

PREDICTING FAILURES IN CYLINDRICAL PRESSURIZED VESSELS FROM TESTS CARRIED OUT ON FLAT PLATES

by

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ABSTRACT

Utilizing the theory of thin shell structures, a failure criterion is presented that one may use to predict, analytically, catastrophic failures, in cylindrical pressurized vessels, based on the fracture toughness profile K obtained from test data carried out on flat plates of the same material and thickness. These test data are plotted as a function of the characteristic ratio (h / c), where h represents the specimen thickness and c one half the crack length. Comparison with carefully controlled experimental data substantiates its validity and its potential use. The advantage of such an approach is that considerable amount of time and money can be saved for it is easier and safer to perform experiments on flat plates.

1. INTRODUCTION

In nature, shells are the rule rather than the exception. The list of natural shell-like structures is long, and the strength properties of some of them are remarkable. It is logical, therefore, for man to utilize them in man-made structures. But to do this safely, we must understand the fundamental laws which govern the strength and displacement behavior of such structures for they are not immune to failures, particularly in the fracture mode.

The engineering community has long recognized that large, thin-walled, pressurized cylindrical vessels resemble balloons and like balloons are subject to puncture and explosive loss. For a given material, under a specified stress field due to an internal pressure q_0 , there will be a crack length in the material which will be self-propagating. Crack lengths less than the critical value will cause leakage but not destruction. However, if the critical crack length is ever reached, either by penetration or by the growth of a small fatigue crack, explosion and complete loss of the structure may occur. Therefore, to ensure the integrity of the structure, the designer must be cognizant of the relationship that exists between fracture load, flaw shape and size, material

properties, and global cylindrical geometry. A relationship of this kind is referred to as a fracture criterion and can be derived by the application of the theory of fracture mechanics.

2. GENERAL THEORY

Let us consider a portion of a thin, shallow cylindrical vessel, of constant thickness h , that is subjected to a uniform internal pressure q_0 and contains a through-the-thickness crack of length $2c$. In this paper, we shall limit our considerations to elastic, isotropic and homogeneous segments of cylindrical vessels that are subjected to small deformations.

The basic variables in the theory of cylindrical pressurized vessels are, the displacement function $W(x, y)$ in the direction of the z -axis, and the stress function $F(x, y)$ which represents the stress resultants tangent to the middle surface of the shell. Following Marguerre (1938), the couple differential equations governing W and F , with x and y the rectangular Cartesian coordinates of the base plane (see Fig. 1) are given by :

$$\nabla^2 F + \frac{Eh}{R} \frac{\partial^2 w}{\partial x^2} = 0 \quad (1)$$

$$\nabla^2 W - \frac{1}{RD} \frac{\partial^2 F}{\partial x^2} = \frac{q_0}{D} \quad (2)$$

where ∇^4 is the biharmonic operator, E Young's modulus, h the thickness of the vessel, D the flexural rigidity, and q_0 the internal pressure.

For a cylindrical vessel, three observations are worthy of note. First, the coupled nature of the differential equations clearly suggests that there exists an interaction between bending and stretching. That is, a bending load will generally induce both bending and extensional stresses, and similarly a stretching load will also induce both bending and extensional stresses. Second, the radius of curvature varies all the way from zero to a constant as one sweeps from the axial to the peripheral direction. Third, if one lets the cylinder radius R to tend to infinity, the special case of a flat plate is recovered. It is this last characteristic property on which the present paper is based on.

Analytical investigations of the above equations in the presence of discontinuities such as cracks have been carried out by Folias (1964, 1974), Copley and Sanders (1969), Erdogan and Kibler (1969). Moreover, Folias (1964, 1974) was able to show that there exists a correlation function between a cylindrical pressure vessel and the corresponding case of a flat plate, i.e.

$$\sigma_{cylinder} = \{1 + f\}^{-\frac{1}{2}} \sigma_{plate}, \quad (3)$$

where the function f represents a geometrical correction factor

$$f = 0.317\lambda^2 \quad (4)$$

with

$$\lambda^2 = \{12(1 - \nu^2)\}^{\frac{1}{2}} \frac{c^2}{Rh}. \quad (5)$$

Although this correlation function was derived on the basis of a homogeneous and isotropic material, the author believes that it reflects the dominant term of a geometrical inherent property that exists between shallow cylindrical shell structures and similarly loaded flat plates of the same material. Such property is of great practical value for it may now be used to predict failing stresses in pressurized cylinders by simply performing experiments on flat plates. This represents a great advantage because (i) it is relatively easy to carry out experiments on flat plates and (ii) it is less costly.

3. THE FRACTURE TOUGHNESS CURVE

Recent experiments carried out by Royce Foreman and his group (NASA) on cracked plates, made of aluminum 2219-T87, provide us with the fracture toughness profile curve of this material. This material is to be used in the construction of the US space station Manned Module. Foreman's experimental data may now be used, in conjunction with the theory developed by the author (1974), to predict catastrophic failures in pressurized vessels of the same material. In order to accomplish this, however, one must examine the above experimental data in a unique way.

It has long been recognized by the fracture mechanics community that the fracture toughness curve is not really a material constant, and that its true value depends strongly on the specimen thickness. Its expected behavior may be seen schematically in Fig. 2. The reader may note that beyond a certain thickness h_s , a state of plane strain prevails whereby the toughness reaches the value of K_{Ic} . Alternatively, there exists an optimum thickness h_o , where the toughness reaches its highest value. This value is referred to as plane stress fracture toughness and its failure is characterized as *shear* or *slant* fracture. Moreover, for a thickness smaller than h_o there has, in the past, been some uncertainty about the toughness. In some cases, a horizontal level was found (Allen 1971, Feddersen 1970), while in other cases a decreasing toughness was observed (Broek 1966, Christensen 1961, Weiss 1965). It may also be noted that for a thickness $h < h_o$, the plastic zone is approximately equal to the specimen thickness and yielding in the thickness direction is unconstrained. As a result, a state of plane stress can fully develop in such regions. Presently, it is widely accepted that for $h < h_o$ the fracture toughness decreases almost linearly to the value of zero.

Additionally, recent 3D analytical studies on flat plates carried out by the author and his co-workers reveal that the stress concentration factor (1989, 1990), as well as the stress intensity factor (1975), are indeed functions of the radius to half thickness ratio (a/h), and half crack size to half thickness ratio (c/h), respectively. Thus, motivated by these findings, we plot Foreman's experimental data of the fracture toughness K , as a function of the parameter (h/c) (see Fig. 3). The profile of the curve appears to follow the theoretically expected trend. The method which is described in this paper works best for experiments carried out on flat plates of the same material thickness as that of the vessel. Unfortunately, the tests carried out by Foreman were on plates of different thickness. Be that as it may, the reader may notice that tests A show a slightly higher rise than those of tests 2K. This occurs in the region that is characterized with a 45° plane fracture or more commonly referred to as '*shear fracture*', and it occurs to the left of the ratio (h/c) = 0.1

Moreover, the thickness for the tests A was $h = 0.191$ in., while for tests K it was $h = 0.083$ in. even though the ratio h / c remained the same. This observation brings the following two questions to mind. Why is there a need for the presence of two different paths? Moreover, at what value of the ratio (h / c) does this bifurcation take place ?

A partial answer to the above two questions may be obtained from the results of the work on the 3D stress field of a plate weakened by the presence of a circular hole (Folias et. al., 1990). Recently, utilizing a more sophisticated numerical analysis, these results have further been sharpened (Folias 1997) and the maximum, 3D, stress concentration factor versus the radius to half thickness ratio is shown in Fig. 4. The results of this figure provide us with a definite answer, at least for the case of a circular hole, of the regions in which a plate is considered to be in a state of plane stress, i.e. $a / h > 10$, or in a state of plane strain, i.e. for $a / h < 0.10$. Moreover, the transition region occurs between the values of $0.10 < a / h < 10$, whereby the 3D effects become more pronounced. Thus, if one assumes that similar trends also prevails in the case of a, 3D, cracked plate, one may conjecture that a state of plane stress exists for ratios of $h / c < 0.1$. This result appears to be in line with the experimental curves of Fig. 3. Thus, it is not unusual to have a fan, or a series of paths, for different thickness that fan out to the left of the critical ratio $h / c = 0.1$. Such conjecture, may also explain the experimental uncertainty noted by researchers in the past (see discussion of Fig. 2).

4. FAILURE PREDICTION IN PRESSURIZED VESSELS

The fracture toughness curve, as given by Fig. 4, may now be used to predict *the static* failing pressures in cylindrical vessels made of Al 2219-T87. For static considerations, the results by Folias (1974) should be applicable. More specifically, under the assumption that bending and bulging effects are negligible, Folias's general failure criterion (1974) may then be approximated by the simple relationship

$$\frac{q_0}{1000} \frac{R}{h} \sqrt{\pi C} \sqrt{1 + 0.317\lambda^2} = K \quad , \quad (6)$$

where q_0 is expressed in psi. Alternatively, if the effects of bending and bulging are sufficiently large, then the approximate eq.(6) can no longer be used and the general failure criterion must be utilized.

Recently, two very carefully controlled experiments on pressurized vessels were carried out by Nemat Nasser and his group at UCSD. The work was done in order to assess the structural characteristics of the United States Manned Module of the International Space Station. The tests were carried out on cylindrical pressurized vessels, of 20.75 in. in diameter and for two different values of thickness, $h = 0.05$ in. and $h = 0.086$ in. respectively. The cylinders were made out of Al 2219-T87.

A crack length of $2c = 5.5$ in. was very cleverly introduced by the rapid impact of a knife edge. The crack resembled a very sharp ellipse with crack tips sharp enough to represent a crack. Subsequently, the vessels were sealed internally with a thick hard rubber sheet, in the area of the crack, and were then pressurized incrementally with pressure differentials of 5 psi. This

incremental 'pressure stepping' clearly suggests that the bending and bulging effects may no longer be neglected since they represent a cumulative effect.

Let us examine next, how well may one now be able to predict the failing pressures for the UCSD static tests # 1 & # 2. A closer examination of the geometrical and loading characteristics of test # 1 reveal that, because λ is greater than 5.5 and because the bending effects present are not negligible, one must use the general failure criterion by including the pertinent bending terms which in this case are significant. In view of this, eq. (6) now becomes

$$\frac{q_o}{1000} \frac{R}{h} \sqrt{\pi C} F(\lambda) = K \quad , \quad (7)$$

where

$$F(\lambda) = \left\{ f_e(\lambda) + f_b(\lambda) \frac{\sigma_{appl.bend}}{\sigma_{appl.hoop}} \right\} \quad (8)$$

and $f_e(\lambda), f_b(\lambda)$ are given numerically in the above reference. Without going into the numerical details, the predicted results for tests # 1 and # 2 become respectively :

(i) static test # 1 :

For this test, $h = 0.05$ in. and $h / c = 0.02$, and from Fig. 3 one may read a value of K that is closest to that thickness, thus

$$K = K_{lower} = 62 \text{ ksi. in.}, \quad \text{which gives } q_o = 31.2 \text{ psi.} \quad (9)$$

The failing pressure reported was 34.4 psi.

It should, furthermore, be noted that for test # 1, the hard rubber sheet used to seal the crack was well bonded to the interior surface of the vessel. Consequently, the rubber did carry some load and this was substantiated after the failure since it fractured along the same direction of the crack and at a 45° plane through its thickness. In carrying out test # 2, however, the rubber sheet was not bonded to the vessel. We have estimated the load carrying capacity of the rubber sheet for test # 1 to be approximately 1.6 psi. Thus, accounting for the rubber, we predict a failing pressure for test # 1 to be, approximately, 32.8 psi.

(ii) static test # 2 :

For this test, $h = 0.086$ in. and $h / c = 0.03$, and from Fig. 3, one may read a value of K that is closest to that thickness

$$K = K_{lower} = 65 \text{ ksi. in.}, \quad q_o = 59.6 \text{ psi.} \quad (10)$$

The failing pressure reported was 59.9 psi.

Perhaps it is appropriate here to emphasize that in both tests, the upper and lower faces of the crack were not touching.

5. CONCLUSIONS

Although the correlation function (eq. 3) was derived on the basis of a homogeneous and isotropic material, the author believes that it reflects the dominant term of a geometrical inherent property that exists between shell structures and similarly loaded flat plates of the same material. Needless to say that such a property is of great practical significance to the designer for, it is now possible to successfully predict catastrophic failures in cylindrical structures from experimental data accumulated in the laboratory on flat plates of the same thickness. This is an advantage that has not only important economic implications but also the experiments that need to be carried out are simpler.

Comparison between the theoretically predicted values and the UCSD experimental data shows a very good agreement. Moreover, to substantiate the above hypothesis, we have also compared to the case of a composite material system. Unlike the fracture of metals, which is well characterized by the principle of fracture mechanics, the fracture of composite structures involves a complex interaction of fiber breaks, matrix cracks and inter-ply delaminations. Without going into the details, in Fig. 5 we compare the results of eq. 3 with experimental data available in the literature (Graves and Lagace, 1985) on pressurized graphite / epoxy cylinders. The cylinders were slit in the longitudinal direction and were pressurized. Here again the agreement is fairly good.

6. REFERENCES

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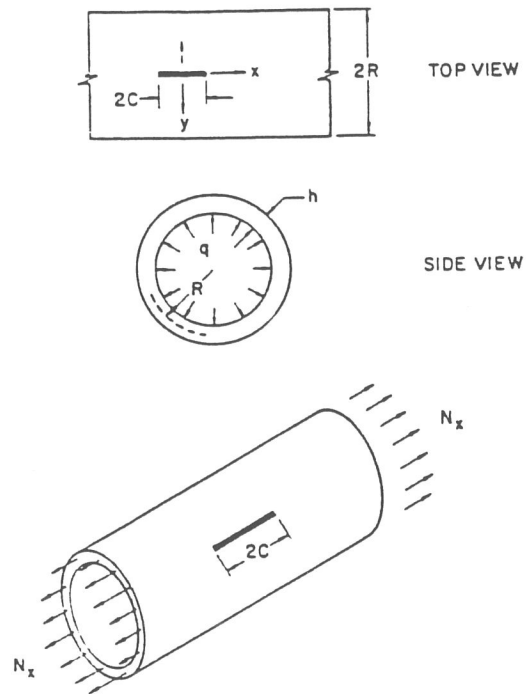


Fig. 1. Geometry and coordinates of an axially cracked cylindrical vessel.

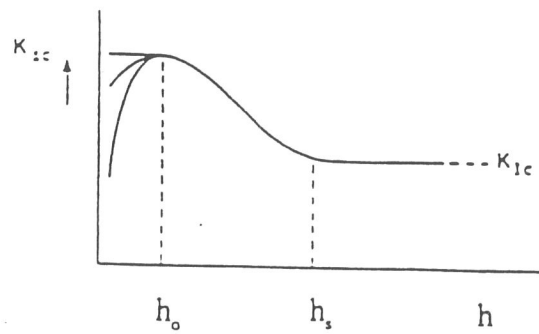


Fig. 2. Fracture toughness versus specimen thickness.

FRACTURE TOUGHNESS CURVE FOR Al 2219-T87 PLATES

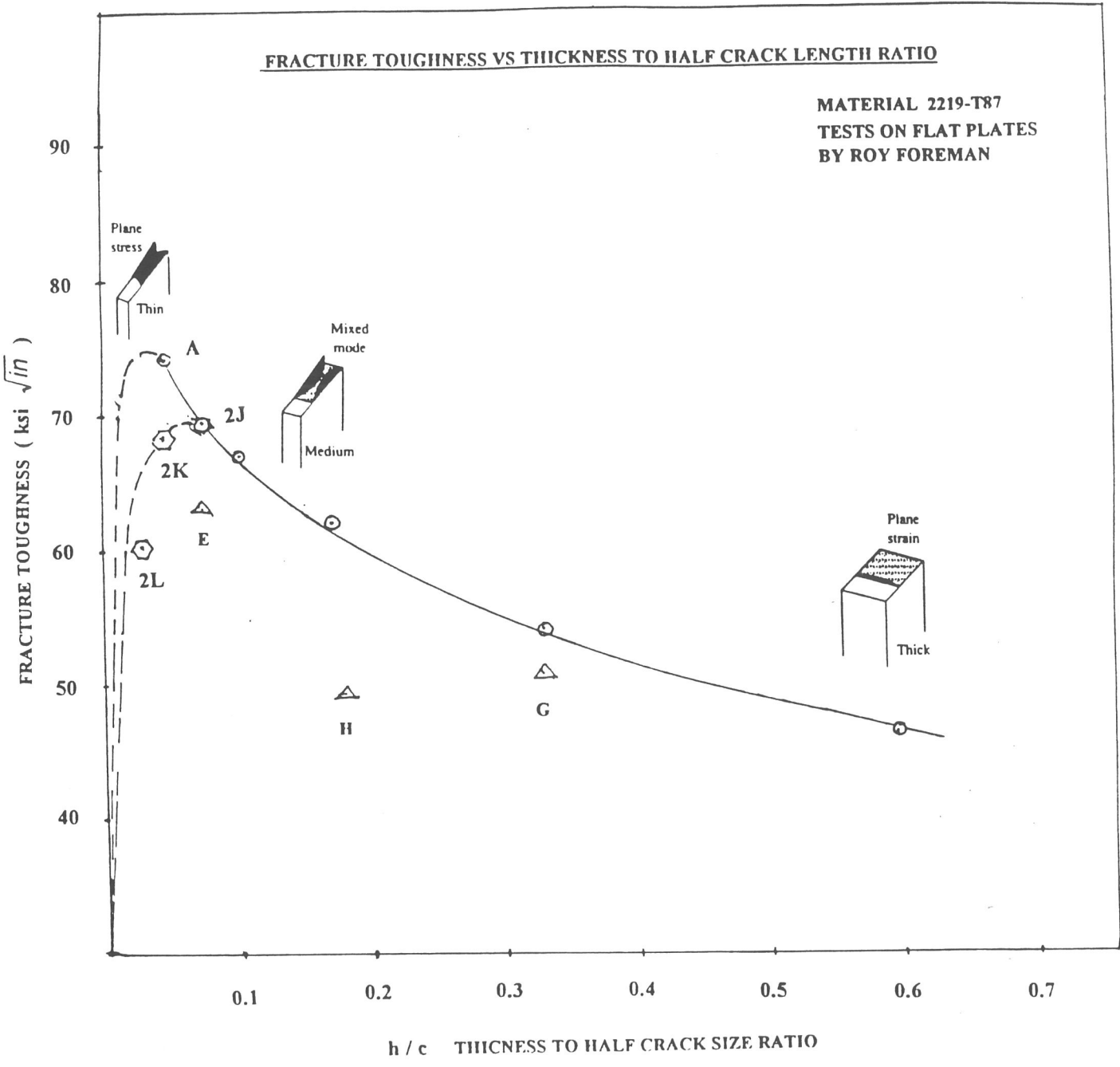


Fig. 3. Fracture toughness versus thickness to half crack length ratio for 2219-T87 aluminum alloy plates.

REVIEW OF SOME 3D ANALYTICAL RESULTS

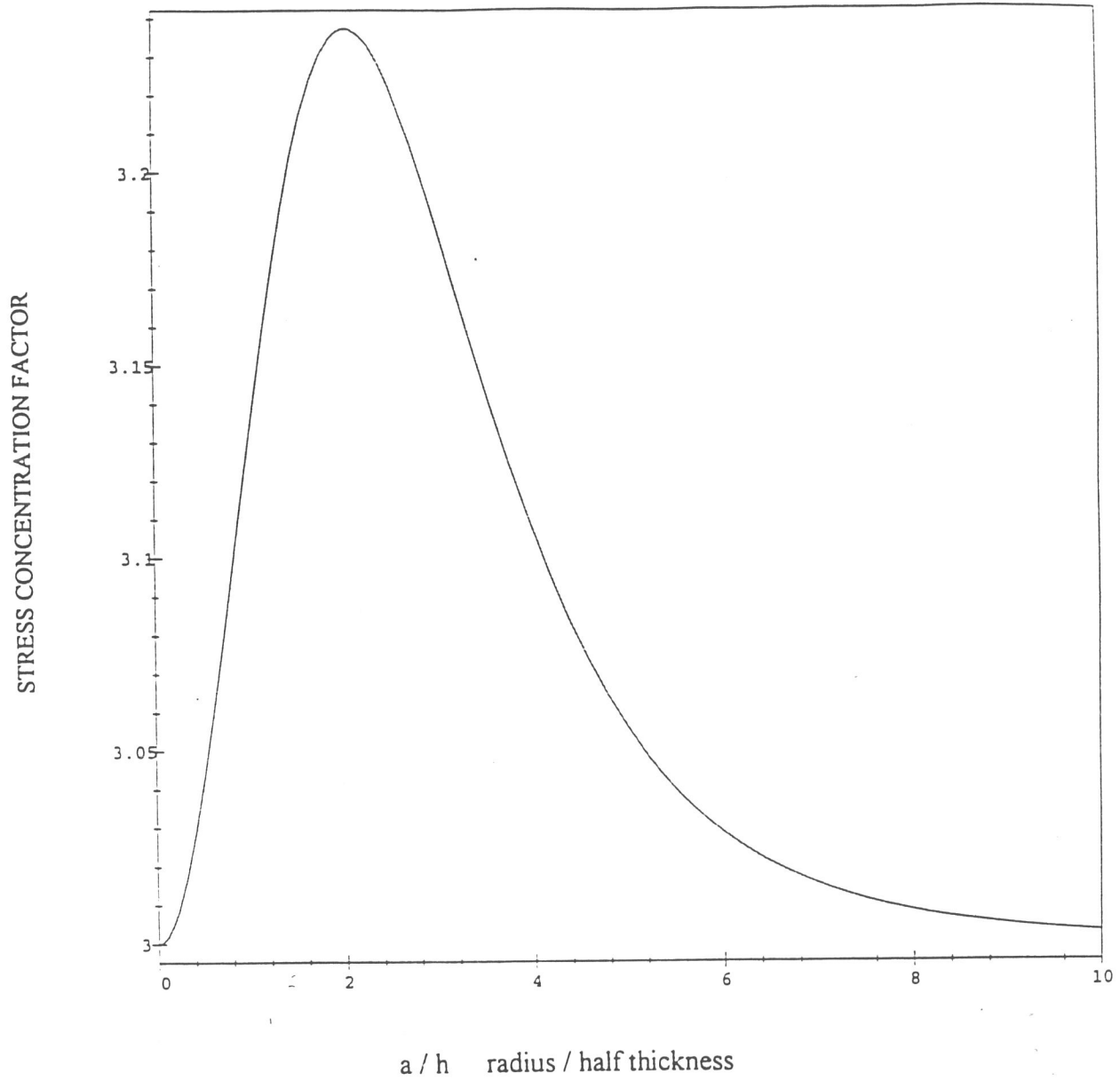


Fig. 4. Maximum stress concentration factor versus diameter to thickness ratio.

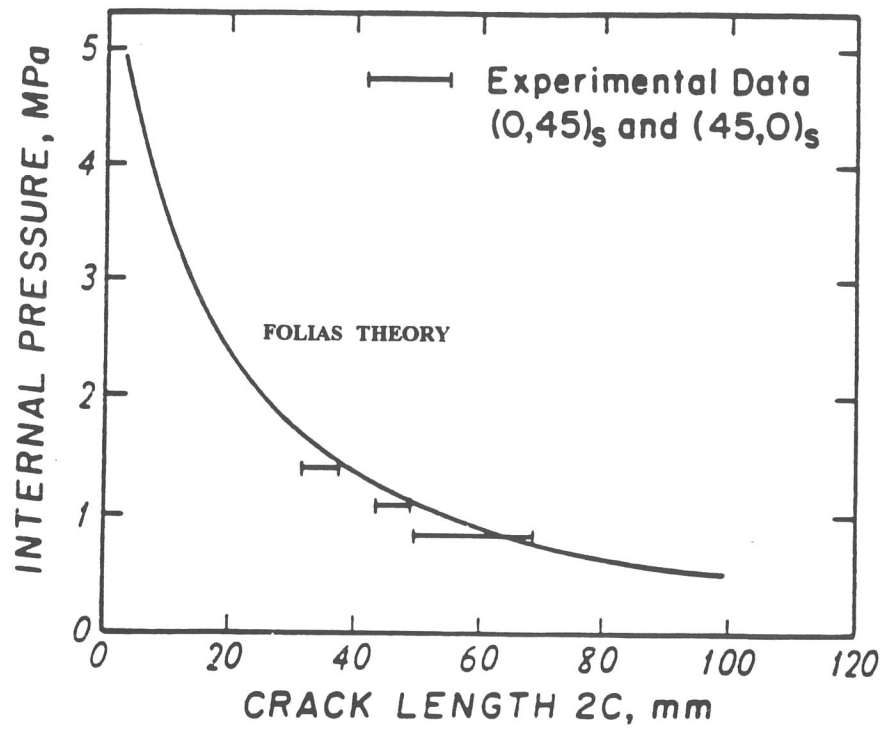


Fig. 5 Comparison between theory and experiment for graphite / epoxy cylinders with axial slits.