Homework 5
Due Wednesday, Mar. 6th at 11 PM
Answers should be written in LaTeX.

1. Assume that X, Y and Z are connected topological spaces and let $p_0 \colon X \to Y$ and $p_1 \colon Y \to Z$ be maps and let $p = p_1 \circ p_0$. If p and p_1 are covering maps show that p_0 is a covering map. (**Hint:** Given a point $y \in Y$ find a connected neighborhood U of $p_1(z)$ that is evenly covered for both p and p_1. If V is the component of $p^{-1}(U)$ that contains y and W is a component of $p^{-1}(U)$ show that either $p_0(W)$ is disjoint from V or p_0 restricted to W is a homeomorphism to V. Make sure you show that p_0 is surjective.)

2. Let
$$p \colon (E, e_0) \to (B, b_0)$$
be a covering space with both E and B path connected and locally path connected. If the induced homomorphism
$$p_* \colon \pi_1(E, e_0) \to \pi_1(B, b_0)$$
is an isomorphism show that p is a homeomorphism. (**Hint:** Apply the final lifting lemma to the identity map from B to itself. Use the previous problem to show that the lifted map is a covering map and hence surjective.)

3. For $i = 0, 1$, let
$$p_i \colon (E_i, e_i) \to (B, b_0)$$
be covering maps and assume that
$$(p_0)_*(\pi_1(E_0, e_0)) = (p_1)_*(\pi_1(E_1, e_1)).$$
Show that there is a homeomorphism from (E_0, e_1) to (E_1, e_1).