International Journal of Fracture 26 (1984) 247–249.

Foreword

In closing this 20th year of publication of the International Journal of Fracture (the International Journal of Fracture Mechanics until 1973), the editors have chosen to devote this issue to reprints of papers published during the life of the journal. The selections have been difficult inasmuch as there have been well over 800 contributions, excluding consideration of the 600 Reports of Current Research. Of necessity, therefore, a certain element of subjectivity was injected in the final presentation of the Editors' selection. One quantitative element, however, was included based upon references in Science Citation Index. We have found that the two most frequently cited works, which were also among the Editors' recommendations, were to S.N. Zhurkov, *Kinetic Concept of the Strength of Solids* [1 (1965) 311] and A.G. Evans and S.M. Weiderhorn, *Proof Testing of Ceramic Materials – An Analytical Basis for Failure Prediction* [10 (1974) 379].

Many of the contributions generated a succession of related papers. One example is that of E.S. Folias, *An Axial Crack in a Pressurized Cylindrical Shell* [1 (1965) 104], which along with his own related work on cracked spherical shells [1 (1965) 20], circumferentially cracked cylinders [3 (1967) 1], initially curved flat shells [5 (1969) 327] and spherical vibrations [7 (1971) 23] stimulated work on axial cylinders by Copley and Sanders [5 (1969) 117] and circumferential cracking by Sanders and Duncan-Fama [8 (1972) 15]. Erdogan and Ratwani followed with contributions to cracked cylinders having fatigue and circumferential cracks [6 (1970) 379], torsion loading [8 (1972) 87], plasticity [8 (1972) 413] and two colinear cracks [10 (1974) 463]. Erdogan and Kibler also discussed cracked cylindrical and spherical shells [5 (1969) 229], while Ratwani and Yuceoglu combined with Erdogan to treat orthotropy [10 (1974) 369]. Another contributor to papers on the axial crack in cylinders during this time was Murthy, Rao and Rao [8 (1972) 287].

Another example of a stimulus for a branch of work was provided by the paper by Malyshev and Salganik [this issue and 1 (1965) 114] which presented a novel debonding test in the form of a circular metal plate bonded to a base plate which was then separated from it by a point load inserted through the base plate. This specimen was subsequently extended to employ an internal pressurization – a "blister test" having limited, environmentally controllable volume – and applied to various engineering situations such as the debonding of thin films as discussed in the analytical and numerical adhesive fracture work of S.J. Bennett et al. [10 (1974) 33], G.P. Anderson et al. [10 (1974) 565], and including the Updike [12 (1976) 815] numerical analysis for a finite adhesive interlayer. S.S. Wang et al. [14 (1978) 39] contributed a numerical analysis also involving an interlayer, but in a double cantilevered beam, wherein a special crack tip finite element was used. In the same year Wool [14 (1978) 597] presented a simple analysis and test for the peeling of thin tapes.

Problems associated with angled cracks were discussed by J.G. Williams and Ewing [8 (1972) 441] from the experimental point of view as reprinted in this issue; Goldstein and Salganik [10 (1974) 507] also commented on the analytical aspects of non-straight cracks. Other amplifications related to angled-cracks in cylinders, were provided by Ewing and Williams [10 (1974) 537] and in flat plates by Cotterell and Rice [16 (1980) 155]. Aspects of numerical analysis were contributed by Lakshminarayana et al. [19 (1982) 257]. Portions of the angled crack problem expanded from the then current literature for mode III were treated by Simonson and Jones [6 (1970) 65] and later corrected by Smith [9

(1973) 181]. An important problem still remaining is to show any association between elastic analysis applied at the "kinking point", where the local conditions are no doubt highly inelastic, and the actual behavior in real material.

The general matter of running cracks has been one of continuing interest, but many analyses only apply to a crack tip moving at constant velocity. One exception is the reprinted paper by Kostrov [11 (1975) 47] in which the velocity may vary with time, but in general it seems that additional work is needed to settle the crack initiation conditions, both as to the critical load and the time dependent movement of the tip, especially in real materials. Another exception is the approximation by Rose [12 (1976) 829] used for an anti-plane deformation analysis. Shirrer and Pixa [this issue and 11 (1975) 1003] exhibited some good experimental techniques in velocity measurement in a photoelastic material using the Cranz-Schardin camera, as has Doll [12 (1976) 595] in a combined experimental analytical paper dealing with PMMA.

Other dynamic phenomena relate to shock waves and spallation, e.g. Tuler and Butcher [this issue and 4 (1968) 431], fracture created by dilatation waves authored by Achenbach and Nuismer [7 (1971) 77], and supersonic crack propagation by Winkler et al. in two papers [6 (1970) 151–158, 271–278]. An interesting subsonic crack propagation problem in which a rigid slender wedge penetrated an elastic body at a speed exceeding the Rayleigh velocity was given by Barenblatt and Goldstein [8 (1972) 427]. No open crack was found ahead of the wedge. From an overall standpoint, the most recent pertinent review article in the Journal was by Rose [12 (1976) 799]. At the low end of the rate scale, there have been many papers relating to the fatigue process although this subject, except for the two special review issues edited by H.W. Liu for non-metals [16 (1980) 481] and metals [17 (1981) 19], has not been emphasized in the International Journal of Fracture. Indeed, while dislocation models of the fracture process have also not been emphasized, this approach is quite popular, e.g. Weertman [this issue and 2 (1966) 460], and may be of special value in analyzing fatigue.

Papers from the Journal have also contributed to the literature over a wide variety of topical problems in fracture, as for example special finite elements in numerical work by Byskov [this issue and 6 (1970) 159] and later work, e.g. Holston [12 (1976) 887]; the Smith [this issue and 11 (1975) 39] comparison of various crack extension criteria such as those by Griffith, Barenblatt and Elliott; the generalized energy failure criterion by Atkinson and Eshelby [this issue and 4 (1968) 3]; mixed crack mode failure by Sih [10 (1974) 305]; viscoelastic fracture analysis, e.g. M.L. Williams [1 (1965) 292, 4 (1968) 69], Knauss [6 (1970) 7], J.G. Williams [8 (1972) 393], four papers by Schapery [11 (1975) 141–159, 369–388, 549–562, 14 (1978) 293], and Christensen [15 (1979) 3]. The probabilistic theory of fracture has not been neglected. It was described by Evans and Weiderhorn [this issue and 10 (1974) 379] with application to ceramics, experimental data from Bartenev [5 (1969) 179] on flawed and unflawed glasses, certain theoretical observations by Batdorf [this issue and 13 (1977) 5] and the series of papers by Phoenix, Harlow and Pitt [17 (1981) 347, 17 (1981) 631, 20 (1982) 291].

One of the more unusual subjects introduced to the western world through the Journal was the Zhurkov paper [1 (1965) 311] presented to the First International Congress of Fracture at Sendai. His paper, reprinted in this issue, concerned the connection between gross macroscopic applied stress and microscopic bond separation as deduced by electron spin resonance spectroscopy. It sparked many follow-up research extensions and qualifications to his basic premise, e.g. Kausch [6 (1970) 301] and De Vries [7 (1971) 197] and formed much of the basis of the special 60th Zhurkov Anniversary Issue [11 (1975) 721] prepared by his world-wide colleagues and published in the October 1975 issue.

As a personal observation, there seems to be a need for increased attention on the part of the fracture community to what might be called "non-standard" problem areas, experimental, analytical, and application. Some immediate candidates come to mind, along with illustrative contributions from some of our Journal's contributors. First, there are the materials and applications areas that have not normally been treated extensively in our papers, but might benefit from the synergism of related fields, e.g. rock mechanics, e.g. Hoek [this issue and 1 (1965) 137], concrete, e.g. Varder-Finnie [11 (1975) 495], and even medical applications of fracture – both elastic and viscoelastic – as in the cracking of human bones, teeth, and tearing in skin and muscle tissue. Second, three dimensional fracture analysis, in the sense of an elastic stress singularity equivalent to the inverse square root character of the stress in two dimensions, although, for example, the work of Cruse and Van Buren [7 (1971) 1], Yamamoto and Sumi [14 (1978) 17], and Folias [16 (1980) 335] is duly noted. Third, new approaches such as might be contained in statistical, inelastic and finite deformation theory may be needed in order to provide the next major breakthrough. Without attempting any preferential evaluation, the next ideas may follow from such examples as the nonlinear fracture mechanics work of Burns et al. [14 (1978) 311], non-local theory of elasticity applied to fracture as proposed by Eringen [14 (1978) 367], statistical analysis of thermally activated fracture by Petrov and Orlov [12 (1976) 231], finite elastic analysis using the fully non-linear equilibrium theory as advanced by Knowles [13 (1977) 611] or the addition of small deformations to a basic large deformation theory, e.g. Selvadurai [16 (1980) 327].

Unfortunately space does not permit elaboration upon other important contributions over the past 20 years, emphasizing the earlier ones as might be expected, while the later papers are becoming disseminated and digested. One item does, however, intrigue us very much. It is the 600 Reports on Current Research since 1971 and the impact they may have had upon setting trends and stimulating further investigations. This matter will at least be monitored in the future.

On behalf of the entire Board of Editors, I wish to offer my personal thanks to our contributors and reviewers over the past 20 years and to our publisher, from its first antecedent, Noordhoff Publishers, to its present successor, Martinus Nijhoff Publishers. In conjunction with the dedicated editorial and administrative assistance provided by our Associate Editor, Melba C. Williams, to whom we acknowledge a special note of appreciation, the technical editors wish to express their indebtedness for the high caliber of input from all the participators.

M.L. Williams

University of Pittsburgh
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AN AXIAL CRACK IN A PRESSURIZED CYLINDRICAL SHELL

E. S. Folias*

ABSTRACT

Following an earlier analysis of a line crack in a spherical cap, the stresses in a cylindrical shell containing an axial crack are presented. The inverse square root singular behavior of the stresses peculiar to crack problems is obtained in both the extensional and bending components. This singularity may be related to that found in an initially flat plate by

$$\frac{\sigma_{shell}}{\sigma_{plate}} \approx 1 + \left(a + b\,\ell n\frac{c}{\sqrt{Rh}}\right)\frac{c^2}{Rh} + \ldots$$

where the quantity in parentheses is positive. An approximate fracture criterion, based on Griffith's Theory, is also deduced, and bending-stretching interaction curves for this case are presented.

INTRODUCTION

In a recent paper[1], the stress fields in the vicinity of a line crack in a spherical cap were determined. It was pointed out that bending loads induce extensional stresses, and vice versa, so that the subject of eventual concern is the simultaneous stress fields produced in an initially curved sheet containing a crack. Of the two simple geometries which may first come to mind, a spherical shell, and a cyclindrical shell, the former was studied first because the radius of curvature is constant in all directions, affording considerable mathematical simplification. In the latter case, however, the radius varies between a constant and infinity as one considers different angular positions with respect to the point of a crack aligned parallel to the cylinder axis. In a previous treatment of this problem, Sechler and Williams[2] suggested an approximate equation based upon the behavior of a beam on an elastic foundation, and were able to obtain reasonable agreement with experimental results. Using techniques developed earlier, the author has been able to investigate this problem analytically, and the results are given below; certain details of this work have been omitted here but may be found elsewhere[3].

FORMULATION OF THE PROBLEM

Consider a portion of a thin, shallow cylindrical shell of constant thickness h and subjected to an internal pressure q_o. The material of the shell is assumed to be homogeneous and isotropic; parallel to the axis there exists a cut of length 2c. Following Marguerre[4], the coupled differential equations governing the displacement function W and the stress function F, with x and y as dimensionless rectangular coordinates of the base plane (see Figure 1) are given by

$$\frac{Ehc^2}{R}\frac{\partial^2 W}{\partial x^2} + \nabla^4 F = 0 \qquad (1)$$

$$\nabla^4 W - \frac{c^2}{RD}\frac{\partial^2 F}{\partial x^2} = \frac{q_o}{D}c^4 \qquad (2)$$

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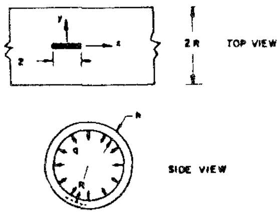

Figure 1. Geometry and Coordinates

where R is the radius of the cylinder. As to boundary conditions, one must require that the normal moment, equivalent vertical shear, and normal and tangential in-plane stresses vanish along the crack. However, suppose that one has already found* a particular solution satisfying eqns 1 and 2, but that there is a residual normal moment M_y, equivalent vertical shear V_y, normal in-plane stress N_y, and in-plane tangential stress N_{xy}, along the crack $|x| < 1$ of the form:

$$M_y^{(P)} = - D\, m_o / c^2 \tag{3}$$

$$V_y^{(P)} = 0 \tag{4}$$

$$N_y^{(P)} = - n_o / c^2 \tag{5}$$

$$N_{xy}^{(P)} = 0 \tag{6}$$

where m_o and n_o will be considered constants for simplicity.

Assuming, therefore, that a particular solution has been found, we need to find two functions of the dimensionless coordinates (x, y), $W(x, y)$ and $F(x, y)$, such that they satisfy the partial differential equations 1 and 2 and the following boundary conditions.

At $y = 0$ and $|x| < 1$:

$$M_y(x, o) = - \frac{D}{c^2}\left[\frac{\partial^2 W}{\partial y^2} + \upsilon \frac{\partial^2 W}{\partial x^2}\right] = \frac{D\, m_o}{c^2} \tag{7}$$

$$V_y(x, o) = - \frac{D}{c^3}\left[\frac{\partial^3 W}{\partial y^3} + (2 - \upsilon) \frac{\partial^3 W}{\partial x^2 \partial y}\right] = 0 \tag{8}$$

$$N_y(x, o) = \frac{1}{c^2} \frac{\partial^2 F}{\partial x^2} = \frac{n_o}{c^2} \tag{9}$$

$$N_{xy}(x, o) = - \frac{1}{c^2} \frac{\partial^2 F}{\partial x \partial y} = 0 \tag{10}$$

At $y = 0$ and $|x| > 1$ we must satisfy the continuity requirements, namely

* As an illustration of how the local solution may be combined in a particular case see Reference 3.

$$\lim_{|y|\to 0}\left[\frac{\partial^n}{\partial y^n}(W^+) - \frac{\partial^n}{\partial y^n}(W^-)\right] = 0 \tag{11}$$

$$\lim_{|y|\to 0}\left[\frac{\partial^n}{\partial y^n}(F^+) - \frac{\partial^n}{\partial y^n}(F^-)\right] = 0 \tag{12}$$

$n = 0, 1, 2, 3.$

Furthermore, we shall limit ourselves to large radii of curvature, i.e., small deviations from flat sheets; we thus require that the displacement function W and the stress function F together with their first derivatives be finite far from the crack. In this manner, we avoid infinite stresses and displacements in the region far away from the crack. These restrictions at infinity simplify the mathematical complexities of the problem considerably, and correspond to the usual expectations of St. Venant's principle.

METHOD OF SOLUTION

We construct the following integral representations which have the proper symmetrical behavior with respect to x, with $\lambda^4 = Ehc^4/R^2 D$

$$W(x, y^\pm) = \int_0^\infty \left\{P_1 e^{-\sqrt{s(s-\lambda\alpha)}|y|} + P_2 e^{-\sqrt{s(s+\lambda\alpha)}|y|} + P_3 e^{-\sqrt{s(s-\lambda\beta)}|y|} + P_4 e^{-\sqrt{s(s+\lambda\beta)}|y|}\right\} \cos xs \, ds \tag{13}$$

$$F(x, y^\pm) = -i\sqrt{EhD}\int_0^\infty \left\{P_1 e^{-\sqrt{s(s-\lambda\alpha)}|y|} + P_2 e^{-\sqrt{s(s+\lambda\alpha)}|y|} - P_3 e^{-\sqrt{s(s-\lambda\beta)}|y|} - P_4 e^{-\sqrt{s(s+\lambda\beta)}|y|}\right\} \cos xs \, ds \tag{14}$$

where the P_i $(i = 1, 2, 3, 4)$ are arbitrary functions of s to be determined from the boundary conditions, and the $\pm$ signs refer to $y > 0$ and $y < 0$, respectively. Also $\alpha = i^{\frac{1}{2}}$, $\beta = (-i)^{\frac{1}{2}}$.

Assuming that we can differentiate under the integral sign, formally substituting eqns 13 and 14 into eqns 7-10 yields respectively:

$$\lim_{|y|\to 0}\int_0^\infty -\left\{P_1 s(\nu_0 s - \lambda\alpha) e^{-\sqrt{s(s-\lambda\alpha)}|y|} + P_2 s(\nu_0 s + \lambda\alpha) e^{-\sqrt{s(s+\lambda\alpha)}|y|} + P_3 s(\nu_0 s - \lambda\beta) e^{-\sqrt{s(s-\lambda\beta)}|y|} + P_4 s(\nu_0 s + \lambda\beta) e^{-\sqrt{s(s+\lambda\beta)}|y|}\right\} \cos xs \, ds = m_0; \tag{15}$$

$|x| < 1$

$$\lim_{|y|\to 0} \pm\int_0^\infty \left\{P_1 s(\nu_0 s + \lambda\alpha)\sqrt{s(s-\lambda\alpha)}\, e^{-\sqrt{s(s-\lambda\alpha)}|y|} + P_2 s(\nu_0 s - \lambda\alpha)\sqrt{s(s+\lambda\alpha)}\, e^{-\sqrt{s(s+\lambda\alpha)}|y|} + P_3 s(\nu_0 s + \lambda\beta)\sqrt{s(s-\lambda\beta)}\, e^{-\sqrt{s(s-\lambda\beta)}|y|} + P_4 s(\nu_0 s - \lambda\beta)\sqrt{s(s+\lambda\beta)}\, e^{-\sqrt{s(s+\lambda\beta)}|y|}\right\} \cos xs \, ds = 0; \tag{16}$$

$|x| < 1$

$$\pm \lim_{\substack{|y|\to 0\\ |x|<1}} i\sqrt{EhD}\int_0^\infty \Big\{P_1 e^{-\sqrt{s(s-\lambda\alpha)}\,|y|} + P_2 e^{-\sqrt{s(s+\lambda\alpha)}\,|y|} - P_3 e^{-\sqrt{s(s-\lambda\beta)}\,|y|} - P_4 e^{-\sqrt{s(s+\lambda\beta)}\,|y|}\Big\}\, s^2 \cos xs\, ds = n_o; \tag{17}$$

$$\lim_{\substack{|y|\to 0\\ |x|<1}} i\sqrt{EhD}\int_0^\infty \Big\{P_1\sqrt{s(s-\lambda\alpha)}\, e^{-\sqrt{s(s-\lambda\alpha)}\,|y|} + P_2\sqrt{s(s+\lambda\alpha)}\, e^{-\sqrt{s(s+\lambda\alpha)}\,|y|} - P_3\sqrt{s(s-\lambda\beta)}\, e^{-\sqrt{s(s-\lambda\beta)}\,|y|} - P_4\sqrt{s(s+\lambda\beta)}\, e^{-\sqrt{s(s+\lambda\beta)}\,|y|}\Big\}\, s \sin xs\, ds = 0; \tag{18}$$

where again the $\pm$ signs refer to $y > 0$ and $y < 0$, respectively, and $\upsilon_o = 1-\upsilon$. A sufficient condition for 16 and 18 to be satisfied is to set the integrands equal to zero. This leads to:

$$\sqrt{s(s-\lambda\beta)}\,P_3 = -\left(\frac{\upsilon_o s}{\lambda\beta} - \frac{1}{2}\right)\left[\sqrt{s(s-\lambda\alpha)}\,P_1 + \sqrt{s(s+\lambda\alpha)}\,P_2\right] - \frac{\alpha}{2\beta}\left[\sqrt{s(s-\lambda\alpha)}\,P_1 - \sqrt{s(s+\lambda\alpha)}\,P_2\right] \tag{19}$$

$$\sqrt{s(s+\lambda\beta)}\,P_4 = \left(\frac{\upsilon_o s}{\lambda\beta} + \frac{1}{2}\right)\left[\sqrt{s(s-\lambda\alpha)}\,P_1 + \sqrt{s(s+\lambda\alpha)}\,P_2\right] + \frac{\alpha}{2\beta}\left[\sqrt{s(s-\lambda\alpha)}\,P_1 - \sqrt{s(s+\lambda\alpha)}\,P_2\right] \tag{20}$$

Next, it may easily be shown that the continuity conditions are satisfied if we consider the following combinations to vanish

$$\int_0^\infty \Big\{P_1\sqrt{s(s-\lambda\alpha)}\left(1+\frac{\lambda\alpha}{s}\right) + P_2\sqrt{s(s+\lambda\alpha)}\left(1-\frac{\lambda\alpha}{s}\right)\Big\}\cos xs\, ds = 0;\ |x|>1 \tag{21}$$

$$\int_0^\infty \Big\{P_1\sqrt{s(s-\lambda\alpha)}\left(1-\frac{\lambda\alpha}{s}\right) + P_2\sqrt{s(s+\lambda\alpha)}\left(1+\frac{\lambda\alpha}{s}\right)\Big\}\cos xs\, ds = 0;\ |x|>1 \tag{22}$$

We have thus reduced our problem to solving the dual integral equations 15, 17, 21, and 22 for the unknown functions $P_1(s)$ and $P_2(s)$. These may be transformed to a set of coupled singular integral equations of the Cauchy type, a solution of which may be found in a series form for small values of the parameter λ. Details of the method of solution may be found in Reference 3. It is an easy matter to show that the physical range of λ is $0 \le \lambda < 20$ and for most practical cases $0 \le \lambda < 2$, depending upon the size of the crack.

Without going into the details, the displacement and stress functions are:

$$W(x,y^{\pm}) = \int_0^{\infty}\Bigg\{\left(\frac{A_o}{s}+\frac{B_o}{\lambda\alpha}\right)\frac{J_1(s)}{2\sqrt{s(s-\lambda\alpha)}}\,e^{-\sqrt{s(s-\lambda\alpha)}\,|y|} + \left(\frac{A_o}{s}-\frac{B_o}{\lambda\alpha}\right)\frac{J_1(s)}{2\sqrt{s(s+\lambda\alpha)}}\,e^{-\sqrt{s(s+\lambda\alpha)}\,|y|} - \left[\left(\frac{\upsilon_o s}{\lambda\beta}-\frac{1}{2}\right)\frac{A_o}{s}+\frac{B_o}{2\lambda\beta}\right]\frac{J_1(s)}{\sqrt{s(s-\lambda\beta)}}\,e^{-\sqrt{s(s-\lambda\beta)}\,|y|} + \left[\left(\frac{\upsilon_o s}{\lambda\beta}+\frac{1}{2}\right)\frac{A_o}{s}+\frac{B_o}{2\lambda\beta}\right]\frac{J_1(s)}{\sqrt{s(s+\lambda\beta)}}\,e^{-\sqrt{s(s+\lambda\beta)}\,|y|} + \ldots\Bigg\}\cos xs\,ds \tag{23}$$

$$F(x,y^{\pm}) = -i\sqrt{EhD}\int_0^{\infty}\Bigg\{\left(\frac{A_o}{s}+\frac{B_o}{\lambda\alpha}\right)\frac{J_1(s)\,e^{-\sqrt{s(s-\lambda\alpha)}\,|y|}}{2\sqrt{s(s-\lambda\alpha)}} + \left(\frac{A_o}{s}-\frac{B_o}{\lambda\alpha}\right)\frac{J_1(s)\,e^{-\sqrt{s(s+\lambda\alpha)}\,|y|}}{2\sqrt{s(s+\lambda\alpha)}} + \left[\left(\frac{\upsilon_o s}{\lambda\beta}-\frac{1}{2}\right)\frac{A_o}{s}+\frac{B_o}{2\lambda\beta}\right]\frac{J_1(s)\,e^{-\sqrt{s(s-\lambda\beta)}\,|y|}}{\sqrt{s(s-\lambda\beta)}} - \left[\left(\frac{\upsilon_o s}{\lambda\beta}+\frac{1}{2}\right)\frac{A_o}{s}+\frac{B_o}{2\lambda\beta}\right]\frac{J_1(s)\,e^{-\sqrt{s(s+\lambda\beta)}\,|y|}}{\sqrt{s(s+\lambda\beta)}} + \ldots\Bigg\}\cos xs\,ds \tag{24}$$

where

$$A_o = -\frac{n_o}{\sqrt{EhD}}\,\frac{\lambda^2}{32\upsilon_o(4-\upsilon_o)}\left\{\frac{42-37\upsilon_o}{3}+(12-10\upsilon_o)\left(\gamma+\ln\frac{\lambda}{8}\right)\right\} - \frac{m_o}{\upsilon_o(4\,\upsilon_o)}\left\{1+\frac{12\upsilon_o-5\upsilon_o^2-8}{4\upsilon_o(4-\upsilon_o)}\cdot\frac{\pi\lambda^2}{16}\right\} + O(\lambda^4\ln\lambda) \tag{25}$$

$$B_o = \frac{n_o}{i\sqrt{EhD}}\Bigg\{1+\frac{\pi\lambda^2}{16}\left[\frac{5}{4}+\frac{12\upsilon_o-5\upsilon_o^2-8}{4\upsilon_o(4-\upsilon_o)}\right] + \frac{\lambda^2\beta^2}{32\upsilon_o(4-\upsilon_o)}\Bigg[\frac{7\upsilon_o^2}{3} + (10\upsilon_o^2-14\upsilon_o)+4\pi i+(10\upsilon_o^2-18\upsilon_o)\left(\gamma+\ln\frac{\lambda\alpha}{8}\right)+6\upsilon_o\left(\gamma+\ln\frac{\lambda\beta}{8}\right)\Bigg]\Bigg\} + \frac{m_o}{\upsilon_o(4-\upsilon_o)}\Bigg\{\upsilon_o+\upsilon_o\frac{\pi\lambda^2}{16}\left[\frac{5}{4}+\frac{12\upsilon_o-5\upsilon_o^2-8}{4\upsilon_o(4-\upsilon_o)}\right]+\frac{\lambda^2\alpha^2}{16}\Bigg[\frac{37\upsilon_o-42}{6} + 5\upsilon_o\left(\gamma+\ln\frac{\lambda\alpha}{8}\right)-6\left(\gamma+\ln\frac{\lambda}{8}\right)\Bigg]\Bigg\} + O(\lambda^4\ln\lambda) \tag{26}$$

having used $\gamma = 0.5768\ldots$ = Euler's constant. Furthermore, it may be shown, that this series form solution converges to the exact solution for small values of the parameter λ.

STRESS DISTRIBUTION NEAR THE CRACK POINT

The bending and extensional stress components are defined in terms of the displacement function W and stress function F as:

$$\sigma_{x_b} = -\frac{Ez}{(1-\upsilon^2)c}\left[\frac{\partial^2 W}{\partial x^2} + \upsilon\frac{\partial^2 W}{\partial y^2}\right] \tag{27}$$

$$\sigma_{y_b} = -\frac{Ez}{(1-\upsilon^2)c}\left[\frac{\partial^2 W}{\partial y^2} + \upsilon\frac{\partial^2 W}{\partial x^2}\right] \tag{28}$$

$$\tau_{xy_b} = \frac{2Gz}{c}\frac{\partial^2 W}{\partial x\,\partial y} \tag{29}$$

$$\sigma_{x_e} = \frac{1}{hc^2}\frac{\partial^2 F}{\partial y^2} \tag{30}$$

$$\sigma_{y_e} = \frac{1}{hc^2}\frac{\partial^2 F}{\partial x^2} \tag{31}$$

$$\tau_{xy_e} = -\frac{1}{hc^2}\frac{\partial^2 F}{\partial x\,\partial y} \tag{32}$$

where z is the dimensionless distance through the thickness h of the shell, measured from the middle surface. Then in view of eqns 23 and 24, the stresses can be expressed in an integral form. When evaluated, these give the following results, where $\epsilon e^{i\theta} = x - 1 + iy$:

Bending Stresses: On the surface $z = h/2c$

$$\sigma_{x_b} = \frac{P_b}{\sqrt{2\epsilon}}\left(-\frac{3-3\upsilon}{4}\cos\frac{\theta}{2} - \frac{1-\upsilon}{4}\cos 5\frac{\theta}{2}\right) + O(\epsilon^o) \tag{33}$$

$$\sigma_{y_b} = \frac{P_b}{\sqrt{2\epsilon}}\left(\frac{11+5\upsilon}{4}\cos\frac{\theta}{2} + \frac{1-\upsilon}{4}\cos 5\frac{\theta}{2}\right) + O(\epsilon^o) \tag{34}$$

$$\tau_{xy_b} = \frac{P_b}{\sqrt{2\epsilon}}\left(-\frac{7+\upsilon}{4}\sin\frac{\theta}{2} - \frac{1-\upsilon}{4}\sin 5\frac{\theta}{2}\right) + O(\epsilon^o) \tag{35}$$

where

$$P_b = \frac{3\,n_o\,\lambda^2}{16(3+\upsilon)\sqrt{12(1-\upsilon^2)}\,hc^2}\left[\frac{5+37\upsilon}{3} + 2(1+5\upsilon)(\gamma + \ln\lambda/8)\right]$$
$$+ \frac{6\,m_o\,D}{(3+\upsilon)h^2c^2}\left[1 - \frac{1+2\upsilon+5\upsilon^2}{4(3+\upsilon)(1-\upsilon)}\frac{\pi\lambda^2}{16}\right] + O(\lambda^4\ln\lambda) \tag{36}$$

Similarly we find through the thickness

Extensional Stresses:

$$\sigma_{x_e} = \frac{P_e}{\sqrt{2\epsilon}} \left(\frac{3}{4} \cos \frac{\theta}{2} + \frac{1}{4} \cos 5\frac{\theta}{2}\right) + O(\epsilon^{o}) \tag{37}$$

$$\sigma_{y_e} = \frac{P_e}{\sqrt{2\epsilon}} \left(\frac{5}{4} \cos \frac{\theta}{2} - \frac{1}{4} \cos 5\frac{\theta}{2}\right) + O(\epsilon^{o}) \tag{38}$$

$$\tau_{xy_e} = \frac{P_e}{\sqrt{2\epsilon}} \left(\frac{1}{4} \sin \frac{\theta}{2} - \frac{1}{4} \sin 5\frac{\theta}{2}\right) + O(\epsilon^{o}) \tag{39}$$

where

$$P_e = \frac{n_o}{hc^2}\left[1 + \frac{5\pi}{64}\lambda^2\right] + \frac{\sqrt{12(1-\upsilon^2)}\, m_o D \lambda^2}{32(3+\upsilon)(1-\upsilon) h^2 c^2}\left[\frac{5+37\upsilon}{3} + 2(1+5\upsilon)(\gamma + \ell n\, \lambda/8)\right] + O(\lambda^4 \ell n\, \lambda) \tag{40}$$

As a result of the Kirchhoff boundary condition, the bending shear stress τ_{xy_b} does not vanish in the free edge. For the flat sheet this difficulty was discussed by Knowles and Wang who considered Reissner bending(5). Furthermore, it is apparent from the above equations that there exists an interaction between bending and stretching, except that in the limit as $\lambda \to 0$ the stresses of a flat sheet are recovered and coincide with those obtained previously for bending(6) and extension(7). We are thus in a position to correlate, at least locally, flat sheet behavior with that of initially curved specimens.

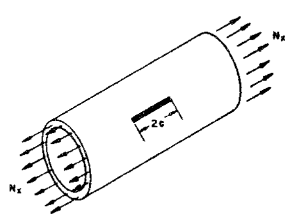

Figure 2. Cracked Shell under Uniform Axial Extension N_x and Internal Pressure q_o

As a practical matter, consider a shell subjected to a uniform internal pressure q_o with an axial extension $N_x = \frac{q_o R}{2}$, $M_y = 0$ far away from the crack (see Figure 2). The stresses along the line of crack prolongation are found for $\upsilon = 1/3$ and $\lambda = 0.98$ to be:

$$\sigma_{y_{total}}(\epsilon, o) \approx \frac{0.79}{\sqrt{\epsilon}}\, q_o\, R/h \tag{41}$$

$$\sigma_{x_{total}}(\epsilon, o) \approx \frac{0.97}{\sqrt{\epsilon}}\, q_o\; R/h \qquad (42)$$

where, based on the Kirchhoff theory, the stresses σ_x and σ_y have the same sign but differ in magnitude. This difference is due to the fact that in a cylindrical shell the curvature varies between zero and a constant as one considers different angular positions with respect to the line of the crack. On the other hand, for a spherical shell[2] and for a flat plate[6], for which the curvature remains constant in all directions, we obtain a "hydrostatic tension" stress field.

FRACTURE CRITERION

In precisely the same manner used for a fracture criterion for a spherical cap, we derive the following approximate criterion for a cylindrical panel, at $\upsilon = 1/3$ and the Griffith stress $\sigma^* = (16G\gamma^*/\pi c)^{\frac{1}{2}}$:

$$\begin{aligned}(1 + 0.49\,\lambda^2)\,(\bar{\sigma}_e/\sigma^*)^2 + 0.21\,(1 - 0.10\,\lambda^2)\,(\bar{\sigma}_b/\sigma^*)^2 \\ - (0.04 - 0.10\,\ell n\,\lambda)\,\lambda^2\,(\bar{\sigma}_e/\sigma^*)\,(\bar{\sigma}_b/\sigma^*) + O\,(\lambda^4\,\ell n\,\lambda) = 1\end{aligned} \qquad (43)$$

where the barred quantities denote applied stress. This equation represents a family of ellipses which are plotted in Figure 3. Note that the curves

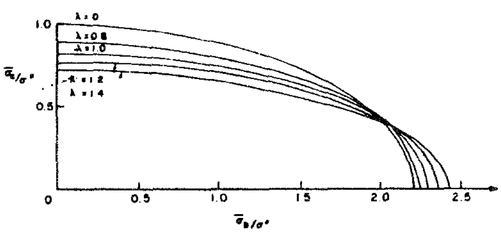

Figure 3. Extension-Bending Interaction Curves for a Cylindrical Shell Containing a Crack, for $\upsilon = 1/3$; $\lambda = \sqrt[4]{12(1-\upsilon^2)}\; c/\sqrt{Rh}$

cross each other, which did not occur in the case of a spherical shell (see Figure 4). The author conjectures that this is due to the slower rate of convergence of the former case and that, when higher orders of λ are used in the solution, the curves will correct themselves to give the same trend.

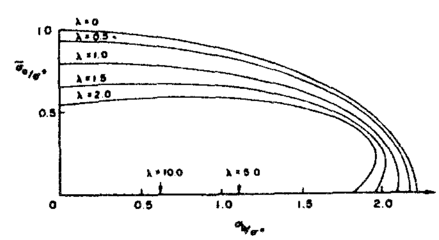

Figure 4. Extension-Bending Interaction Curves for a Shallow Spherical Shell Containing a Crack, for $\upsilon = 1/3$; $\lambda = \sqrt[4]{12(1-\upsilon^2)}\; c/\sqrt{Rh}$

For the special case $\bar{\sigma}_b = 0$ eqn 43 reduces to:

$$(1 + 0.49\,\lambda^2)\,(\bar{\sigma}_e/\sigma^*)^2 + O\,(\lambda^4\,\ell n\,\lambda) = 1 \qquad (44)$$

Specializing further to a cracked shell under uniform axial extension $N_x = q_0 R/2$ and internal pressure q_0

$$\left(\frac{q_0 R/h}{\sigma^*}\right)^2 \approx 1 - 0.49\,\lambda^2 \tag{45}$$

which gives the maximum internal pressure that the shell may withstand before fracture. A plot of eqn 45 is given in Figure 5. Similar results were obtained experimentally by Sechler and Williams[3] for pressurized monocoque cylinders.

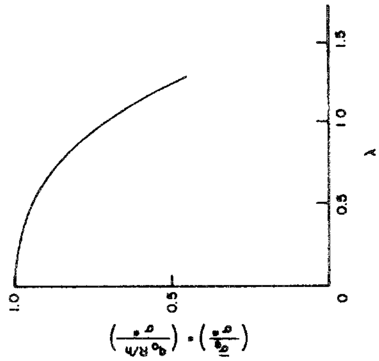

Figure 5. Critical (Fracture) Pressure in a Cylindrical Shell, for $\nu = 1/3$; $\lambda = \sqrt[4]{12(1-\nu^2)}\, c/\sqrt{Rh}$

CONCLUSIONS

As in the case of a spherical shell,
(i) the stresses are proportional to $1/\sqrt{\epsilon}$
(ii) the stresses have the same angular distribution as that of a flat plate
(iii) an interaction occurs between bending and stretching
(iv) the stress intensity factors are functions of R; in the limit as $R \to \infty$ we recover the flat plate expressions. Thus we may write

$$\frac{\sigma_{shell}}{\sigma_{plate}} \approx 1 + \left(a + b\,\ell n\,\frac{c}{\sqrt{Rh}}\right)\frac{c^2}{Rh} + O\left(\frac{1}{R^2}\right) \tag{46}$$

where the expression in parentheses is a positive quantity. From this and the corresponding result for a spherical cap, it would appear that the general effect of initial curvature is to increase the stress in the neighborhood of the crack point. It is also of some practical value to be able to correlate flat sheet behavior with that of initially curved specimens. In experimental work on brittle fracture for example, considerable effort might be saved since, by eqn 46, we would expect to predict the behavior of curved sheets from flat sheet tests.

In conclusion it must be emphasized that the classical bending theory has been used in deducing the foregoing results. Hence only the Kirchhoff shear condition is satisfied along the crack, and not the vanishing of both individual shearing stresses. While outside the local region the stress distribution should be accurate, one might expect the same type of discrepancy to exist near the crack joint as that found by Knowles and Wang in comparing Kirchhoff and Reissner bending results for the flat plate case. In this case the order of the stress singularity remained unchanged but the circumferential distribution around the crack changed so as to be precisely the same as that due to solely extensional loading. Pending further investigation of this effect for initially curved plates, one is tempted to conjecture that the bending amplitude and angular distribution would be the same as that of stretching.

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RÉSUMÉ - A la suite d'une analyse antérieure d'une fissure dans une coquille sphérique, on a présenté les contraintes dans une coque cylindrique contenent une fente axiale. Le comportement singulier de l'inverse de la racine carrée des contraintes particulières aux problèmes de rupture, est obtenu en même temps selon les composantes d'extension et de flexion. Cette singularité peut-être rapprochée de celle trouvée dans une plaque initialement plate, à l'aide de:

$$\frac{\sigma_{coque}}{\sigma_{plaque}} \approx 1 + (a + b\,\ln\frac{c}{\upsilon Rh})\,\frac{c^2}{Rh} + \ldots$$

où la quantité entre parenthèses est positive. On a déduit un critère de rupture, basé sur la theorie de Griffith, et on a présenté les courbes d'interaction entre flexion et tension.

ZUSAMMENFASSUNG - In Fortsetzung der frueheren Analyse eines geradlinigen Risses in einer kugelfoermigen Kappe werden jetzt die Spannungen in einer Zylinderschale, die einen Riss in axialer Richtung aufweist, gegeben. Die diesen Problemen eigene Singularitaet der Spannungen, die umgekehrt proportional der Wurzel aus der Entfernung von der Riss-spitze sind, wird auch hier fuer die Zug- und Biegekomponenten erhalten. Diese Singularitaet kann zu derjenigen, die in einer urspruenglich ebenen Platte gefunden wird, in folgenden Zusammenhang gebracht werden

$$\frac{\sigma_{Schale}}{\sigma_{Platte}} \approx 1 + (a + b\,\ln\frac{c}{\upsilon Rh})\,\frac{c^2}{Rh} + \ldots.$$

Wobei die Groesse in Klammern positive ist. Ein angenaehertes Bruchkriterium, das auf Griffith's Theorie beruht, ist abgeleitet worden und Biegungs-Zug Wechselwirkungskurven sind fuer diesen Fall bestimmt worden.