Sample Path Properties of Anisotropic Gaussian Random Fields

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Abstract

The class of anisotropic Gaussian random fields includes fractional Brownian sheets, certain operator-self-similar Gaussian random fields with stationary increments and the random string process. They arise naturally in many areas and can serve as more realistic models than fractional Brownian motion.

We will discuss sample path properties of an (N, d)-fractional Brownian sheet $B^H = \{B^H(t),\ t \in \mathbb{R}^N_+\}$ with Hurst index $H = (H_1, \ldots, H_N) \in (0,1)^N$. Our interest lies in characterizing the anisotropic properties of B^H in terms of H. Many other anisotropic Gaussian random fields such as the random string processes share similar properties with B^H.

First we determine the Hausdorff dimension of the image set $B^H([0,1]^N)$, graph set $\mathrm{Gr}B^H([0,1]^N) = \{(t, B^H(t)) : t \in [0,1]^N\}$ and the level set $L_u = \{t \in (0,1]^N : B^H(t) = u\}$, where $u \in \mathbb{R}^d$.

Secondly, we prove that B^H has the property of *sectorial local non-determinism*. By using it as a technical tool, we prove that, if $\sum_{j=1}^N H_j^{-1} > d$, then B^H has a jointly continuous local time.

Prerequisites: Some basic knowledge on Hausdorff dimension and capacity.