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nels are given by

$$_0I_1 - K_1I_0) + \xi^2R^2(K_0I_1 - K_1I_0)$$

$$- z)]d\xi \qquad \text{(A1)}$$

$$- K_0I_0) + 2(1-\nu)K_1I_1]$$

(A2)

$$s - z)]d\xi \qquad \text{(A3)}$$

and $K_1 = K_1(\xi R)$. These integrals are
mowitz and Stegun 1972).

The 3D Stress Field at the Intersection of a Partial Through Crack and a Free Surface

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ABSTRACT

The 3D stress field ahead of a partially through the thickness crack is investigated analytically. Particular emphasis is placed in determining the behavior of the stress field in the neighborhood of the intersection of the crack front with the free surface of the plate. The stress field is shown to possess the usual $\rho^{-1/2}$ singular behavior over the entire length of the crack. Moreover, all stresses are singular. The result is identical to that obtained by Williams which was based on 2D elastic considerations.

KEYWORDS

Stress intensity factor; stress singularities; partial through crack; 3D stress field.

INTRODUCTION

The question of the order of the prevailing singularity at the neighborhood of the intersection of a crack front with the free surface of a plate has so far defied analysis. However, utilizing a general, 3D, analytical solution for the equilibrium of linear elastic plates (Folias 1975, Folias et. al. 1987a), the author recently was able to investigate analytically the stress field in the neighborhood of the intersection of the surface of a hole with that of the free surface of a plate (Folias, 1987b). The analysis revealed that the stress field there is non-singular. Moreover, the result was shown to be identical to that obtained by Williams (1952) on re-entrant corners which was based on 2D considerations.

The above analysis has also been extended to the case of a plate containing a partial through the thickness crack (Folias, 1988). Special attention was given particularly to the neighborhood where the crack front meets the free surface of the plate. In this paper, the author summarizes some of the important findings which are of practical interest.

THE MATHEMATICAL MODEL

Consider the equilibrium of a homogeneous, isotropic, elastic plate that occupies the space $|x| < \infty$, $|y| \infty$, $o < z < h$ and contains a partial through the thickness circular crack. The crack is assumed to be of radius a and its plane is perpendicular to the plate surface z = o (see Fig. 1). The plate is subjected to a uniform tensile load σ_o along the y-axis and parallel to the bounding planes.

In the absence of body forces, the coupled differential equations governing the displacement functions u, v and w are

$$\frac{m}{m-2}\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) e + \nabla^2 (u, v, w) = o \qquad (1)\text{-}(3)$$

where ∇^2 represents the Laplacian operator, $m \equiv 1/\nu$, ν is Poisson's ratio,

$$e = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \qquad (4)$$

and the stress- displacement relations are given by Hooke's law as:

$$\sigma_{xx} = 2G \left\{ \frac{\partial u}{\partial x} + \frac{e}{m-2} \right\}, \ldots, \tau_{xy} = G \left\{ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right\}, \ldots \qquad (5)\text{-}(10)$$

with G being the shear modulus.

THE STRESS FIELD AHEAD OF THE CRACK TIP

Guided by a general, 3D, solution for the equilibrium of linear elastic plates (Folias et. al., 1987a), the author was able to construct an asymptotic solution which is valid at the base of a partial through the thickness crack. Suppressing the long and tedious mathematical details, the complementary displacement and complementary stress fields are found in terms of the local to the crack coordinate system to be (Fig. 1):

(i) <u>the displacement field:</u>

$$u = \rho^{1/2} \cos\phi \, C(\phi) \left\{ (1-2\nu) \cos\left(\frac{\theta}{2}\right) + \frac{1}{2} \sin\theta \sin\left(\frac{\theta}{2}\right) \right\} + o(\rho^{3/2}) \qquad (11)$$

$$v = \rho^{1/2} C(\phi) \left\{ 2(1-\nu) \sin\left(\frac{\theta}{2}\right) - \frac{1}{2} \sin\theta \cos\left(\frac{\theta}{2}\right) \right\} + o(\rho^{3/2}) \qquad (12)$$

$$w = \rho^{1/2} \sin\phi \, C(\phi) \left\{ (1-2\nu) \cos\left(\frac{\theta}{2}\right) + \frac{1}{2} \sin\theta \sin\left(\frac{\theta}{2}\right) \right\} + o(\rho^{3/2}) \qquad (13)$$

(ii) <u>the stress field:</u>

$$\tau_{xy} = \rho^{-1/2} G \cos\phi \, C(\phi) \left\{ \frac{1}{4} \sin\left(\frac{5}{2}\theta\right) - \frac{1}{4} \sin\left(\frac{\theta}{2}\right) \right\} + o(\rho^{\circ}) \qquad (14)$$

$$\tau_{yz} = \rho^{-1/2} G \sin\phi \, C(\phi) \left\{ \frac{1}{4} \sin\left(\frac{5\theta}{2}\right) - \frac{1}{4} \sin\left(\frac{\theta}{2}\right) \right\} + o(\rho^{\circ}) \qquad (15)$$

$$\tau_{xz} = \rho^{-1/2} G \sin(2\phi) \, C(\phi) \left\{ \frac{1}{8} \cos\left(\frac{5\theta}{2}\right) + \left(\frac{3}{8} - \nu\right) \cos\left(\frac{\theta}{2}\right) \right\} + o(\rho^{\circ}) \qquad (16)$$

$$\sigma_{xx} = \rho^{-1/2} 2G \cos^2\phi \, C(\phi) \left\{ \frac{1}{8} \cos\left(\frac{5\theta}{2}\right) + \left(\frac{1}{8} - \nu\right) \cos\left(\frac{\theta}{2}\right) \right\}$$
$$+ \rho^{-1/2} \nu \, 2G \, C(\phi) \cos\left(\frac{\theta}{2}\right) + o(\rho^{\circ}) \qquad (17)$$

$$\sigma_{yy} = \rho^{-1/2} 2G \, C(\phi) \{ \ldots$$
$$+ \rho^{-1/2} \nu \, 2G \, C(\phi) \ldots$$
$$\sigma_{zz} = \rho^{-1/2} 2G \sin^2\phi \ldots$$
$$+ \rho^{-1/2} \nu \, 2G \, C(\phi) \ldots$$

where

$$r - a = \rho \cos\theta$$

$$y = \rho \sin\theta$$

and

$$C(\phi) = \sum_{n=1}^{\infty} d_n (\sin\phi)^{\beta+n}$$

where the d_n's are constants and β a c
constants are to be determined from the bc

CONCLUSIONS

The foregoing results clearly show that th
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Williams (1952) based on 2D consideratic

Finally, the author would like to emphasi
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making its explicit construction possible.

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HE CRACK TIP

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$$\theta \cos(\frac{\theta}{2}) \} + o(\rho^{3/2}) \qquad (12)$$

$$+ \frac{1}{2} \sin\theta \sin(\frac{\theta}{2}) \} + o(\rho^{3/2}) \qquad (13)$$

$$) - \frac{1}{4} \sin(\frac{\theta}{2}) \} + o(\rho^{\circ}) \qquad (14)$$

$$- \frac{1}{4} \sin(\frac{\theta}{2}) \} + o(\rho^{\circ}) \qquad (15)$$

$$) + (\frac{3}{8} - \nu) \cos(\frac{\theta}{2}) \} + o(\rho^{\circ}) \qquad (16)$$

$$\frac{\theta}{2}) + (\frac{1}{8} - \nu) \cos(\frac{\theta}{2}) \}$$

$$) + o(\rho^{\circ}) \qquad (17)$$

$$\sigma_{yy} = \rho^{-1/2}\, 2G\, C(\phi) \{ -\frac{1}{8} \cos(\frac{5\theta}{2}) + (\frac{7}{8} - \nu) \cos(\frac{\theta}{2}) \}$$

$$+ \rho^{-1/2}\, \nu\, 2G\, C(\phi) \cos(\frac{\theta}{2}) + o(\rho^{\circ}). \qquad (18)$$

$$\sigma_{zz} = \rho^{-1/2}\, 2G \sin^2\phi\, C(\phi) \{ \frac{1}{8} \cos(\frac{5\theta}{2}) + (\frac{5}{8} - \nu) \cos(\frac{\theta}{2}) \}$$

$$+ \rho^{-1/2}\, \nu\, 2G\, C(\phi) \cos(\frac{\theta}{2}) + o(\rho^{\circ}), \qquad (19)$$

where

$$r - a = \rho \cos\theta \qquad (20)$$

$$y = \rho \sin\theta \qquad (21)$$

and

$$C(\phi) = \sum_{n=1}^{\infty} d_n (\sin\phi)^{\beta+n}. \qquad (22)$$

where the d_n's are constants and β a constant which is greater than minus one. These constants are to be determined from the boundary conditions away from the crack.

CONCLUSIONS

The foregoing results clearly show that the stress field in the neighborhood of the corner point, i.e. the point where the crack front meets the free of stress boundary plane, has the usual $\rho^{-1/2}$ singular behavior. It is rather interesting to note that the result is identical to that obtained by Williams (1952) based on 2D considerations.

Finally, the author would like to emphasize the importance of the general 3D solution, (Folias et. al., (1987a), for it reveals the inherent form of the solution at such corner points thus making its explicit construction possible.

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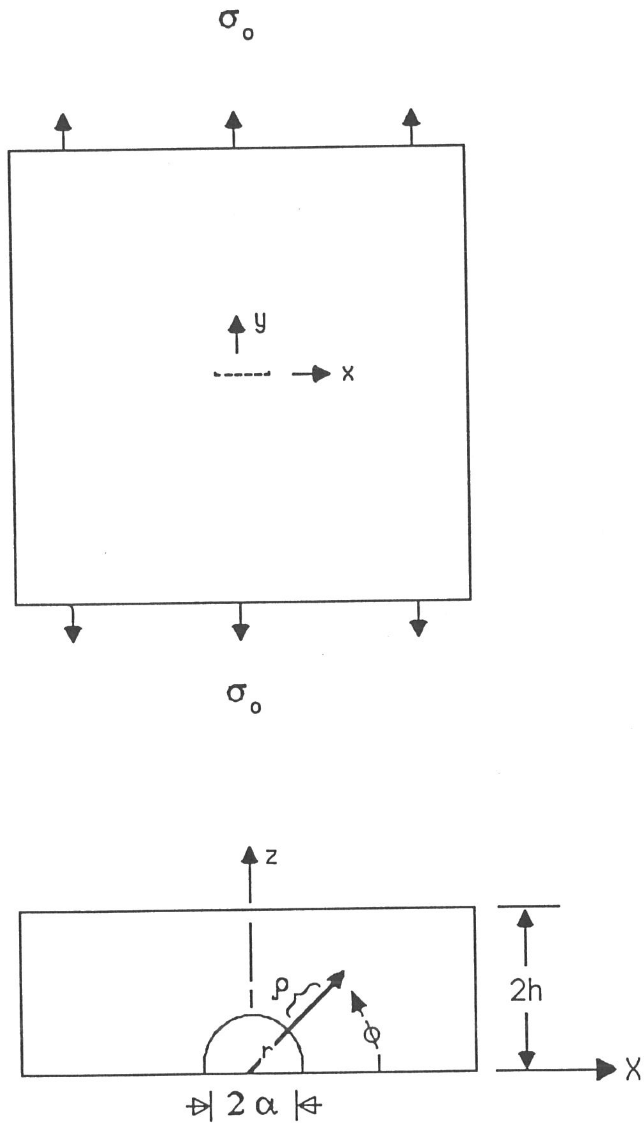

Fig. 1 Geometrical and loading configuration of a partial through-the-thickness crack.

Stress Intensity Fact
Ellipsoidal Cap in

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N.S.W. 2

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ABSTRACT

The paper calculates first the stress fie
cut over an ellipsoidal cap by solving a
discontinuity across the cut. This field
factors along the edge of the cut.

KEYWORDS

Ellipsoidal cut; displacement discontinu

INTRODUCTION

The problem under consideration arise
parameters of an ellipsoidal inclusion or
of its boundary from the surrounding
the medium which has undergone an in
inhomogeneity refers to a region of mat
those of the medium. In this paper we
in an infinite elastic body and will calc
a singular integral equation for the disp
also indicate how these results can be i
ellipsoidal inclusion. The latter problem
toughening of ceramics (Evans and Ca
to calculate the stress intensity factors
dimensional problems of partially debo
and plane strain conditions were recent
a,b,c).