REU Evaluation
Jeremy Pecharich
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Over the summer we decided that the best way to spend the time would be to go over in detail foundations of CAT (0) spaces and Bridson's flat torus theorem. We believed this because it would give me a better foundation for proving theorems of my own. The main text that we used was Bridson entitled "Metric Spaces of Non-Positive Curvature." Reading through this text we also classified isometries CAT (0) spaces, read into detail about translation length and projections in CAT (0) spaces. We then looked again at Steve Gerseten's article for an example of an automorphism of F_3 that cannot act geometrically on any CAT (0) space. We were successful in generalizing his results to the following automorphisms of F_3,

$G = \left\langle a,b,c,t \mid tat^{-1} = a, tbt^{-1} = ba^m, tct^{-1} = ca^n \right\rangle$ where m=-1,0,1. We found if $n \neq m, m+1$ then these are automorphisms of F_3 that cannot act geometrically on any CAT (0) space. For the case when $n = m, m+1$ they act on CAT (0) square complexes. We have not yet been successful in our attempts with $m \neq -1,0,1$. We are now trying a new tactic to decide when these can act geometrically on a CAT (0) space. The idea is to look at the following group $H = \left\langle a,b,t \mid tat^{-1} = a, tbt^{-1} = ba^m \right\rangle$. This group, H, is a subgroup of G so if H cannot act geometrically on a CAT (0) space then G cannot either. This idea will lead us to investigating three-manifolds which are homeomorphic to a mapping torus of an automorphism of a torus with one boundary component. We also read an article by John Stallings to give us an easy way of detecting when certain automorphisms of a free group cannot arise from a compact two-manifold.

Next year I will be a graduate student at the University of Utah. I would like to continue my research in this area under the supervision of Ken Bromberg. The REU that I have done the past three semesters has given the opportunity to look at mathematics not in a classroom setting. This will help me in graduate school because it has shown me what being a mathematician is about.