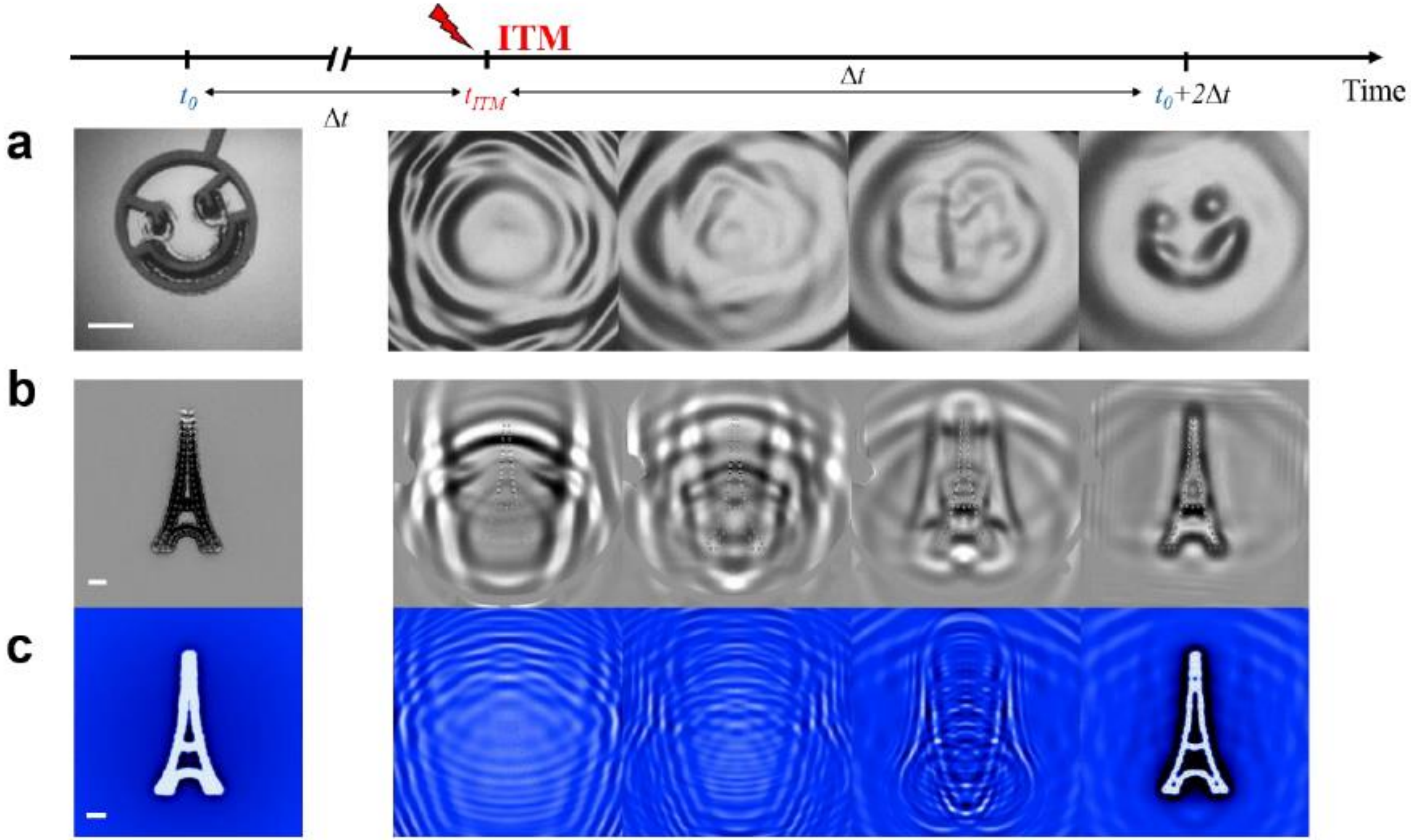
Pure time interface

Dynamic composites



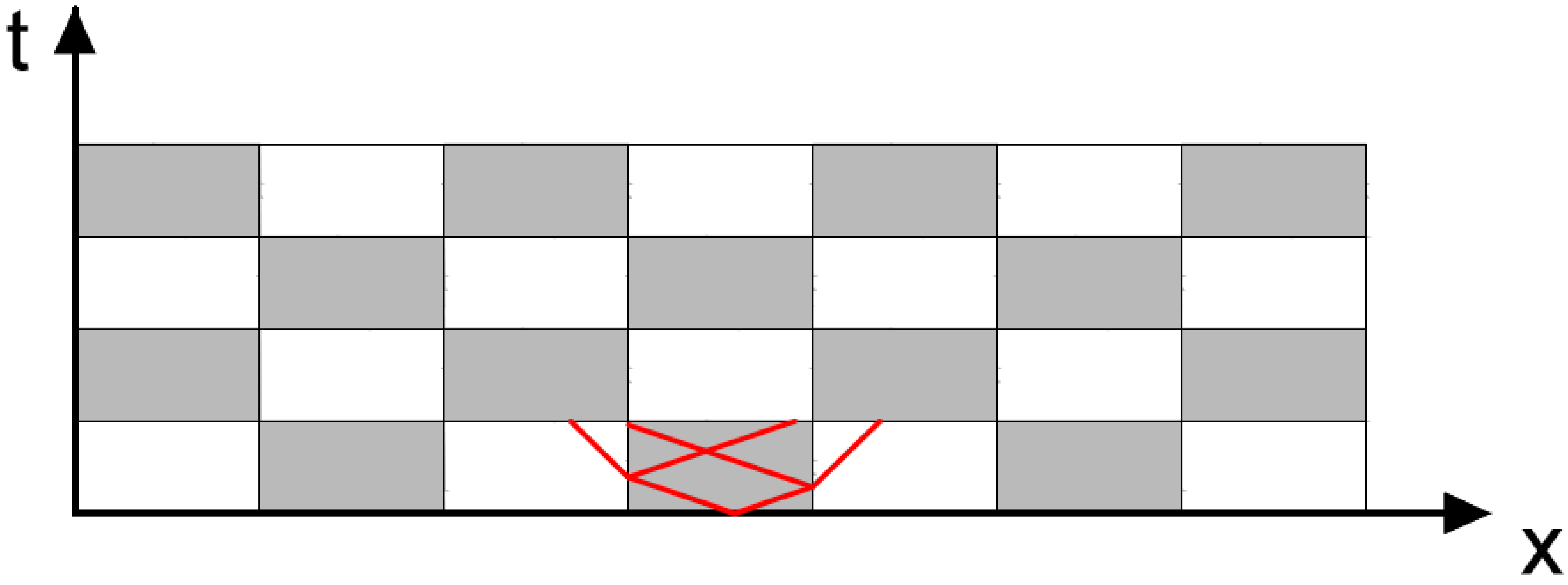


What happens at a time interface?



Bacot, Labousse, Eddi, Fink, and Fort, Nature 2016

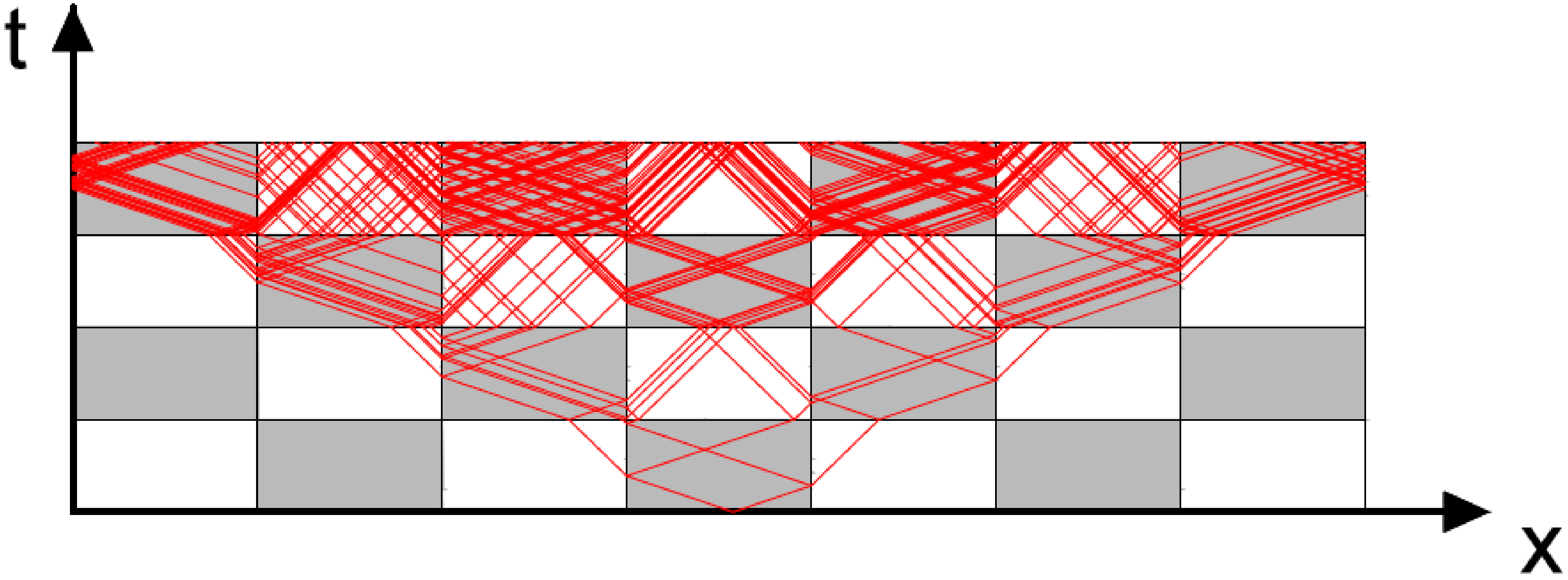
Evolution of a disturbance in a space-time checkerboard



Transmission conditions:

$$
\begin{cases}
V_1 = V_2 \\
\mathbf{n} \cdot \boldsymbol{\sigma}_1 \nabla V_1 = \mathbf{n} \cdot \boldsymbol{\sigma}_2 \nabla V_2
\end{cases}
$$

Evolution of a disturbance in a space-time checkerboard

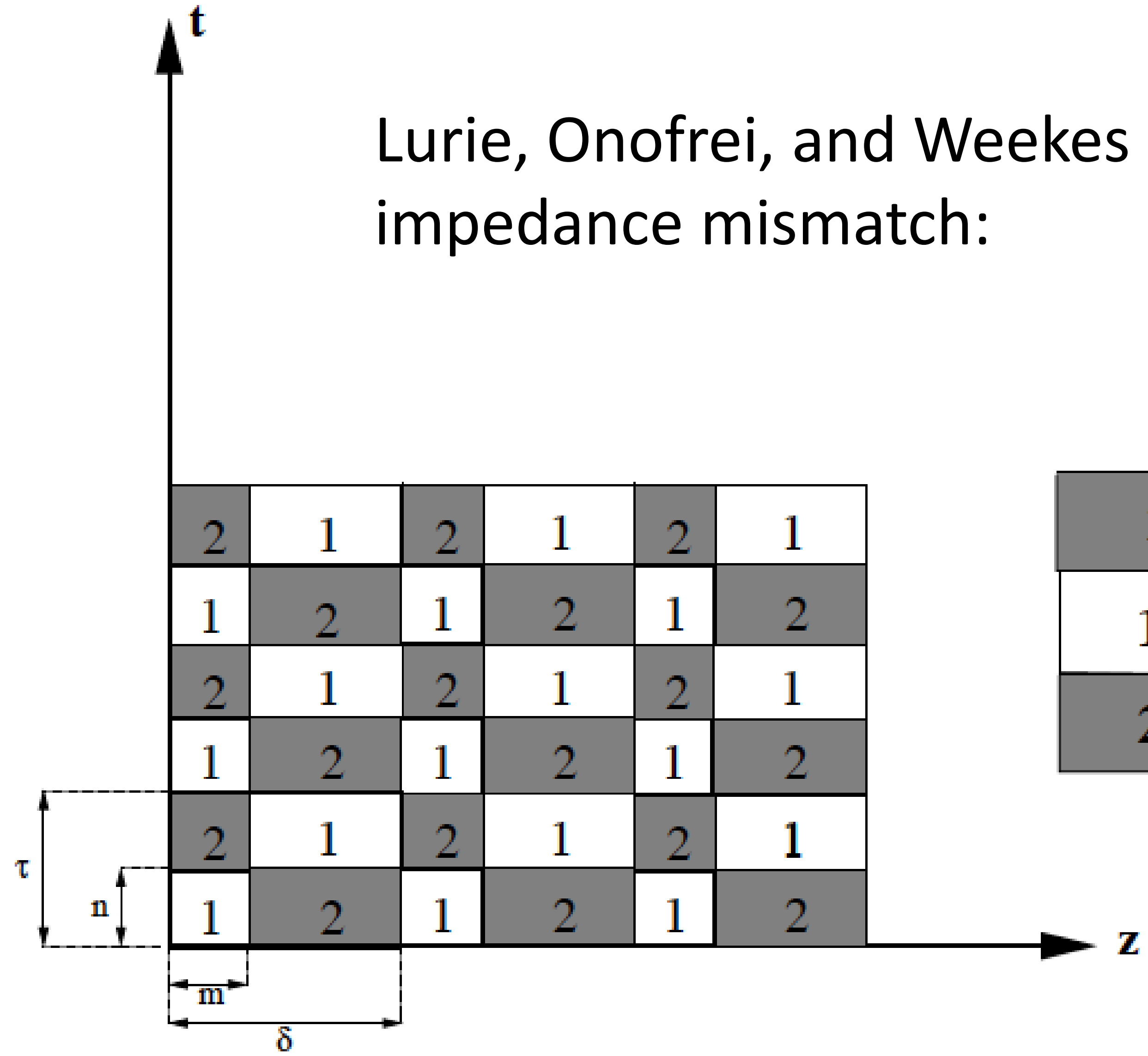


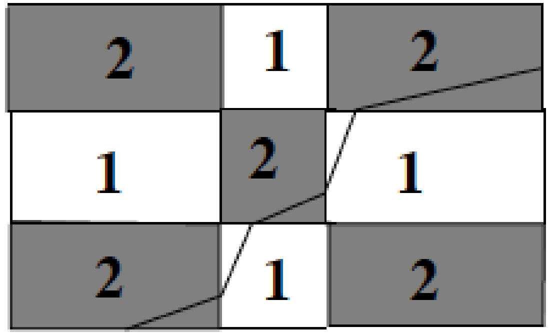
Transmission conditions:

$$\begin{cases} V_1 = V_2 \\ \mathbf{n} \cdot \boldsymbol{\sigma}_1 \nabla V_1 = \mathbf{n} \cdot \boldsymbol{\sigma}_2 \nabla V_2 \end{cases}$$

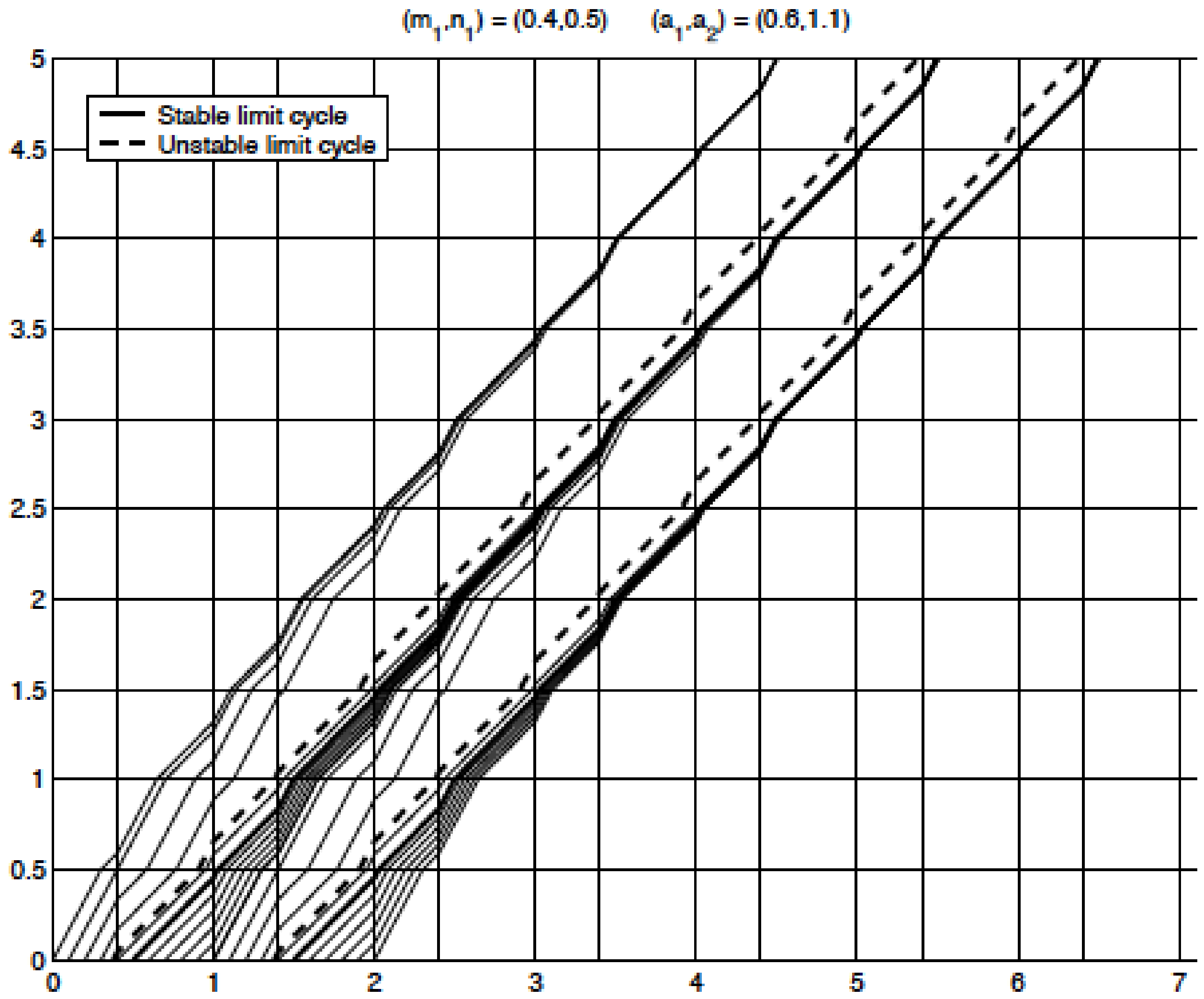
How to avoid this complicated cascade?

Lurie, Onofrei, and Weekes (2009) suggested having a zero impedance mismatch:





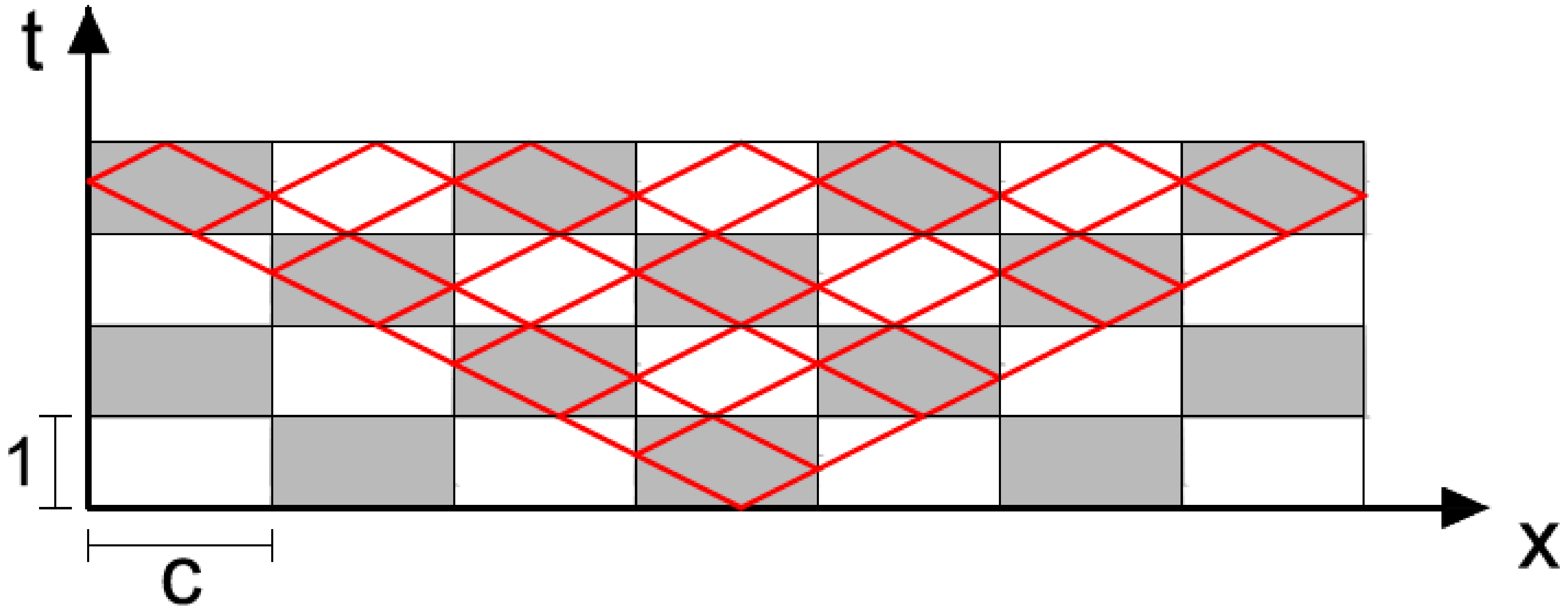
Curiously they found accumulations of the characteristic lines:



A bit like a shock but in a linear medium!

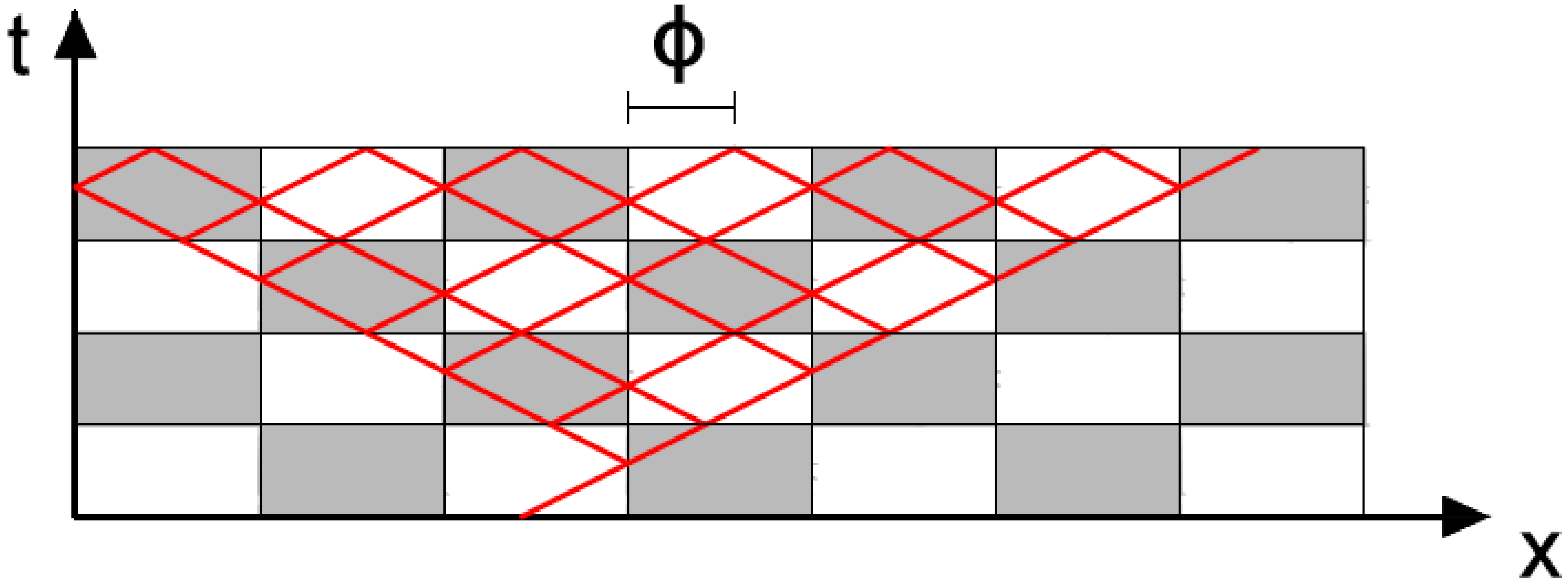
Field patterns in a space-time checkerboard

$$c_1 = c_2 = c \quad \Rightarrow \quad \frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2}$$



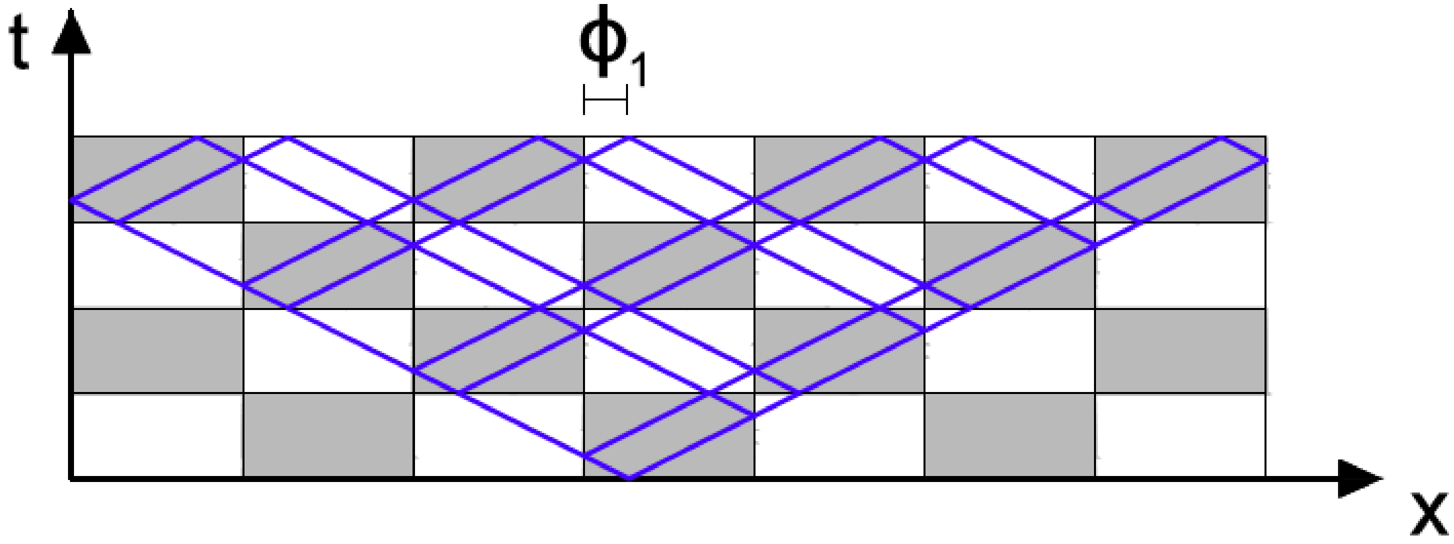
Families of field patterns

$$c_1 = c_2 = c \quad \Rightarrow \quad \frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2}$$



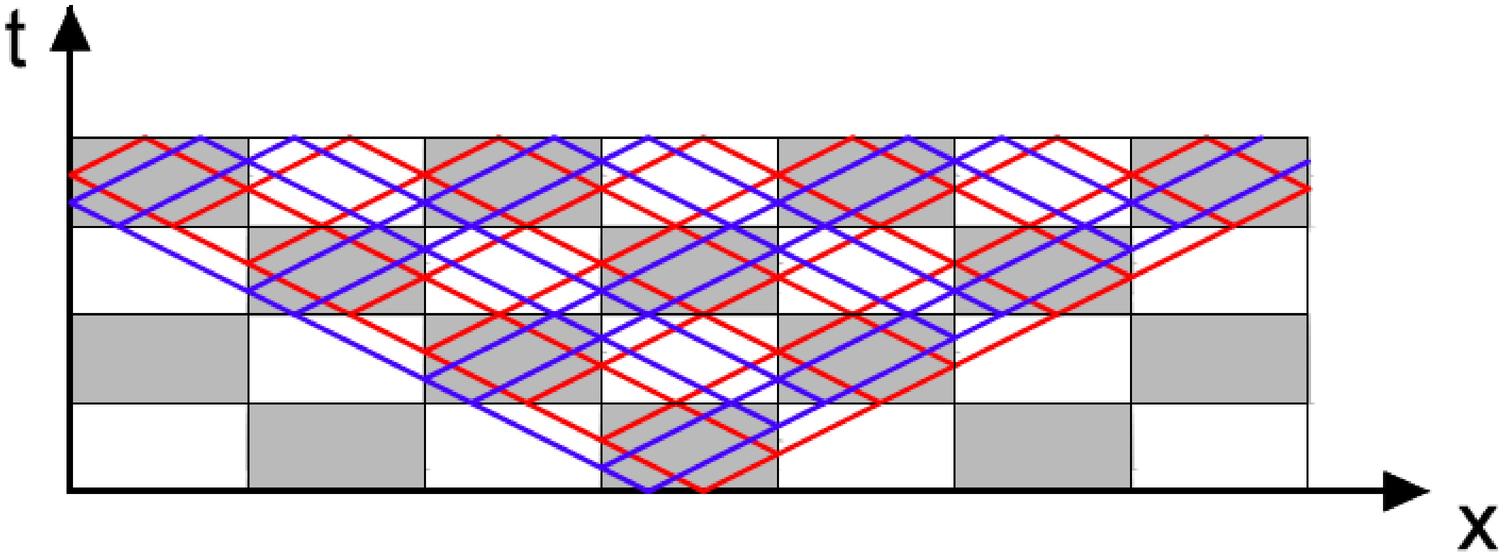
Families of field patterns

$$c_1 = c_2 = c \quad \Rightarrow \quad \frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2}$$



Field patterns are a new type of wave propagating along orderly patterns of characteristic lines which arise in specific space-time microstructures whose geometry in one spatial dimension plus time is somehow commensurate to the slope of the characteristic lines.

Multidimensional nature of field patterns

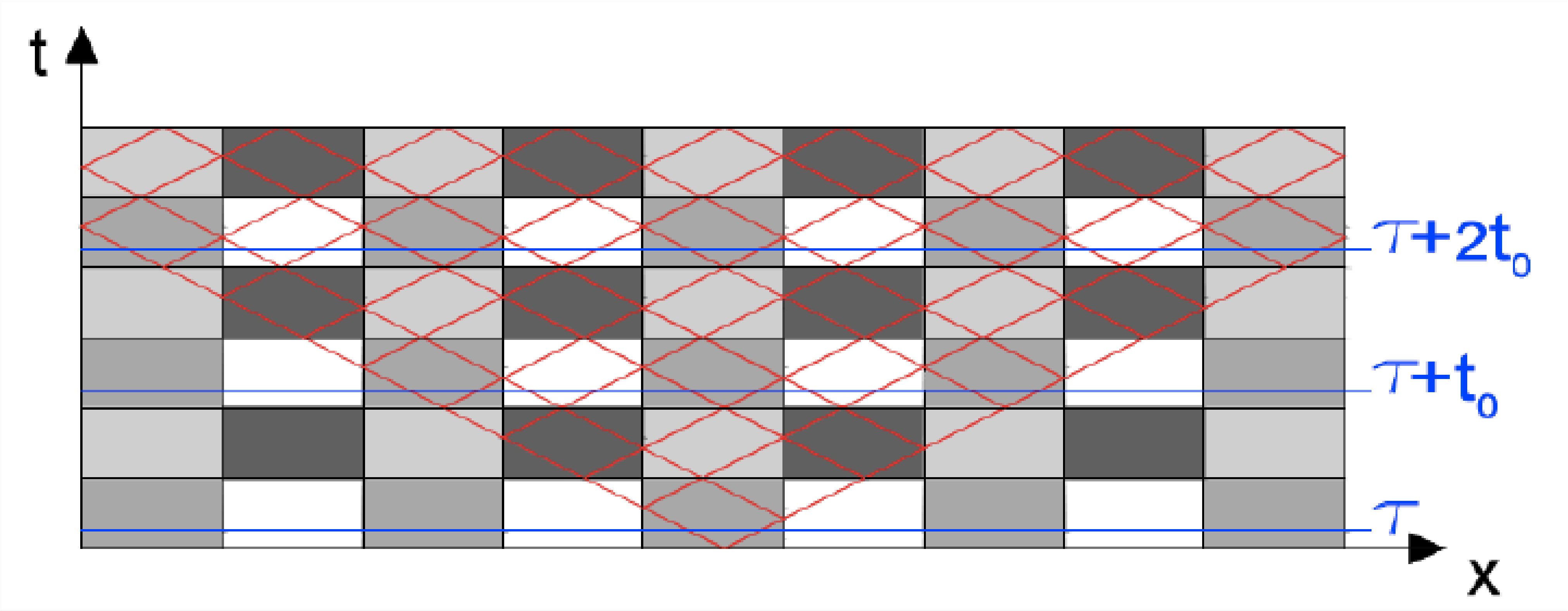


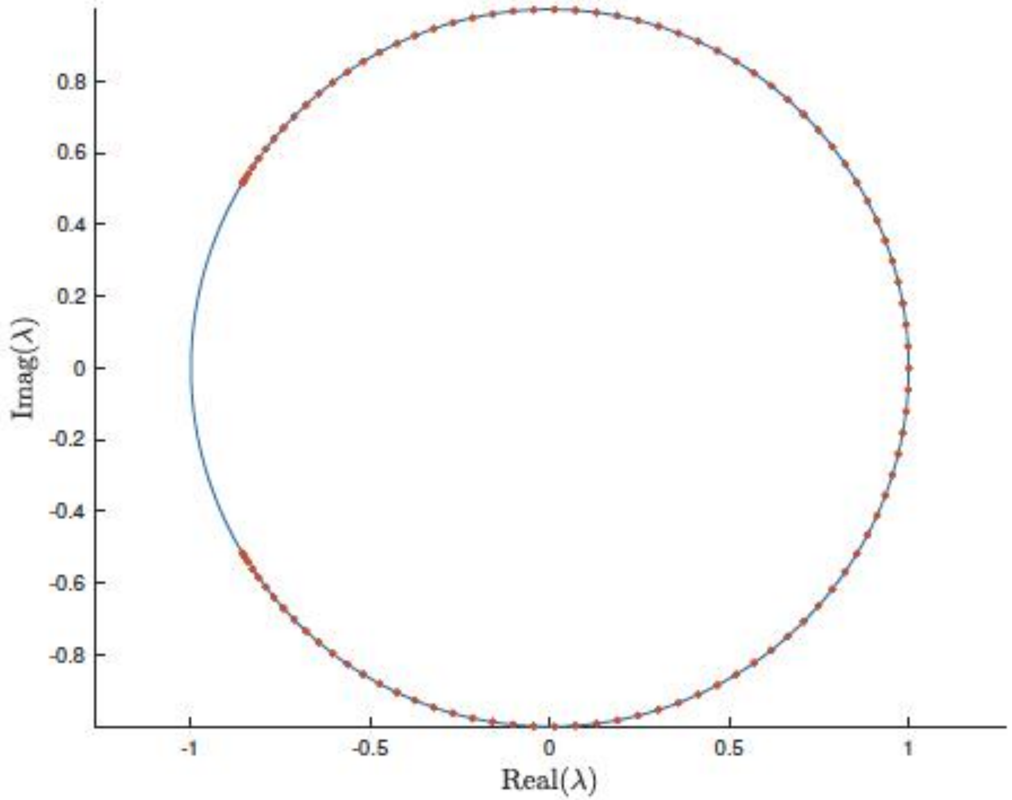
$$V(x,t) = \sum_{i=1}^{m} V_{\phi_i}(x,t)$$

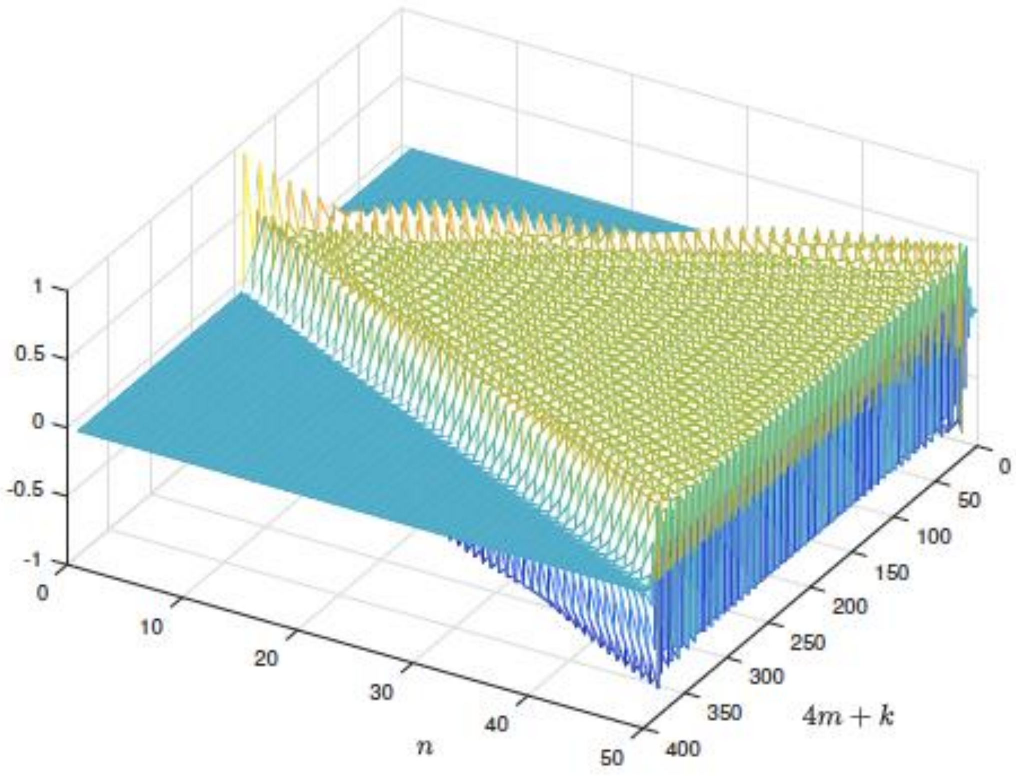
Multidimensional space: $V(x_1, x_2, \ldots, x_m) = \sum_{i=1}^{m} V_{\phi_i}(x_i, t)$

Multicomponent potential: $\mathbf{V}(x,t)$

Checkerboard geometries where there is no blow up

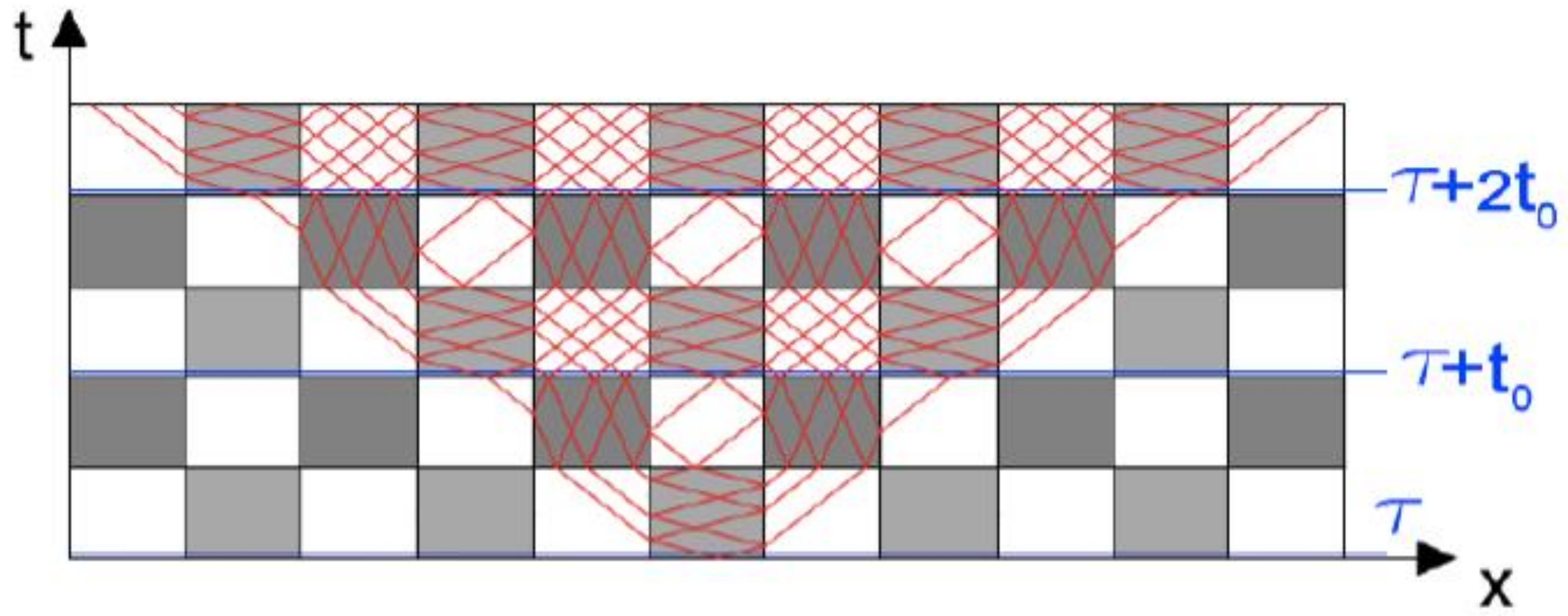




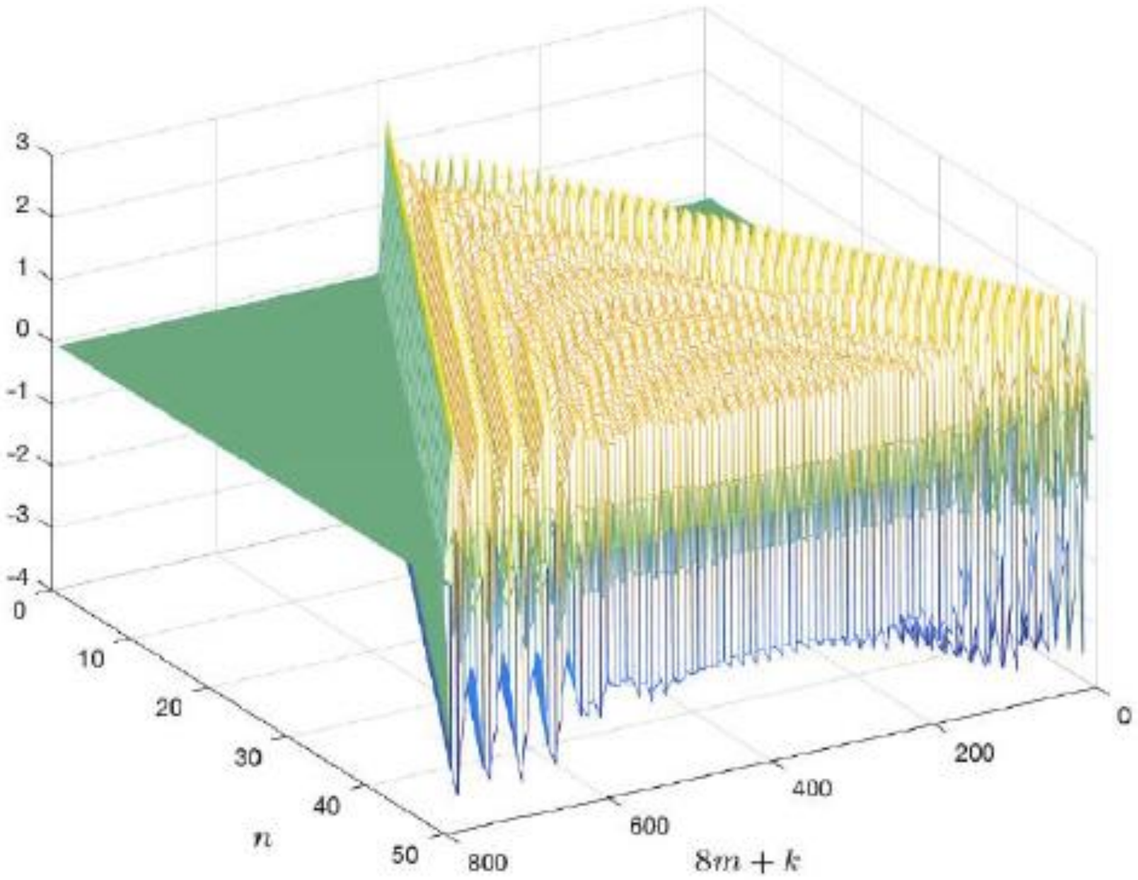


A New Wave

Checkerboard geometries where there is no blow up

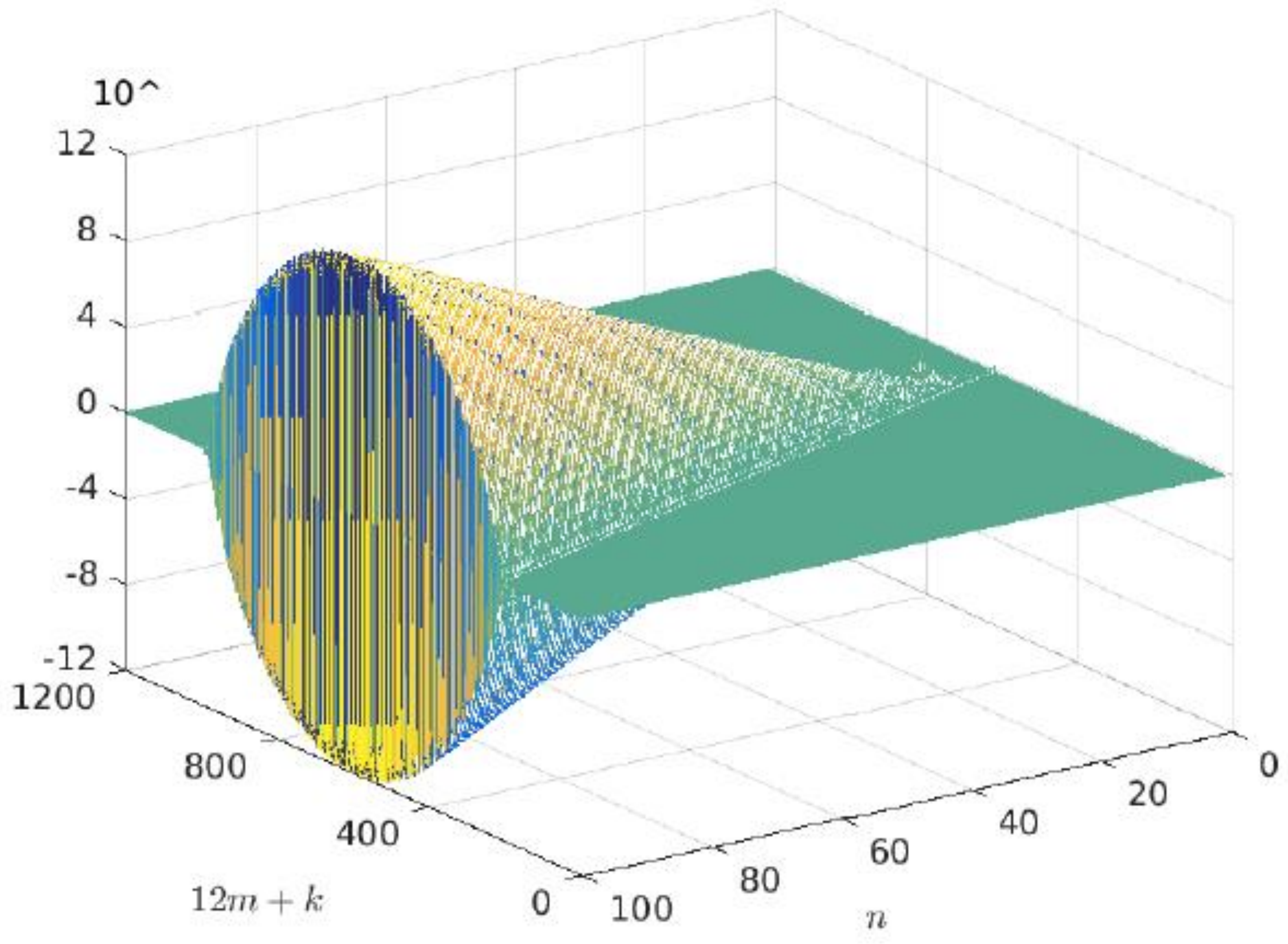


Checkerboard geometries where there is no blow up

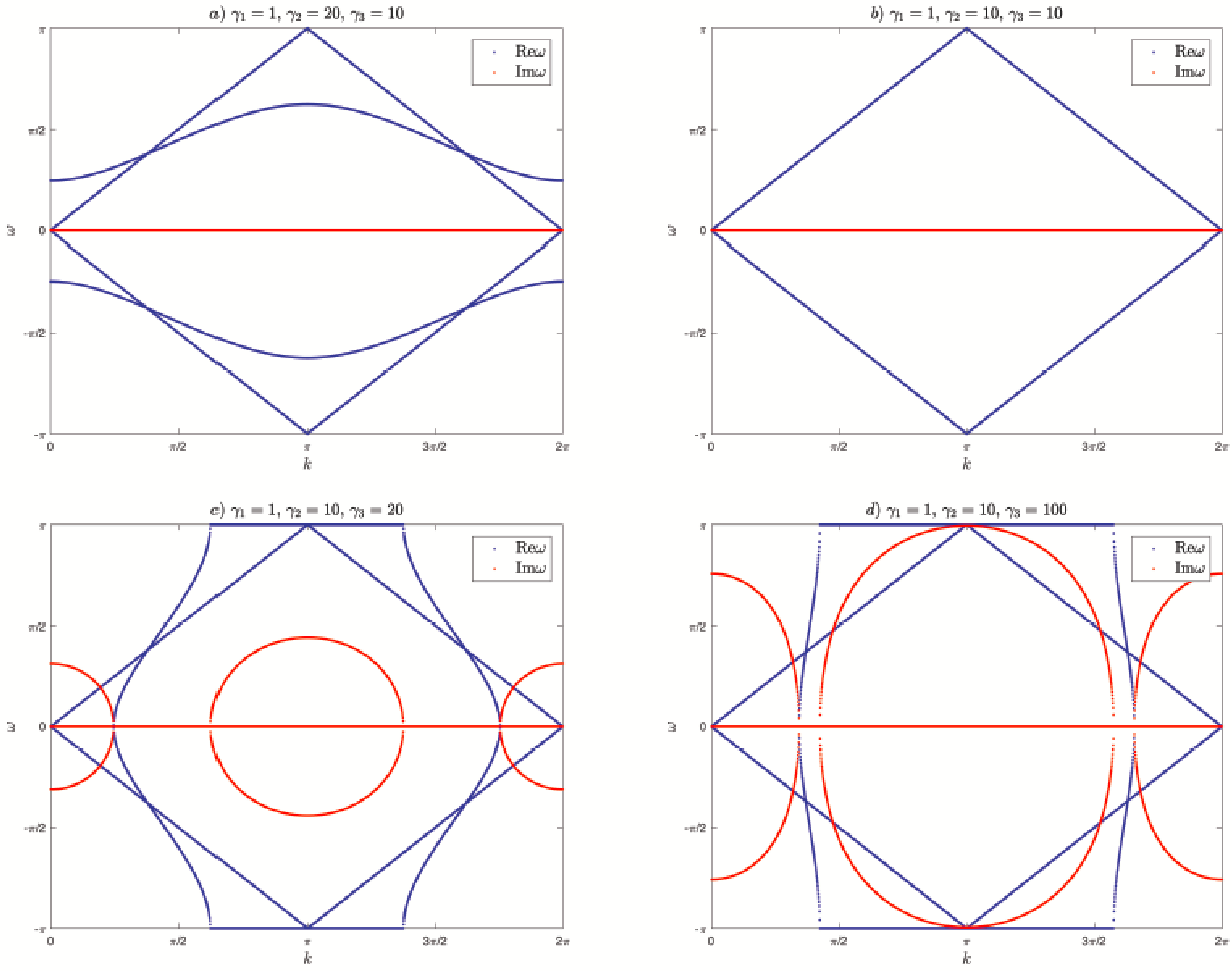


Breaking time's arrow, ala Boltzmann.

Blow up

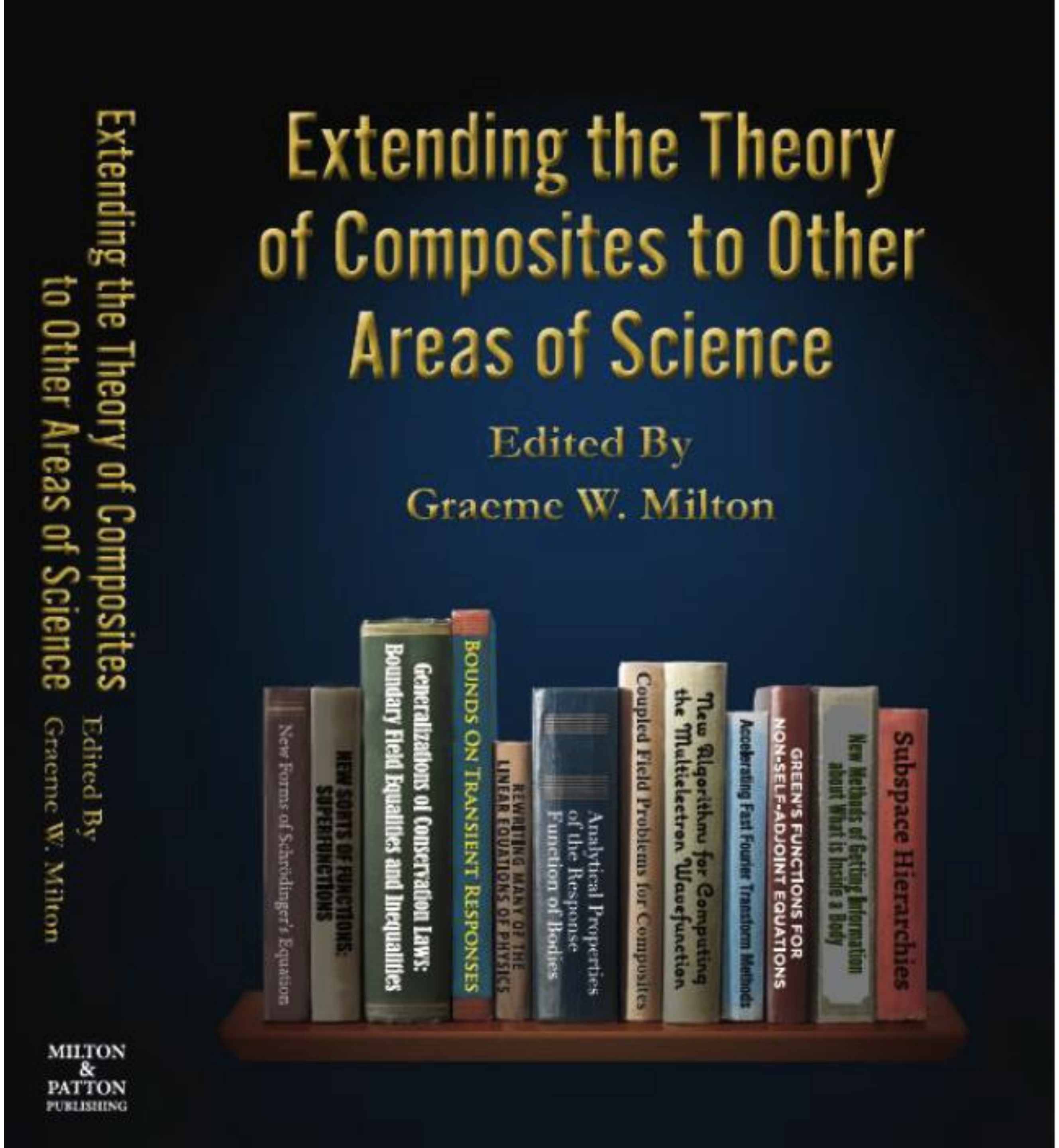


Dispersion diagrams for the three–phase checkerboard



Bloch Waves are:

Infinitely Degenerate!



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Aaron Welters

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