Authentication of Musical Compositions with Techniques from Information Theory.

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Abstract

It is an oft-quoted fact that there is much in common between the fields of music and mathematics. This paper seeks to examine that commonality through the use of information theory techniques, specifically, entropy as defined by Shannon. The analysis will be presented in the form of an authentication problem with an entropy table being constructed and presented for a specific composer. Time allowing, zero, first, and second order "typical" models will be presented as well.

1. Introduction

One of the most fascinating aspects of mathematics is its ability to describe so many different models all with the same basic machinery or techniques. In this vein of thinking I sought to further explore the application of this machinery to models from fields that have not commonly or rhetorically been thought of as scientific–models from the fine arts.

Given my own personal background in ballet, the original choices for this analysis were actually going to be the works of a choreographer. The mathematical approach was to be a statistical analysis yielding a probability distribution for each step (in the sense of a dance step) used in the ballet, and that would allow a rudimentary authentication of the choreographer's work to be approached. Choreography was soon abandoned in favor of musical compositions. The reason for this was the lack of well-defined discrete steps in any given piece of choreography. Though a reasonable argument could be made that choreography, particularly in the classical genre, is indeed composed of a finite number of discrete steps, the real problem lies in the breakdown of the larger work into these discrete steps. Such a process would be largely subjective and would introduce a considerable margin of error into the analysis. Thus musical compositions were chosen because they are readily available as scores consisting of a finite number of clearly defined components–that is, music notes.

At the same time musical compositions were adopted to be the raw material for analysis, stronger, and more specific mathematical techniques were adopted as well. Entropy, as defined by Shannon in "The Mathematical Theory of Communication", has already been used to great effect in authentication problems where works of literature were the subject of examination. It became clear at that point that the task at hand was to understand the relevant machinery of information theory and entropy, and then to adapt the method as needed to work with musical compositions.

2. Entropy

The definition of entropy is

$$H(X) = -\sum_{x \in \chi} p(x) \log p(x) \tag{1.1}$$

where X is a discrete random variable taking on values from the set χ which depends on the information source being analyzed. For the purposes of this analysis, the elements of the set χ are all the musical notes used by a given composer in any of his compositions. If the analysis were to be one regarding the works of a famous literary author, the corresponding set would likely consist of all the words used by that author in his or her writings. The logarithm is base 2, which is important as it produces an entropy value with units of bits. This turns out to be a particularly useful because much of the work of information theory and entropy is relevant to the efficiency of coding schemes. There are many properties that arise from the manipulation of this definition that will not be treated here, but can be found in Cover and Thomas.

In the interest of adding an intuitive definition to the one above, we may think of entropy as the average number of yes or no questions needed to match a given X_i with an element from a set χ. This is a somewhat weak definition as it is important to note that the scheme for asking yes or no questions must be the most efficient possible scheme to provide $H(X)$ as the actual average number of questions needed. In many cases the best possible scheme may not be known. Regardless, it is a useful definition for intuitive purposes. A particularly illuminating example is Shannon's finding of a value of 1.34 bits for the entropy of letters in the English language. What this means, is that the bare minimum amount of information needed to identify each letter in a string of English text is 1.34 bits. Now this may seem counterintuitive because there are 26 letters in English and if we include the character "space" we have 27 symbols. It is quite obvious that any single symbol taken at random cannot definitely be identified by 2 yes or no questions, or 2 bits of information, much less 1.34 bits of information. Entropy though doesn't describe the uncertainty in a single independent symbol. It describes the uncertainty in the whole sample space in which the symbols occur, in this case, English. The entropy of English is as low as it is because the probability of a great many sequences of letters is zero. Likewise some letters are followed by all but one symbol with very low probability, "q" being a prime example as it is overwhelmingly followed by "u". That knowledge is due to the structure of English and so on average it takes less than 1.34 bits to identify the symbol following a "q". Also, after the first few letters of many words have been identified, the rest of the word can be guessed easily, again with much less information than 1.34 bits per symbol. As Cover and Thomas point out, "... a large number of guesses will usually be needed at the beginning of words or sentences." but that shouldn't be surprising considering those symbols are much more independent than are the symbols following them. Thus we may think of entropy as the average number of bits needed to identify a symbol that occurs with known probabilities and transition probabilities.

Back to the task of authentication, entropy is calculated with respect to a relevant table of probabilities, an entropy table. Continuing with the well-studied example of English, the corresponding entropy table is a table of probabilities of the occurrences of letters or strings of letters or words depending on the preference and rigor of the experimenter creating the table. With a complete table in hand, equation 1.1 can be computed directly to attain the entropy of English based on that given entropy table. Now that there is a known value for entropy, a sample of text of uncertain language can be analyzed. Using the same entropy table, the uncertain text will be processed, either from a sampling or completely depending on its size, and the two values for entropy shall be compared as a ratio as shown in equation 1.2 with output u being the uncertainty.

$$\frac{H_x(Y)}{H_x(X)} = u \tag{1.2}$$

A value of one would indicate a perfect match meaning the random variable Y representing elements of the sample in question required the same number of bits on average to match each Y_i with an element from the set χ as the random variable X representing the initial known sampling. A value higher or lower than one would indicate inconsistency in the entropies of the two variables and we would then need to examine the error margin associated with the sampling methods used to create the know sample space. If the error margin were large enough, that could account for a value other than one for two variables that did indeed have the same entropy with respect to the distribution of x. Thus finally, we will be able to say whether or not the entropy of the sample in question is the same as that of the known sampling within the associated margin of error.

3. The Musical Compositions

The first task regarding music was the selection of a composer. Some considerations had to be taken into account when identifying a composer with a plausible sampling of work. First and foremost, the composer must have a large number of works that are known to be their own. Without this stipulation, there would be no way to take a sampling large enough to discern the typical elements of that composer's work, thus an entropy from a small sampling would carry with it an uncertainty too large to be of much interest. Secondly, we would ideally find a composer who had at least one or two works which were contested regarding their authorship. Third and finally we would hope for a composer of piano music or solo pieces for other instruments. The reason for this is again to provide a stronger entropy measure from the sampling. As more instruments are added to a score, the number of elements in χ increase substantially. This is because when one instrument is being played, fewer notes can be played in any time step. As more instruments are added many more combinations of notes can be played at any given time step. The problem caused by this is that it is much more likely that no single set of notes played within a time step will be repeated. We could of course choose to analyze only one instrument, but it is much more likely that the stylistic elements that make a composer unique are not broken up neatly into their treatment of each instrument but rather can be found in the instruments interactions and the melodic structure. Under that assumption we could of course try to follow only the melody of a score, but there would be instances in which that could in fact be a tall order. A prime example of this would be canonical structure in which multiple instruments all play the melody but are displaced in their timing much like the singing of "Row Row Row Your Boat" in a round where each singer starts the song after the first measure has passed. Thus we would prefer, at least in our first analysis, to tackle a set of composition with a small number of instruments.

Consultation with a professor of music theory provided one such composer who fit the rather difficult requirement of being known for having several compositions wrongly attributed to him. The composer, Giovanni Battista Pergolesi, would thus have fulfilled our second requirement. However, Pergolesi died a young man and thus didn't leave nearly enough musical compositions to construct a sample space of decent size. Chopin became the next choice, and also the one whose compositions were finally chosen. Again, like Pergolesi, Chopin did not meet all three criteria, but he did meet the first and third, lacking only in that his authorship is fairly certain in the musical world.

Ninety-three pages of Chopin's Nocturnes for piano make up the initial sample space with which I am now working. I am furthermore analyzing only the right hand or treble clef of each stave considering it is the one that generally plays the melody. I chose to begin this way as a means of simplifying the number of note combinations that would need to be recorded. The scheme for choosing notes at random is to chose a page at random, then a stave from that page, a measure from that stave, and a note from that measure, also all at random. All of this is done with a table of random numbers produced by random.org. In this analysis a note can be either a single tone with a corresponding length of time, or a chord with several tones and time values. The important factor in determining a note is where it begins. For instance a chord consisting of three tones, two held for a beat and one for two beats would be recorded as such if all three tones commence on the same beat. If however a chord with three tones were to begin with one tone and then have the other two added on a subsequent beat, they would be considered two different notes because they commenced at two different times. This of course is not musical theory or even notational custom; I merely needed to choose a method for handling such instances to be used systematically throughout the analysis. As I mentioned it was not only the tone of notes that was recorded but also the length. Originally I had planned on recording whether each note was a whole, half, quarter, sixteenth note, et cetera. However, given that each of those notes would be held for different numbers of beats depending on the time signature of the music, I chose instead to record the number of beats each tone was held.

Carrying out the recording of these notes is now the task at hand with regard to this research. It is indeed a slow and tedious process to be done by hand and eye but already some notes have revealed themselves more regularly than others.

4. Current and Future Steps

Currently I am in the process of creating the entropy table from Chopin's Nocturnes. Until this step is completed there will be little other forward progress that can be made. After it's completion there are some interesting further analyses that could be made. The original intent of this research is to use entropy as a means to discern probable authorship of a musical work. The process could also be used to give a rough date for compositions that have not been properly dated. For such a project the method would be adapted to take advantage of the fact that many composers, particularly those that were long lived, went through stylistically distinctive musical phases in their careers. The task then would be to create an entropy table for each period and then compare the undated work with each table to find the closest match. Such and adaptation will not be attempted here due to time constraints, but it regardless, a decent example of another way in which these entropy techniques may be utilized.

References

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