MATH 6720
Homework # 7
Due Date: 04/21/2017

1. Show that if

$$f(x) \sim a(x-x_0)^{-b} \quad \text{as} \quad x \to x_0^+,$$

then

$$\int_{x_0}^{x} f(t)dt \sim \frac{a}{1-b}(x-x_0)^{1-b} \quad \text{as } x \to x_0^+ \quad \text{if } b<1.$$

2. (a) Give an example of an asymptotic relation $f \sim g$ as $x \to \infty$ that cannot be exponentiated, i.e. $\exp(f(x)) \sim \exp(g(x))$ as $x \to \infty$ is false.
   (b) Show that if $f(x) - g(x) \ll 1$ as $x \to \infty$, then $\exp(f(x)) \sim \exp(g(x))$ as $x \to \infty$.

3. Find the leading behavior as $x \to 0^+$ of
   (a) $\int_0^1 e^{-x/t}dt$;
   (b) $\int_x^1 \cos(xt)dt$;
   (c) $\int_0^{1/x} e^{-t^2}dt$;
   (d) $\int_1^\infty \cos(xt)t^{-1}dt$.

4. Let $I(x) = \int_0^\infty e^{-t}/(1+xe^{t^2})dt$. Show that $I(x) - 1 \sim -\exp(\sqrt{-\ln x})$ as $x \to 0^+$.

5. Use Laplace's method to determine the leading behavior of
   (a) $\int_0^{\pi/2} \sqrt{t}e^{-x\sin^4 t}dt$ as $x \to \infty$;
   (b) $\int_0^1 \sqrt{\tan t}e^{-xt^2}dt$ as $x \to \infty$.

6. **Problem 10.3.1** Use Watson's lemma to obtain an asymptotic expansion of $E_1(x) = \int_x^\infty e^{-t}/tdt$. HINT: Show that $E_1(x) = e^{-x}\int_0^\infty e^{-xt}/(1+t)dt$.

7. **Problem 10.3.9** The modified Bessel function $I_n(x)$ has the integral representation

$$I_n(x) = \frac{1}{\pi}\int_0^\pi \exp(x\cos\theta)\cos(n\theta)d\theta.$$

Show that $I_n(x) \sim e^x/\sqrt{2\pi x}$.