**Math 6510 - Homework 4**

Due at 4 PM on 10/13/04

1. (a) The projective plane, $\mathbb{R}P^2$, is the space of lines through the origin in $\mathbb{R}^3$. There is a natural map $\pi : \mathbb{R}^3 \backslash \{0\} \longrightarrow \mathbb{R}P^2$. Show that $\mathbb{R}P^2$ has a differentiable structure such that $\pi$ is a submersion.

   (b) A function $f : \mathbb{R}^3 \longrightarrow \mathbb{R}$ is *homogeneous* if $f(\lambda x) = f(x)$ for all $\lambda \in \mathbb{R}$. For any homogeneous function there is unique function $\bar{f} : \mathbb{R}P^2 \longrightarrow \mathbb{R}$ with $f = \bar{f} \circ \pi$.
   Define
$$f(x, y, z) = \frac{x^2 + 2y^2}{x^2 + y^2 + z^2}.$$
   Show that $f$ is homogeneous and that the corresponding function $\bar{f}$ is a Morse function on $\mathbb{R}P^2$.

2. GP 1.7 #16,17,18; 1.8 #15