Extra problems for Homework 1.

Let $f : \mathbb{C} \longrightarrow \mathbb{C}$ be a complex valued function. Recall that we have defined

$$\frac{\partial f}{\partial z} = f_z = \frac{1}{2}\left(\frac{\partial f}{\partial x} - \imath \frac{\partial f}{\partial y}\right)$$

and

$$\frac{\partial f}{\partial \overline{z}} = f_{\overline{z}} = \frac{1}{2}\left(\frac{\partial f}{\partial x} + \imath \frac{\partial f}{\partial y}\right).$$

Let $g : \mathbb{C} \longrightarrow \mathbb{C}$ be another complex valued function and define h to be the composition $h = f \circ g$.

1. Show that $h_z(w)$ and $h_{\overline{z}}(w)$ only depend on $g_z(w)$, $g_{\overline{z}}(w)$, $f_z(g(w))$ and $f_{\overline{z}}(g(w))$. (Hint: Why does a similar statement hold for the partial derivatives of h with respect to x and y?)

2. Show that:

   (a) $h_z = (f_z \circ g)g_z + (f_{\overline{z}} \circ g)\overline{g_{\overline{z}}}$

   (b) $h_{\overline{z}} = (f_z \circ g)g_{\overline{z}} + (f_{\overline{z}} \circ g)\overline{g_z}$

   (Hint: Use (1) to reduce the calculation to the special functions $f(z) = az + b\overline{z}$ and $g(z) = cz + d\overline{z}$.)