MATH 6750
Homework # 2
Due Date: October 15, 2019 at the beginning of class.

1. When the wind blows over a chimney, vortices are shed into the wake. The frequency of vortex shedding f depends on the chimney diameter D, its length L, the wind velocity V and the kinematic viscosity of air ν. Express the nondimensional shedding frequency in terms of its dependence on the other nondimensional groups.

2. A cone and plate viscometer consists of a cone with a very small angle α which rotates above a flat surface. The torque required to spin the cone at a constant speed is a direct measure of the viscous resistance. The torque T is a function of the radius R, the cone angle α, the fluid viscosity μ, and the angular velocity ω.
   (a) Use dimensional analysis to express this information in terms of a functional dependence on nondimensional groups.
   (b) If α and R are kept constant, how will the torque change if both the viscosity and the angular velocity are doubled?

3. Two incompressible viscous fluids of the same density ρ flow, one on top of the other, down an inclined plane making an angle α with the horizontal. Their viscosities are μ_1 and μ_2, the lower fluid is of depth h_1 and the upper fluid is of depth h_2. Show that

$$u_1(y) = \left[(h_1 + h_2)y - \frac{1}{2}y^2\right] \frac{g \sin\alpha}{\nu_1}.$$

4. A viscous flow is generated in $r \geq a$ by a circular cylinder $r = a$ which rotates with constant angular velocity Ω. There is also a radial inflow which results from a uniform suction on the (porous) cylinder, so that $u_r = -U$ on $r = a$. Show that

$$u_r = -\frac{Ua}{r} \quad \text{for} \quad r \geq a,$$

   and that

$$r^2 \frac{d^2 u_\theta}{dr^2} + (\mathrm{Re} + 1) r \frac{du_\theta}{dr} + (\mathrm{Re} - 1) u_\theta = 0,$$

   where $\mathrm{Re} = Ua/\nu$. Show that if $\mathrm{Re} < 2$ there is just one solution of this equation which satisfies the no-slip condition on $r = a$ and has finite circulation $\Gamma = 2\pi r u_\theta$ at infinity, but that if $\mathrm{Re} > 2$ there are many such solutions.
   The circulation around a cylinder of radius a is $\Gamma = \oint \mathbf{u} \cdot d\mathbf{r} = \oint u_\theta a d\theta = 2\pi u_\theta a$.

5. Consider two parallel plates located at $y = \pm L$. Assume that the pressure gradient in the x-direction oscillates in time, i.e $\frac{\partial p}{\partial x} = P_x \cos(nt)$, where P_x is constant representing the magnitude of the pressure-gradient oscillations. Assuming no-slip and no-penetration boundary conditions and one dimensional flow, show that the solution to this unsteady problem is

$$u(y,t) = \mathrm{Re}\left(i\frac{P_x}{\rho n} \left[1 - \frac{\cosh\left((1+i)\sqrt{n/(2\nu)}y\right)}{\cosh\left((1+i)\sqrt{n/(2\nu)}L\right)} \right] e^{int} \right)$$

6. Show that the dispersion relation for waves on the interface between two fluids, the upper fluid being of density ρ_2 and the lower being of density ρ_1 with $\rho_1 > \rho_2$ is

$$c^2 = \frac{\omega^2}{k^2} = \frac{g}{|k|} \left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} \right).$$