Math 6510 - Homework 3

Due in class on 10/7/14

1. Let M be compact and $f : M \to N$ a differentiable map that is transverse to a submanifold $X \subset N$. If $f_t : M \to N$ is a homotopy show that there exists and $\epsilon > 0$ such that f_t is transverse to X for all $t < \epsilon$.

2. Let M be an n-dimensional manifold embedded in $\mathbb{R}^m$. If $I = \{i_1, \dots, i_n\} \subset \{1, \dots, m\}$ define $\pi_I : \mathbb{R}^m \to \mathbb{R}^n$ by $\pi_I(x_1, \dots, x_m) = (x_{i_1}, \dots, x_{i_n})$. Show that for all $x \in M$ there exists an I and an neighborhood of U of x in M such that (U, π_I) is a chart.

3. Let M be an n-dimensional manifold embedded in $\mathbb{R}^m$. Show that for any $k \geq m - n$ there exists a k-dimensional hyperplane P such that $M \cap P$ is a non-empty smooth manifold of dimension of $n + k - m$. If $k < m - n$ show that there exists k-dimesnional hyperplane that is disjoint from M.

4. Let $f : M \to M$ be a smooth map and $x \in M$ a fixed point ($f(x) = x$). Show that if 1 is not an eigenvalue of $f_*(x)$ that x has a neighborhood U such that if $y \in U$ and $f(y) = y$ then $y = x$. (Hint: Look at the graph of f in $M \times M$ and show that it intersects the diagonal transversally at (x, x).)