Midterm 1, Math 3210
September 16, 2015
You must write in complete sentences and justify all of your work.

1. (10 pts.) Use induction to prove that $8^n - 5^n$ is divisible by 3 for all $n \in \mathbb{N}$.

2. Let F be a field as defined in the book (and in the notes). Given $x, y, z \in F$ show that:

   (a) (10 pts.) If $x + z = y + z$ then $x = y$.

   (b) (5 pts.) $x \cdot 0 = 0$.

   In your proofs you can only use the properties of a field given in the notes. Make sure you clearly indicate which field properties you are using as you use them.

3. (10 pts.) Let $L = \{r \in \mathbb{Q} | r^3 < 2\}$. Show that L is a Dedekind cut.

Scratchwork