

Digital Cleaning of Old Paintings

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Problem:

- Many old paintings have become darkened or faded due to centuries of exposure to harmful chemicals and/or sunlight

Goal:

- To digitally recover original color of old paintings using inverse techniques



Problem Formulation:

- In 1995, Dr. Cortelazzo, Dr. Geremia and Dr. Mian of the University of Padova, Italy suggested that an appropriate model would follow this form...

$$X \Rightarrow \begin{bmatrix} D \end{bmatrix} \Rightarrow Y \Rightarrow \begin{bmatrix} V \end{bmatrix} \Rightarrow X_{\text{new}}$$

X is the original painting

Y is the dirty painting

X_{new} is the digitally cleaned painting

Different Approaches:

- Linear Model
- Quadratic Model
- “Bleeding” Pixel Model

Linear Model:

Forward model is:

$$\begin{pmatrix} X_r & X_g & X_b \end{pmatrix} \begin{pmatrix} C_{rr} & C_{rg} & C_{rb} \\ C_{gr} & C_{gg} & C_{gb} \\ C_{br} & C_{bg} & C_{bb} \end{pmatrix} = \begin{pmatrix} Y_r & Y_g & Y_b \end{pmatrix}$$

Backward model is:

$$X_{new} = Y * C^T (C * C^T)^{-1}$$

Quadratic Model:

Forward model is:

$$\begin{pmatrix} X_r & X_g & X_b & X_r^2 & X_g^2 & X_b^2 \end{pmatrix} \begin{pmatrix} C_{rr} & C_{rg} & C_{rb} \\ C_{gr} & C_{gg} & C_{gb} \\ C_{br} & C_{bg} & C_{bb} \\ C_{rr}^2 & C_{rg}^2 & C_{rb}^2 \\ C_{gr}^2 & C_{gg}^2 & C_{gb}^2 \\ C_{br}^2 & C_{bg}^2 & C_{bb}^2 \end{pmatrix} = \begin{pmatrix} Y_r & Y_g & Y_b \end{pmatrix}$$

Backward model is:

$$X_{new} = Y * C^T (C * C^T)^{-1}$$

“Bleeding” Pixel Model:

Forward model is:

$$\begin{pmatrix} X_r^c & X_g^c & X_b^c & X_r^u & X_g^u & X_b^u & \dots \end{pmatrix} \begin{pmatrix} C_{rr}^c & C_{rg}^c & C_{rb}^c \\ C_{gr}^c & C_{gg}^c & C_{gb}^c \\ C_{br}^c & C_{bg}^c & C_{bb}^c \\ C_{rr}^u & C_{rg}^u & C_{rb}^u \\ C_{gr}^u & C_{gg}^u & C_{gb}^u \\ C_{br}^u & C_{bg}^u & C_{bb}^u \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{pmatrix} = \begin{pmatrix} Y_r & Y_g & Y_b \end{pmatrix}$$

Backward model is:

$$X_{new} = Y * C^T (C * C^T)^{-1}$$

Results:

- Linear Model
- Quadratic Model
- “Bleeding” Pixel Model

Results From Linear Model:

Before Chemically Cleaned ... After Chemically Cleaned

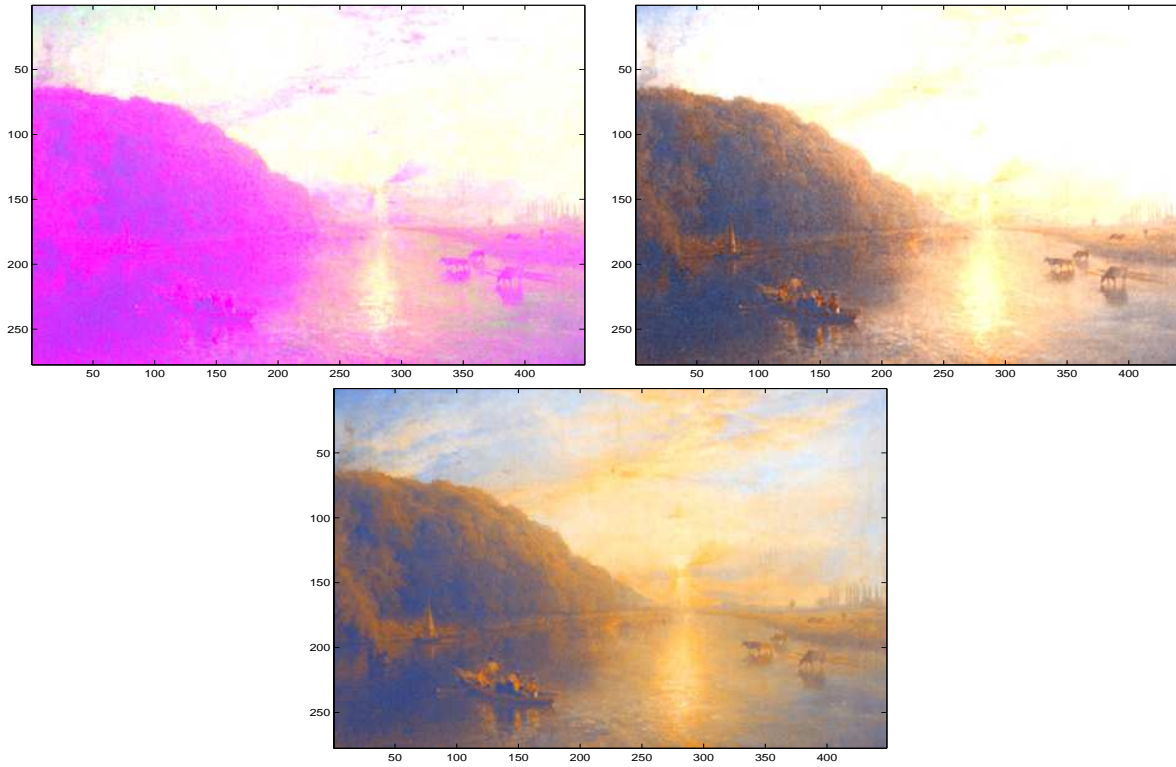


After Digitally Cleaned (Xnew)



Problems With Linear Model:

Sample Size...???



Results From Quadratic Model:

Before Chemically Cleaned ... After Chemically Cleaned

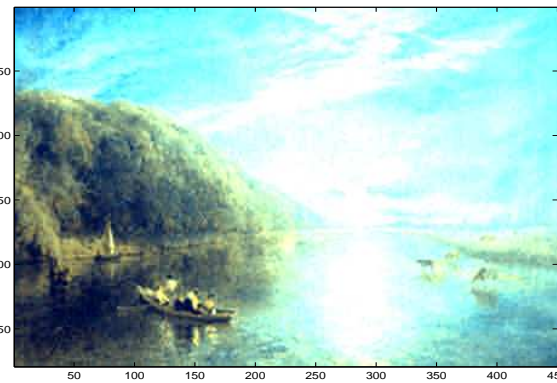
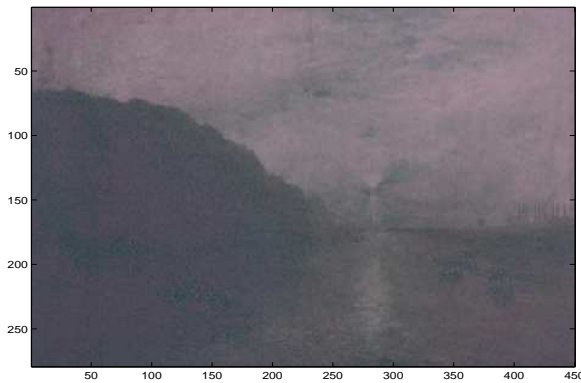


After Digitally Cleaned (Xnew)



Problems With Quadratic Model:

Sample Size...???...Again...



Results From “Bleeding” Pixel Model: NO SUCCESSFULL RESULTS

- Forward Model Was Successful, Indicating Correct “C” matrix values
- Condition Number of $C * C^T$ Was Ridiculous
- Taking R^1 and Attempting to Project into R^9 is a Bad Thing

Possible Future Questions/Objectives:

1. Find a better way to recover X_{new} values.
2. In particular, can the quadratic method produce better results.

