Math 2270 Homework, due Oct 21, 2015.

(1) Let U and V be vector spaces with bases $u_1, \cdots, u_n$ and $v_1, \cdots, v_m$ respectively. The product

$$U \times V = \{(u, v) \mid u \in U, v \in V\}$$

is a vector space under the operations $(u, v) + (u', v') = (u + u', v + v')$ and $r(u, v) = (ru, rv)$. Show that the vectors

$$(u_1, 0), \cdots, (u_n, 0), (0, v_1), \cdots, (0, v_m)$$

form a basis of $U \times V$. Deduce that

$$\dim U \times V = \dim U + \dim V$$

For example, $\dim K^m \times K^n = \dim K^{m+n} = m + n$.

(2) Let W be the subspace of $\mathbb{R}^3$ spanned by the vector $(1, 2, 3)$. Find an orthogonal basis for the orthogonal complement $W^\perp$.

(3) Consider the linear system

$$x - 2y - z = 0$$

$$x + y + z = 0$$

Write the system in the form $Ax = 0$ for a matrix A. What is the rank of A, what is the dimension of $Ker(A)$, and what is the dimension of the solution set? Explicitly find the solution set using the reduced echelon form.

(4) p.94 #10 (b)

(5) p.94 #11

(6) p.138 #1.