The set of non-uniquely ergodic d-IETs has Hausdorff codimension 1/2 (with H. Masur) (Inventiones Mathematicae)

This paper shows that in every hyperelliptic stratum, the set of minimal and not uniquely ergodic IETs has codimension $\frac{1}{2}$. This establishes the Hausdorff d-dimension of not uniquely ergodic IETs has codimension $\frac{1}{2}$ for all $d \geq 4$. (For $d = 2, 3$ minimality implies unique ergodicity and the set of not uniquely ergodic d-IETs has codimension 1. The case of $d = 4$ was established by J. Athreya and me.) The upper bound was proven earlier by Masur. To prove the lower bound we construct a large family of paths in the relevant Rauzy diagrams. These paths are chosen so that every IET corresponding to such a path is not uniquely ergodic. Bounding the Hausdorff dimension of the set of IETs coming from these paths is technical, as it is the intersection of the unions of highly distorted simplices. (These simplices are the projective images of the simplex under matrices of Rauzy induction coming from our paths.)

A prime system with many self-joinings (with B. Kra) (Journal of Modern Dynamics)

We build an explicit example of a prime transformation with some novel properties: Its self-joinings form a Poulsen simplex. That is, the ergodic self-joinings are weak-* dense in the simplex of self-joinings. It is rank 1 but not quasi-simple (nor even quasi-distal) and thus can not arise as a factor of a simple system. As a side effect we show that there is a residual set of measure preserving transformations so that its elements are not quasi-simple (nor even quasi-distal) strengthening a result of Ageev and answering a question of Danilenko. The methods of this paper are two pronged: on the one hand we develop the methods of *Self-joinings for 3-IETs* to produce many and big joinings. On the other hand, we use different arguments to show that our system can not have factors. Both parts of this paper are technical.

Singularity of the spectrum for smooth area-preserving flows in genus two and translation surfaces well approximated by cylinders (with K. Frączek, A. Kanigowski and C. Ulcigrai) (Communications in Mathematical Physics)

This paper shows that for almost every genus two translation surface, any (finite area) smooth flow with logarithmic singularities that it gives rise to has singular spectrum. The main tool is a criterion of Frączek and Lemanczyk for singularity of spectral measures. We use the special geometry of our situation to verify this criterion.

Uniform distribution of saddle connection lengths (with D. Robertson) (Journal of Modern Dynamics)

This paper shows that almost every translation surface has the property that its saddle connection lengths are uniformly distributed modulo 1 sequence (when ordered by increasing length). The proof uses Nevo, Ruhr, Weiss' result establishing error bounds on the number of saddle connections of length at most R in almost every translation surface. The methods are perturbative and we do not identify a single surface with this property. The basic idea is that a small element of $SL(2, \mathbb{R})$ has a large effect on the length of saddle connections modulo 1. Morally, this is also the key idea in Hubert and my paper *Circle averages and disjointness in typical flat surfaces on every Teichmüeller disc.* In an appendix of the paper, D. El-Baz and B. Huang prove that every torus has that the sequence of the lengths of its periodic trajectories is uniformly distributed modulo 1.

Ergodicity of skew products over linearly recurrent IETs (with D. Robertson) (accepted Journal of the London Mathematical Society)

We show that for any linearly recurrent IET and almost every integral 0 step function, f, the skew product $T_f : [0,1)\times\mathbb{R} \to [0,1)\times\mathbb{R}$ by $T_f(x,s) = (Tx, s+f(x))$ is ergodic with respect to Lebesgue measure on $[0,1) \times \mathbb{R}$. An IET is linearly recurrent if there exists $c > 0$ so that the smallest interval of continuity of T^n has length at least $\frac{c}{n}$ for all n, or equivalently, that a translation surface that it arises from defines a Teichmüller geodesic ray which is precompact in the stratum. A step function is a linear combination of characteristic functions of intervals. Integral 0 step functions with a fixed number of discontinuities have a natural (Lebesgue) measure and almost every is with respect to this measure. The main argument is to gain extra invariance as in Ratner style arguments, for an ergodic measure by finding nearby points which diverge in a controlled way. In our case this is because these two points orbit on other sides of a discontinuity of f. A key tool is a quantitative version of Atkinson's theorem.

Self-joinings for 3-IETs (with A. Eskin) (accepted Journal of the European Mathematical Society)

This paper prove some results on the self-joinings of 3-IETs. First, it shows that typical 3-IETs are not simple, answering a question of Veech in the negative. That is, almost every 3-IET has an ergodic self-joining that is not 1-1 on almost every fiber. Second, we show that the self-joinings of 3-IETs form a Poulsen simplex. That is, the ergodic self-joinings are weak-* dense in the simplex of self-joinings. The construction of the self-joinings is novel and relies on the fact that 3-IETs admit good linked $(n_j, n_j + 1)$ approximation (in the language of Katok) have many ergodic self-joinings.

Stationary coalescing walks on the lattice (with A. Krishnan) (Probability Theory and Related Fields)

We consider translation invariant measures on families of nearest-neighbor semi-infinite walks on the integer lattice. We assume that once walks meet, they coalesce. In 2d, we classify the collective behavior of these walks under mild assumptions: they either coalesce almost surely or form bi-infinite trajectories. Bi-infinite trajectories form measure-preserving dynamical systems, have a common asymptotic direction in 2d, and possess other nice properties. We use our theory to classify the behavior of compatible families of semi-infinite geodesics in stationary first- and last-passage percolation. We also partially answer a question raised by C. Hoffman about the limiting empirical measure of weights seen by geodesics. We construct several examples: our main example is a standard first-passage percolation model where geodesics coalesce almost surely, but have no asymptotic direction or average weight. This last example was further developed by Bakhtin and Li.

A smooth mixing flow on a surface with non-degenerate fixed points (with A. Wright) (Journal of the American Mathematical Society)

We show that there exists a smooth, volume preserving mixing flow on a surface of genus 5 where all the fixed points are non degenerate, answering a question of Katok, Sinai and Stepin. By work of C. Ulcigrai, such a flow is atypical in the set of smooth, volume preserving flows with non-degenerate fixed points. Such flows are given by flows over IETs with (symetric) logarithmic singularities. The construction of the relevant IET is based on modifying a construction of Katok-Stepin/Veech for minimal and not uniquely ergodic $\mathbb{Z}/2\mathbb{Z}$ skew products of rotations. We build a $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$ skew product that *is* uniquely but where the equidistribution is very poor.

There exists an interval exchange with a non-ergodic generic measure (with H. Masur) (Journal of Modern Dynamics)

This paper constructs an interval exchange on 6 intervals that has a non ergodic preserved measure with a generic point. That is, there exists T a 6-IET with a preserved, not ergodic measure μ and a point x so that $\frac{1}{N}\sum_{i=0}^{N} \delta_{T^i x}$ converges to μ in the weak-* topology. The proof uses Rauzy induction to more or less explicitly describe the orbit of the point generic for the non-ergodic measure.

Logarithmic laws and unique ergodicity (with R. Treviño) (Journal of Modern Dynamics)

We relate different logarithm laws and unique ergodicity. Given a translation surface ω, let δ_t be the smallest hyperbolic length of any non-homotopically trivial closed curve on $g_t\omega$. Let δ_t^{flat} be the smallest flat length of any non-homotopically trivial closed curve on $g_t\omega$. Masur showed that for almost every ω,

$$\limsup_{t\to\infty} \frac{-\log(\delta_t)}{\log(t)} = \frac{1}{2} = \frac{-\log(\delta_t^{\text{flat}})}{\log(t)}.$$

We show that

$$\limsup_{t\to\infty} \frac{-\log(\delta_t)}{\log(t)} = \frac{1}{2} \tag{1}$$

does not imply that the vertical flow on ω is uniquely ergodic, but that

$$\limsup_{t\to\infty} \frac{-\log(\delta_t^{\text{flat}})}{\log(t)} = \frac{1}{2} \tag{2}$$

does imply that the vertical flow on ω is uniquely ergodic. To prove that (1) does not imply unique ergodicity, we use an example of a construction of a surface with minimal and not uniquely ergodic vertical flow due to Katok-Stepin/Veech-Masur-Smillie. We use results of Minsky and Rafi to control the smallest hyperbolic length of a simple closed curve at each point along the Teichmüller geodesic. To show that (2) implies unique ergodicity we use fast return times for IETs to find short simple closed curves (not just short saddle connections) and the technique of combining complexes to show that if $\limsup_{t\to\infty} \frac{-\log(\delta_t^{\text{flat}})}{\log(t)} = \frac{1}{2}$ then $\liminf_{t\to\infty} \frac{-\log(\delta_t^{\text{flat}})}{\log(t)} < \frac{1}{2}$. This lets us apply a criterion of Treviño for unique ergodicity.

Horocycle flow orbits and lattice surface characterizations (with K. Lindsey) (Ergodic Theory and Dynamical Systems)

We give new characterizations of lattice surfaces. We also establish that for any translation surface, ω, $\overline{h_s r_\theta \omega} = \overline{SL(2,\mathbb{R})\omega}$ for almost every θ. The main tool is an ergodic theorem for the action of upper triangular matrices on strata of translation surfaces, due to Eskin, Mirzakhani and Mohammadi.

Mobius disjointness for interval exchange transformations on three intervals (with A. Eskin) (Journal of Modern Dynamics)

We show that 3-IETs that satisfy a mild diophantine condition satisfy Sarnak's Möbius disjointness conjecture. That is, if T is such a 3-IET, for any continuous function f and x we have $\sum_{n=1}^{M} f(T^n x)\mu(n) = o(M)$, where μ is the Möbius function. Building on an approach of Katai/Bourgain, Sarnak and Ziegler we do this by showing that enough of the powers of T are pairwise disjoint. The diophantine condition is that a torus with 2 marked points related to the 3-IET is not divergent on average under the action of $\begin{pmatrix} e^t & 0 \\ 0 & e^{-t} \end{pmatrix}$. There are two appendices. One lightly modifies a note of Harper to establish that our result on disjointness of powers implies Möbius disjointness. The other shows a much easier result, that almost every 3-IET has that all of its distinct positive powers are pairwise disjoint, which also establishes the Möbius disjointness result. Note that the set of exceptions for our diophantine condition has Hausdorff codimension $\frac{1}{2}$ by work of Al-Saqban, Apisa, Erchenko, Khalil, Mirzadeh and Uyanik and Apisa and Masur.

On limit sets in PMF for Teichmüller geodesics (with H. Masur and M. Wolf) (Crelle's Journal)

We show that there exist minimal and not uniquely ergodic flows on flat surfaces whose associated Teichmüller geodesics have a unique limit point in $\mathcal{PMF}$. Others do not have a unique limit point. There exists non divergent Teichmüller geodesics whose limit sets are different. Also there are two divergent Teichmüller geodesics that have a shared point in their limit set. There exist ergodic (but not uniquely ergodic) vertical foliations whose limit set is not a point. Our examples come from minimal and not uniquely ergodic $\mathbb{Z}_2$ skew products of rotations introduced by W. Veech and developed by H. Masur and coauthors. C. Leinenger, A. Lenzhen and K. Rafi proved some similar results around the same time.

A dichotomy for the stability of arithmetic progressions (with M. Boshernitzan) (Proceedings of the American Mathematical Society)

We show that a Borel set in $[0, 1]$ contains 3-term arithmetic progressions under any homeomorphism iff it contains arbitrarily long arithmetic progressions under any homeomorphism.

The Hausdorff dimension of not uniquely ergodic 4-IETs is $\frac{5}{2}$ (with J. Athreya) (Geometry and Topology)

We proved that the Hausdorff dimension of not uniquely ergodic 4-IETs has Hausdorff dimension $\frac{5}{2}$. This also show that for almost every surface in $\mathcal{H}(2)$, the flow in a Hausdorff dimension $\frac{1}{2}$ is not uniquely ergodic. This was generalized by H. Masur and me.

The set of uniquely ergodic IETs is path connected (with S. Hensel) (Ergodic Theory and Dynamical Systems)

This paper shows that that the set of uniquely ergodic IETs on more than 3 intervals is path connected. (The sets of uniquely ergodic 2 and 3 IETs are not path connected.) The paths will use IETs corresponding to not arational foliations. We reduce the general case to showing that the set of uniquely ergodic 4-IETs is path connected. Given two uniquely ergodic IETs, the approach is to build sequence of paths, which have the property that all the IETs on them look more and more (effectively) minimal as we go further in the sequence. Much of the paper is devoted to showing contraction of the simplices of Rauzy induction about the points on these paths. This mechanism establishes that the limit of this sequence of paths is a path and that the points on it are uniquely ergodic.

Circle averages and disjointness in typical flat surfaces on every Teichmueller disc (Bulletin of the London Mathematical Society) (with P. Hubert)

We prove that for almost every surface increasing 'circles' about a fixed center equidistribute except possibly on a set of density 0. That is, for almost every ω and for any continuous function and point p on ω we have that there exists $A \subset \mathbb{R}$ with density 1 so that

$$\lim_{t \in A,\, t \to \infty} \frac{1}{2\pi} \int_0^{2\pi} f(F^t_{r_\theta \omega}(p))d\theta = \int f d\lambda^2.$$

This follows from showing that, for almost every translation surface, the product of the flows in a random pair of directions is typically uniquely ergodic. We show this fact by proving a disjointness result, which as a consequence shows that on almost every translation surface every isomorphism class of each flow has measure 0. Perhaps surprisingly, it is still open whether there is a surface that does not have this property.

Every flat surface is Birkhoff and Osceledets generic in almost every direction (Journal of Modern Dynamics) (with A. Eskin)

In this paper we show that for any flat surface ω, for almost every θ under geodesic flow $r_\theta \omega$ equidistributes with respect to the unique measure supported on the closure of $SL_2(\mathbb{R})\omega$ that is not supported on a lower dimensional submanifold. Moreover, we show that the Kontsevich-Zorich cocycle has the appropriate Lyapanov spectrum for a full measure set of directions. This strengthens several known results to hold on every flat surface instead of just a full measure set of flat surfaces. The key techniques come from homogeneous dynamics and recent breakthroughs of Eskin, Mirzakhani and Mohammadi.

Every transformation is disjoint from almost every IET (Annals of Mathematics)

In this paper we show that given any transformation of Lebesgue space, almost every IET is different from it. As a corollary we show that the product of almost every pair of IETs is uniquely ergodic and so every point is recurrent. A key step in the proof is that any sequence of density 1 contains a rigidity sequence for almost every IET, strengthening a result of Veech.

Appendix C of *Right-angled billiards and volumes of moduli spaces of quadratic differentials on* $\mathbb{C}P^1$ by J. Athreya, A. Eskin and A. Zorich (Annales ENS)

This appendix shows that for almost every point in an unstable manifold for the geodesic flow on the space of quadratic differentials the circle through that point equidistributes under geodesic flow. This enables a couple of the theorems of the paper to be asymptotics rather than weak asymptotics. The argument uses Avila-Resende's result that there is a spectral gap for $SL_2(\mathbb{R})$ acting on the space of quadratic differentials and Margulis' argument for using exponential mixing to prove the equidistribution of typical circles.

Diophantine properties of IETs and general systems: Quantitative proximality and connectivity (with M. Boshernitzan) (Invetiones Mathematicae)

This paper shows a variety results on IETs, systems of linear block growth and general systems. It shows that there are no topologically mixing 3-IETs. It shows that if μ is an ergodic measure for an IET T then $T^i(x) \in B(y, \frac{\epsilon}{i})$ for any $\epsilon > 0$ and $\mu \times \mu$ a.e. (x, y). This result is proper. It also proves the following ergodic theorem: Let $f : X \to [0, \infty]$ be μ-measurable and $T : X \to X$ be μ ergodic. if $\{s_i\} \subset \mathbb{R}_+$ is non-decreasing and there exists $c > 1$ such that $\frac{s_{2n}}{s_n} > c$ for all large enough n then $\liminf_{n\to\infty} s_n f(T^n x)$ is $\in \{0, \infty\}$ for μ a.e. x. In particular it takes finite positive values on a set of measure zero. This shows that for any y we have that $\liminf_{n\to\infty} nd(T^n x, y) \in \{0, \infty\}$ for almost every x. Using the Lebesgue Density Theorem, Cassels showed earlier that for any x this was true for almost every y.

Borel-Cantelli Sequences (with M. Boshernitzan) (Journal d'Analyse Mathematique)

We classify the sequences of points $\{x_i\}_{i=1}^{\infty} \subset [0, 1]$ such that for any monotone decreasing sequence of positive reals $\{r_i\}_{i=1}^{\infty}$ with $\sum_{i=1}^{\infty} r_i = \infty$ one has $\lambda(\bigcap_{n=1}^{\infty} \bigcup_{i=n}^{\infty} B(x_i, r_i)) = 1$. We extend this result to Ahlfors regular spaces.

Shrinking targets for IETs (Geometric and Functional Analysis)

In this paper we consider the following problem: Given a point $p \in [0, 1)$ and a monotone decreasing sequence of positive reals $\{r_i\}_{i=1}^{\infty}$, for typical $x \in [0, 1)$ and IET T will $T^i x \in B(y, r_i)$ infinitely often? The Borel-Cantelli Theorem says no if the sum of the radii converge. We show that if $\{r_i\}$ is non-increasing, $\sum_{i=1}^{\infty} r_i = \infty$ then for almost every IET, T and point x, we have $T^i x \in B(y, r_i)$ for infinitely many i. The full measure set of IETs depends on the sequence. For almost every IET there exists a monotone decreasing sequence $\{r_i\}_{i=1}^{\infty}$ where $\sum_{i=1}^{\infty} r_i = \infty$ and yet for almost every x we have $T^i x \in B(y, r_i)$ for only finitely many i. However, if we restrict our sequences of radii by requiring that ir_i is monotone decreasing then one full measure set of IETs has that $T^i x \in B(y, r_i)$ infinitely often for all such $\{r_i\}_{i=1}^{\infty}$. Other related results are considered. The next paper, joint work with D. Constantine, improves this result to consider the frequency of hits to the shrinking balls.

Quantitative shrinking targets for rotations, IETs and billiards in rational polygons (accepted Israel Journal of Mathematics) (with D. Constantine)

We consider how often the orbit of a point under an IET visits a shrinking ball about a fixed point. We show that under some assumptions it is roughly as often as one would expect.

Theorem 1. *For almost every IET T and any $\{r_i\}_{i=1}^{\infty}$ such that $\{ir_i\}$ is monotone decreasing and $\sum_{i=1}^{\infty} r_i = \infty$ we have*

$$\lim_{N\to\infty} \frac{\sum_{i=1}^{N} \chi_{B(p,a_i)}(T^i x)}{\sum_{i=1}^{N} 2a_i} \overset{a.e.x}{\to} 1.$$

We prove a variant of this theorem, in particular establishing an analogous result for any fixed rational billiard. The proof follows the outline of the strong law of large numbers, with quantitative unique ergodicity supplying the necessary independence estimates. Our quantitative unique ergodicty is established by proving a quantitative version of Boshernitzan's criterion for unique ergodicity. Results of

Athreya-Forni and others probably could be used to prove our necessary estimates, but the quantitative Boshernitzan's criterion interfaces better with our setting.

There exists a topologically mixing IET (Ergodic Theory and Dynamical Systems)

This paper shows that there exists an IET with the property that for any $\epsilon > 0$ there exists $N \in \mathbb{N}$ such that $n > N$ implies that for any x we have $T^n B(x, \epsilon)$ is ϵ dense. The next paper improves on this result. In joint work with M. Boshernitzan (*Diophantine properties...*) we showed it is false when $n = 3$.

Topological mixing for residual sets of IETs (with J. Fickenscher) (Communication in Mathematical Physics)

This paper shows that a residual set of IETs with a permutation in a non-degenerate Rauzy class on 4 or more intervals are topologically mixing. This implies that there are topologically mixing uniquely ergodic IETs. It then proves that some billiards in L-shaped polygons are topologically mixing. It uses combinatorial methods to prove the result in the Rauzy class of (4321). It then uses the structure of Rauzy classes to extend it to the general situation.

Winning games for bounded geodesics in moduli spaces of quadratic differentials (with Y. Cheung and H. Masur) (Journal of Modern Dynamics)

This paper shows that the set of bounded geodesics in Teichmüller space is a strong winning set for C. McMullen's variant of Schmidt's game, answering a question of McMullen. We show that the set of badly approximable interval exchanges is also strong winning. Call a direction θ in a Teichmüller disc q *bounded* if there is a compact set of Teichmüller space K_θ such that $g_t r_\theta q \in K_\theta$ for all t. We also show that in every Teichmüller disc the set of bounded directions is absolutely winning for McMulllen's variant of Schmidt's game. The first version of the paper on the arxiv has a simpler proof of the Teichmüller disc result (but does not prove the substantially harder result for Teichmüller space).

The distribution of gaps for saddle connection directions (with J. Athreya) (Geometric and Functional Analysis)

This paper shows a variety of result on the gaps between saddle connection directions for flat surfaces. Three highlights are that for almost every surface there exists a distribution of normalized gaps, this distribution decays quadratically towards 0 at 0 and a surface has no 'small gaps' iff it is a lattice surface. **Note:** There is a mistake in this paper. Lemmas 5.4, 5.5 and Corollary 5.6 are false.

The Gap Distribution for Saddle Connection Directions on the Golden $\mathcal{L}$ (with J. Athreya and S. Leliévre) (Contemp Math 631)

This paper computes the asymptotic distribution of normalized saddle connection gaps for the golden L. This was the first example of such a computation for a flat surface that is not a torus cover.

Skew products over rotations with exotic properties (Geometriae Dedicata)

Answering questions of W. Veech and M. Boshernitzan we show that there exists a $\mathbb{Z}/2\mathbb{Z}$ skew product of a badly approximable rotation that is minimal but not uniquely ergodic. We apply this to show that there is a $\mathbb{Z}$ skew product of a badly approximable rotation where the orbit of Lebesgue almost every point is dense but

Lebesgue measure is not ergodic. The key tool is to view our IET, T, as the limit of non-minimal transformation, T_i, and show the ergodic measures of T are weak-$*$ limits of the ergodic measures of T_i. We explicitly describe the ergodic measures of T_i in terms of the ergodic measures of T_{i-1}.

Omega recurrence in cocycles (with D. Ralston) (Ergodic Theory and Dynamical Systems)

Consider the skew product $\hat{R}_\alpha : [0,1) \times \mathbb{Z} \to [0,1) \times \mathbb{Z}$ by

$$\hat{R}_\alpha(x,i) = \big(x + \alpha - \lfloor x + \alpha \rfloor, i + \chi_{[0,\frac{1}{2})}(x) - \chi_{[\frac{1}{2},1)}(x)\big).$$

We show that for almost every α, x we have that $\hat{R}^i_\alpha(x,0) \in [0,1) \times \{0\}$ fairly often. We show that for some α and any x we have that $\hat{R}^i_\alpha(x,0) \in [0,1) \times \{0\}$ holds rarely.

Schrödinger operators defined by interval exchange transformations (with D. Damanik and H. Krüger) (Journal of Modern Dynamics)

Consider the dynamically defined Shrödinger operator $H_x : l^2(\mathbb{Z}) \to l^2(\mathbb{Z})$ by $H_x(\bar{u})[n] = u(n+1) + u(n-1) + f(T^n x)u(n)$ where T is an IET, f is continuous and $x \in [0,1)$. We show a variety results on the spectrum of this self-adjoint operator. In particular, we use Kotani theory to show the absence of absolutely continuous spectrum in a variety of cases.

Hausdorff dimension for ergodic measures of interval exchange transformations (Journal of Modern Dynamics)

Building on an example of M. Keane this paper shows that a minimal but not uniquely ergodic IET can have a singular ergodic measure that is carried by a Borel set of Hasudorff dimension 0.

Every transformation is disjoint from almost every non-classical exchange (with V. Gadre) (Geometriae Dedicata)

We show that if S is an invertible measure preserving transformation of Lebesgue space and π is a 'permutation' of non-classical exchanges with at least one preserved band then for almost every $\bar{L}$ we have S is different from $T_{\pi,\bar{L}}$.

Submitted

Tremors and horocycle dynamics on the moduli space of translation surfaces (with J. Smillie and B. Weiss)

We show that the horocycle flow on moduli spaces of translations surfaces exhibits a range of behaviors not present for unipotent flows in homogeneous dynamics. There exist points that are not generic for any measure. There exist orbit closures that are not the image of manifolds under immersion and in fact have fractional Hausdorff dimension. There exists uncountable families of properly nested orbit closures. There exist points generic for a measure, but not in the support of the measure. The main tool is a the 'tremor' a new deformation that can be performed on a translation surface whose horizontal flow is not uniquely ergodic. We show that results of Bainbridge, Smillie and Weiss imply that the tremor of a surface in an eigenform locus that has a minimal horizontal flow is generic for the "Lebesgue" measure on that locus. The set of such surfaces is dense and so by the Baire category theorem we obtain the genericity results. The orbit closure results, especially the Hausdorff dimension bound, are more involved.

Weakly mixing polygonal billiards (with G. Forni)

We prove there exists a billiard in a (necessarily irrational) polygon that is weakly mixing with respect to the Liouville measure on the unit tangent bundle of the polygon. We do this by proving that for every pair of finite type translation surface Q_1, Q_2, for almost every θ_1, θ_2 the product of the flow in direction θ_1 on Q_1 and θ_2 on Q_2 is ergodic with respect to the product of the two dimension areas on Q_1 and Q_2. This implies the previous theorem by a standard Baire category argument. To do this we show that for every $\alpha \neq 0$ the set of direction on a given translation surface whose flow has an eigenvalue of α is a zero set. The main tools of this paper are Forni's results on the behavior of the Hodge norm under Teichmüller geodesic flow and Eskin and my Oseledec genericity results. Along the way we prove a new large deviations result for Kontsevich-Zorich cocycle on every Teichmüller disc.

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On the Space of Ergodic Measures for the Horocycle Flow on Strata of Abelian Differentials (with O. Khalil and J. Smillie)

This paper studies for the horocycle flow $u_t := \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}$ on strata of genus two translation surfaces with one singularity of angle 6π. It shows that the set of ergodic, invariant probability measures is not closed in the set of invariant probability measures. As a consequence it shows that there are points that are not u_t-generic for any measure and that the analogue of useful results for unipotent flows in homogeneous spaces by Mozes-Shah and Dani-Margulis do not hold for the horocycle flow on strata of translation surfaces. The main result is proved using renomalization dynamics. Indeed we consider families of long periodic horocycles, that are $g_t := \begin{pmatrix} e^t & 0 \\ 0 & e^{-t} \end{pmatrix}$-pushes of deformations of a length one horocycle on a Teichmüller curve. Using fluctuations in the derivative cocycle for the g_t flow we find periodic horocycles in this family where the uniform measure is weak-* close to a proper convex combination of the Masur-Veech measure on the strata and the Haar measure on the Teichmüller curve.

Path-connectivity of the set of uniquely ergodic and cobounded foliations (with S. Hensel) (hopefully will be submitted by the end of May)

This paper proves that both the set of uniquely ergodic foliations and the set of cobounded foliations on an oriented surface of genus at least 5 are path connected and locally path connected. The starting point is Leininger and Schleimer's construction of paths of arational foliations. We modify this construction to produce paths of cobounded foliations, via an argument from metric diophantine approximation. Using contraction properties of splitting sequences on the Gromov boundary of the curve complex, we show that one can concatenate these paths to connect two arbitrary uniquely ergodic. In particular, the contraction properties give us continuity at the end points, the two arbitrary uniquely ergodic foliations. (Note that cobounded foliations, are uniquely ergodic, so this establishes the path connectivity of the set of cobounded foliations as well.) As in Leininger and Schleimer's work, we first treat punctured surfaces (in our case of genus at least 2) and then use a covering argument to treat closed surfaces (of genus at least 5).

Connectivity of the Gromov Boundary of the Free Factor Complex (with M. Bestvina and S. Hensel)

This paper shows that ∂FF_n, the Gromov boundary of the free factor complex, is path connected and locally path connected in high enough rank. It also shows that the free splitting and cyclic splitting complexes are one-ended. By a theorem of Bestvina and Reynolds, the boundary of the free factor complex is the arational trees in ∂CV_n, quotiented by the map that forgets the measure. Motivated by this, (and the previous paper of myself and Hensel) we start by building paths in copies of $\mathcal{PML}$ in ∂CV_n. At the intersection of two 'adjacent' copies of $\mathcal{PML}$ the points do not project to ∂FF_n. Using a technical argument we 'improve' our path by replacing it in a neighborhood of these bad points by a new chain of $\mathcal{PML}$s. In the limit we produce a path which on a cantor set does not arise from paths in copies of $\mathcal{PML}$ and on an open and dense set does arise from paths in copies of $\mathcal{PML}$. Note that this ignores a subtlety, because we only obtain paths in the projection to ∂FF_n and in particular we do not obtain them 'upstairs' in ∂CV_n.

The typical measure preserving transformation is not an interval exchange transformation (with D. Davis)

This paper shows that a dense G_δ subset of the space of measure preserving transformations with the weak topology are not isomorphic to any IET. The main tool is strengthening Katok's proof that every IET is partially rigid (which he used to show no IET is mixing) to produce a lacunary partial rigidity sequence

Zero Measure Spectrum for Multi-Frequency Schrödinger Operators (with D. Damanik, J. Fillman and P. Gohlke)

This paper shows that the spectrum of some dynamical Schrödinger operators is supported on a zero Lebesgue measure cantor set. These Schrödinger operators have the form $(Hf)(n) = f(n+1)+f(n-1)+V(T^n x)f(n)$ where T is almost every rotation of the 2-torus and V is a step function chosen from a so called "ample" set. To do this we apply a criterion of Damanik and Lenz, which shows that it is sufficient for these rotations of the 2-torus satisfy the so called "Boshernitzan condition." The main ingredient to prove this is a recent paper of Berthé, Steiner and Thuswaldner (or alternately Fogg and Noûs) that gives symbolic codings of linear block growth for almost every rotation of the 2-torus.

Preprints on the arxiv I probably wont publish

On the Hausdorff dimensions of a singular ergodic measure for some minimal interval exchange transformations

This paper proves a variety of sharper versions of theorems in *Hausdorff dimension for ergodic measures of interval exchange transformations.* Additionally, in the appendix it provides the construction of an IET with a weird diophantine property: There exists a minimal and not uniquely ergodic IET T with ergodic measures μ and ν such that

$$\limsup_{n\to\infty}\frac{-\log d(T^n x, y)}{\log n} = 1 \text{ for } \mu\times\nu \text{ a.e. } (x,y)$$

but

$$\limsup_{n\to\infty}\frac{-\log d(T^n x, y)}{\log n} \leq \frac{1}{2} \text{ for } \nu\times\mu \text{ a.e. } (x,y).$$

Homogeneous approximation for flows on translation surfaces

Motivated by a result of L. Marchese this paper shows that for any translation surface, $\{a_i\}$ non-increasing with divergent sum we have that $|F^t_\theta x - x| < a_i$ infinitely often for almost every x and θ. This result is not implied by Marchese's result because it applies to sequences that are just non-increasing (rather than ia_i being non-increasing) and it holds on every flat surface. It does not imply Marchese's result because it does not address the difference between arbitrary pairs of discontinuities. The key tool is Vorobets results on the growth of the number of cylinders on a translation surface with at least a definite area.

The densest sequence in the unit circle (with M. Boshernitzan)

We identify the densest sequence on the unit circle. Not surprisingly it is the most separated one as well.

[0,1] is not a Minimality Detector for $[0,1]^2$

This paper shows that there exists a non-minimal sequence in S^1, $(x_1, x_2, ...,)$, such that for any continuous function $f : S^1 \to [0,1]$ we have $(f(x_1), f(x_2), ...)$ is minimal.

Arithmetic progressions in regular cantor sets

We show that the middle $\frac{1}{N}$ Cantor set contains arithmetic progressions of length $\frac{N}{50 \log N}$.